Electromagnetic Scintillation

II. Weak Scattering

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The first volume on electromagnetic scintillation exploited geometrical optics to describe propagation in random media. That method represents an approximate solution for Maxwell’s equations, which define the electromagnetic field. It was surprisingly successful in two important respects, even though it completely ignores diffraction effects.

Geometrical optics provides an accurate description for the signal-phase fluctuations imposed by a random medium. In this approximation, phase and range variations are caused by random speeding up and slowing down of the signal as it travels along the nominal ray trajectory. The phase variance estimated in this way is proportional to the distance traveled and to the first moment of the spectrum of refractive irregularities. It is therefore primarily sensitive to large eddies and diffraction effects can be safely ignored. This description is confirmed over an unusually wide range of wavelengths and propagation conditions.

The same technique was used to describe the phase difference measured between adjacent receivers. That result is needed in order to interpret observations made with microwave and optical interferometers. A similar expression characterizes the angular resolution of large telescopes in the limit of small separations. In this approach, angular errors are caused by random refractive bending of the rays as they travel through the random medium. The predicted resolution is proportional to the distance traveled and to the spectrum’s third moment. Since aperture averaging suppresses the influence of irregularities smaller than the receiver, angular accuracy depends primarily on the inertial range. Diffraction effects are relatively unimportant for this reason. The resulting description is confirmed over a wide range of wavelengths and propagation conditions.

It was hoped that the same approximation would provide an accurate description for the amplitude and intensity fluctuations. These quantities are usually measured on terrestrial links and during astronomical observations. In the context of geometrical optics, amplitude fluctuations are caused by random bunching and
spreading of the ray trajectories. The scintillation level should be proportional to the third power of the distance and to the spectrum’s fifth moment in this description. The smallest eddies are therefore crucial for such measurements. In establishing the basic framework for geometrical optics, however, we had to insist that all participating irregularities be large relative to the Fresnel length. That was necessary in order that diffraction effects could be ignored. It means that the inner scale length sets the following condition:

\[ \sqrt{\lambda R} < \ell_0 \] (1.1)

This limits the path length to distances less than 100 m for optical experiments and far smaller values for microwave signals.

We are thus confronted with the reality that geometrical optics cannot describe amplitude fluctuations under the circumstances which are usually encountered. That conclusion is confirmed when one compares the predictions of ray theory with a wide range of measurements. This makes clear the need for a description of scintillation that includes diffraction as an essential feature.\(^1\) Such an account is necessarily three-dimensional because the amplitude of a scattered electromagnetic wave is strongly influenced by interference effects in most situations.

Maxwell’s equations are the basis for all descriptions of electromagnetic propagation. We showed in Section 2.1 of Volume 1 that those relationships imply that the electric field must satisfy the following vector wave equation:

\[ \nabla^2 \mathbf{E} + k^2 (1 + \delta \varepsilon) \mathbf{E} = -4\pi i k \mathbf{j}(r) - \nabla [\mathbf{E} \cdot \nabla (\delta \varepsilon)] \] (1.2)

Changes in polarization induced by the random medium are characterized by the last term. We will demonstrate in Chapter 11 that depolarization effects are far below the measurement threshold for line-of-sight transmissions. This means that one can describe the propagation by considering the individual components of the electric-field vector, each of which satisfies a scalar differential equation:

\[ \nabla^2 E + k^2 [1 + \delta \varepsilon(r, t)] E = -4\pi i k j(r) \] (1.3)

Establishing solutions for this random-wave equation is our first challenge. We must solve this equation for specified current sources and a variety of statistical models for the dielectric variations. The same equation describes acoustic propagation in the presence fluctuations in the speed of sound and research effort devoted to that topic can be adapted to the electromagnetic problem.

