Gravitational Solitons

This book gives a self-contained exposition of the theory of gravitational solitons and provides a comprehensive review of exact soliton solutions to Einstein's equations.

The text begins with a detailed discussion of the extension of the inverse scattering method to the theory of gravitation, starting with pure gravity and then extending it to the coupling of gravity with the electromagnetic field. There follows a systematic review of the gravitational soliton solutions based on their symmetries. These solutions include some of the most interesting in gravitational physics, such as those describing inhomogeneous cosmological models, cylindrical waves, the collision of exact gravity waves, and the Schwarzschild and Kerr black holes.

This work will equip the reader with the basic elements of the theory of gravitational solitons as well as with a systematic collection of nontrivial applications in different contexts of gravitational physics. It provides a valuable reference for researchers and graduate students in the fields of general relativity, string theory and cosmology, but will also be of interest to mathematical physicists in general.

VLADIMIR A. BELINSKI studied at the Landau Institute for Theoretical Physics, where he completed his doctorate and worked until 1990. Currently he is Research Supervisor by special appointment at the National Institute for Nuclear Physics, Rome, specializing in general relativity, cosmology and nonlinear physics. He is best known for two scientific results: firstly the proof that there is an infinite curvature singularity in the general solution of Einstein equations, and the discovery of the chaotic oscillatory structure of this singularity, known as the BKL singularity (1968–75 with I.M. Khalatnikov and E.M. Lifshitz), and secondly the formulation of the inverse scattering method in general relativity and the discovery of gravitational solitons (1977–82, with V.E. Zakharov).

ENRIC VERDAGUER received his PhD in physics from the Autonomous University of Barcelona in 1977, and has held a professorship at the University of Barclelona since 1993. He specializes in general relativity and quantum field theory in curved spacetimes, and pioneered the use of the Belinski–Zakharov inverse scattering method in different gravitational contexts, particularly in cosmology, discovering new physical properties in gravitational solitons. Since 1991 his main research interest has been the interaction of quantum fields with gravity. He has studied the consequences of this interaction in the collision of exact gravity waves, in the evolution of cosmic strings and in cosmology. More recently he has worked in the formulation and physical consequences of stochastic semi-classical gravity.

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Contents

Preface

Solitons are some remarkable solutions of certain nonlinear wave equations which behave in several ways like extended particles: they have a finite and localized energy, a characteristic velocity of propagation and a structural persistence which is maintained even when two solitons collide. Soliton waves propagating in a dispersive medium are the result of a balance between nonlinear effects and wave dispersion and therefore are only found in a very special class of nonlinear equations. Soliton waves were first found in some two-dimensional nonlinear differential equations in fluid dynamics such as the Korteweg–de Vries equation for shallow water waves. In the 1960s a method, known as the Inverse Scattering Method (ISM) was developed [111] to solve this equation in a systematic way and it was soon extended to other nonlinear equations such as the sine-Gordon or the nonlinear Schrödinger equations.

In the late 1970s the ISM was extended to general relativity to solve the Einstein equations in vacuum for spacetimes with metrics depending on two coordinates only or, more precisely, for spacetimes that admit an orthogonally transitive two-parameter group of isometries [23, 24, 206]. These metrics include quite different physical situations such as some cosmological, cylindrically symmetric, colliding plane waves, and stationary axisymmetric solutions. The ISM was also soon extended to solve the Einstein–Maxwell equations [4]. The ISM for the gravitational field is a solution-generating technique which allows us to generate new solutions given a background or seed solution. It turns out that the ISM in the gravitational context is closely related to other solution-generating techniques such as different Bäcklund transformations which were being developed at about the same time [135, 224]. However, one of the interesting features of the ISM is that it provides a practical and useful algorithm for direct and explicit computations of new solutions from old ones. These solutions are generally known as soliton solutions of the gravitational field or gravitational solitons for short, even though they share only some, or none, of the properties that solitons have in other nonlinear contexts.

Among the soliton solutions generated by the ISM are some of the most relevant in gravitational physics. Thus in the stationary axisymmetric case the Kerr and Schwarzschild black hole solutions and their generalizations are soliton solutions. In the 1980s there was some active work on exact cosmological models, in part as an attempt to find solutions that could represent a universe which evolved from a quite inhomogeneous stage to an isotropic and homogeneous universe with a background of gravitational radiation. In this period there was also renewed activity in the head-on collision of exact plane waves, since the resulting spacetimes had interesting physical and geometrical properties in connection with the formation of singularities or regular caustics by the nonlinear mutual focusing of the incident plane waves. Some of these solutions may also be of interest in the early universe and the ISM was of use in the generation of new colliding wave solutions. In the cylindrically symmetric context the ISM also produced some solutions representing pulse waves impinging on a solid cylinder and returning to infinity, which could be of interest to represent gravitational radiation around a straight cosmic string. Also some soliton solutions were found illustrating the gravitational analogue of the electromagnetic Faraday rotation, which is a typical nonlinear effect of gravity. Some of this work was reviewed in ref. [288].

In this book we give a comprehensive review of the ISM in gravitation and of the gravitational soliton solutions which have been generated in the different physical contexts. For the solutions we give their properties and possible physical significance, but concentrate mainly on those with possible physical interest, although we try to classify all of them. The ISM provides a natural starting point for their classification and allows us to connect in remarkable ways some well known solutions.

The book is divided into eight chapters. In chapter 1 we start with an overview of the ISM in nonlinear physics and discuss in particular the sine-Gordon equation, which will be of use later. We then go on to generalize and adapt the ISM in the gravitational context to solve the Einstein equations in vacuum when the spacetimes admit an orthogonally transitive two-parameter group of isometries. We describe in detail the procedure for obtaining gravitational soliton solutions. The ISM is generalized to solve vacuum Einstein equations in an arbitrary number of dimensions and the possibility of generating nonvacuum soliton solutions in four dimensions using the Kaluza–Klein ansatz is considered. In chapter 2 we study some general properties of the gravitational soliton solutions. The case of background solutions with a diagonal metric is discussed in detail. A section is devoted to the topological properties of gravitational solitons and we discuss how some features of the sine-Gordon solitons can be translated under some restrictions to the gravitational solitons. Some remarkable solutions such as the gravitational analogue of the sine-Gordon breather are studied.

Chapter 3 is devoted to the ISM for the Einstein–Maxwell equations under the same symmetry restrictions for the spacetime. The generalization of the

ISM in this context was accomplished by Alekseev. This extension is not a straightforward generalization of the previous vacuum technique; to some extent it requires a new approach to the problem. Here we follow Alekseev's approach but we adapt and translate it into the language of chapter 1. To illustrate the procedure the Einstein–Maxwell analogue of the gravitational breather is deduced and briefly described.

In chapters 4 and 5 we deal with gravitational soliton solutions in the cosmological context. This context has been largely explored by the ISM and a number of solutions, some new and some already known, are derived to generalize isotropic and homogeneous cosmologies. Most of the cosmological solutions have been generated from the spatially homogeneous but anisotropic Bianchi I background metrics. Soliton solutions which have a diagonal form can be generalized leading to new solutions and connecting others. Here we find pulse waves, cosolitons, composite universes, and in particular the collision of solitons on a cosmological background. The last of these is described and studied in some detail, and compared with the soliton waves of nonlinear physics. In chapter 5 soliton metrics that are not diagonal or in backgrounds different from Bianchi I are considered. Nondiagonal metrics are more difficult to characterize and study but they present the most clear nonlinear features of soliton physics such as the time delay when solitons interact. Solutions representing finite perturbations of isotropic cosmologies are also derived and studied.

In chapter 6 we describe gravitational solitons with cylindrical symmetry. Mathematically most of the gravitational solutions in this context are easily derived from the cosmological solution of the two previous chapters but, of course, they describe different physics. In chapter 7 we describe the connection of gravitational solitons with exact gravitational plane waves and the head-on collision of plane waves. We illustrate the physically more interesting properties of the spacetimes describing plane waves and the head-on collision of plane waves with some simple examples. The interaction region of the head-on collision of two exact plane waves has the symmetries which allow the application of the ISM. We show how most of the well known solutions representing colliding plane waves may be derived as gravitational solitons.

Chapter 8 is devoted to the stationary axisymmetric gravitational soliton solutions. Now the relevant metric field equations are elliptic rather than hyperbolic, but the ISM of chapter 1 is easily translated to this case. We describe in detail how the Schwarzschild and Kerr metrics, and their Kerr–NUT generalizations are simply obtained as gravitational solitons from a Minkowski background. The generalized soliton solutions of the Weyl class, which are related to diagonal metrics in the cosmological and cylindrical contexts, are obtained and their connection with some well known solutions is discussed. Finally the Tomimatsu–Sato solution is derived as a gravitational soliton solution obtained by a limiting procedure from the general soliton solution.

In our view only some of the earlier expectations of the application of the ISM in the gravitational context have been partially fulfilled. This technique has allowed the generation of some new and potentially relevant solutions and has provided us with a unified picture of many solutions as well as given us some new relations among them. The ISM has, however, been less successful in the characterization of the gravitational solitons as the soliton waves of nongravitational physics. It is true that in some restricted cases soliton solutions can be topologically characterized in a mathematical sense, but this characterization is then blurred in the physics of the gravitational spacetime the solutions describe. Things like the velocity of propagation, energy of the solitons, shape persistence and time shift after collision have been only partially characterized, and this has represented a clear obstruction in any attempt to the quantization of gravitational solitons. We feel that more work along these lines should lead to a better understanding of gravitational physics at the classical and, even possibly, the quantum levels.

As regards to the level of presentation of this book we believe that its contents should be accessible to any reader with a first introductory course in general relativity. Little beyond the formulation of Einstein equations and some elementary notions on differential geometry and on partial differential equations is required. The rudiments of the ISM are explained with a practical view towards its generalization to the gravitational field.

We would like to express our gratitude to the collaborators and colleagues who over the past years have contributed to this field and from whom we have greatly benefited. Among our collaborators we are specially grateful to G.A. Alekseev, B.J. Carr, J. Céspedes, A. Curir, M. Dorca, M. Francaviglia, X. Fustero, J. Garriga, J. Ibáñez, P.S. Letelier, R. Ruffini, and V.E. Zakharov. We are also very grateful to W.B. Bonnor, J. Centrella, S. Chandrasekhar, A. Feinstein, V. Ferrari, R.J. Gleiser, D. Kitchingham, M.A.H. MacCallum, J.A. Pullin, H. Sato, A. Shabat and G. Neugebauer for stimulating discussions or suggestions.

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