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\(^1\) Astronomical telescopes and optical scintillometers often set a higher threshold for ignoring diffraction effects because their aperture size replaces \(\ell_0\) in the condition (1.1).
1.1 The Born Approximation

The first descriptions of weak scattering that included a full account of the diffraction phenomenon were based on the Born approximation. This technique had been developed for solving scattering problems in quantum mechanics. In its applications to electromagnetic scattering by random media, one exploits the smallness of the dielectric fluctuations relative to unity:

$$\delta \varepsilon_{\text{rms}} \ll 1$$  \hspace{1cm} (1.4)

One then expands the electric-field strength in a series of terms that are proportional to successively higher powers of $\delta \varepsilon$:

$$E = E_0 + E_1 + E_2 + E_3 + \cdots$$  \hspace{1cm} (1.5)

This approach has the advantage of providing a clear physical description of the process of scattering. The terms in this series are illustrated by the diagrams in Figure 1.1. The first term, $E_0$, represents the field strength that would be measured if the propagation medium contained no irregularities. The term $E_1$ represents single scattering of the incident plane wave by an irregularity at the point $r$, with the scattered component reaching the receiver as a spherical wave. The term $E_2$ represents double scattering of the transmitted wave and is illustrated by the third

![Figure 1.1: A diagrammatic description of plane-wave scattering by a random medium. The first panel represents the unperturbed plane wave. Subsequent panels portray typical single-, double- and triple-scattering sequences.](image-url)
The fourth term represents triple scattering and is suggested in the last panel of Figure 1.1. Differential equations that define these terms emerge when we substitute the series (1.5) into the random-wave equation (1.3) and group the results according to ascending powers of $\delta \varepsilon$.

The Born-approximation expression for the electric field is considerably more complicated than the corresponding geometrical optics description. Line integrals of the dielectric variation are replaced by volume integrations and the analytical challenge is considerably greater. With the spectrum method, however, we are able to separate the electromagnetic features of the problems from the description of turbulent irregularities. This approach also simplifies the propagation calculations and the measured quantities can be expressed as weighted averages of the wavenumber spectrum for most applications.

Despite its intuitive advantages, the Born approximation suffers from a serious limitation when it is applied to electromagnetic scattering by random media. One would like to be able to use the first few terms in the series to estimate the signal's phase and amplitude. For that to be possible, we must be sure that each term in the Born expansion is smaller than the preceding term. In Chapter 7 we show that the sum of the phase and amplitude variances must satisfy the following condition for the Born series (1.5) to converge rapidly:

$$\langle \varphi^2 \rangle + \langle \chi^2 \rangle < 1 \quad (1.6)$$

We can readily agree that the term $\langle \chi^2 \rangle$ satisfies this condition because we are considering weak scattering in this volume. On the other hand, the phase variance presents a major obstacle. In Section 4.1.5 of Volume 1 we found that measured phase variations are small only for microwave signals at frequencies below 5 GHz. Since the rms phase scales linearly with frequency, it is clear that the Born approximation cannot be used to describe millimeter-wave, infrared and optical signals. To make matters worse, the distribution of amplitude fluctuations predicted by the Born approach is contradicted both by microwave and by optical measurements.

These problems encouraged the search for a description of weak scattering that places a looser limit on phase fluctuations – as geometrical optics does. Fortunately, an approach adequate to this challenge was available for application to the electromagnetic problem.

### 1.2 The Rytov Approximation

A significant improvement on geometrical optics and the Born approximation was discovered in 1937 by Rytov, who was analyzing the diffraction of light by sound waves [1]. That analytical breakthrough was later applied by Obukhov to describe the propagation of electromagnetic waves in random media [2]. This technique
is now known alternatively as the method of smooth perturbations or the Rytov approximation. It is widely used to describe line-of-sight propagation in turbulent media when the amplitude variations are small [3][4].

The Rytov approximation is fundamentally an enlargement of geometrical optics that includes diffraction effects. The essence of this method is to express the field strength as the product of the unperturbed field and the exponential of a surrogate function, which must be determined. That function is similar to the eikonal of geometrical optics but represents a far more complete physical picture. It is a complex function that describes the important influence of diffraction because it is derived from the random-wave equation. To solve specific transmission problems one expands the surrogate function in powers of $\delta e$. The first term in this expansion is simply the single-scattering integral generated by the Born series. Most descriptions of propagation rely on this basic solution. Some estimates require the second-order solution and a few depend on higher-order terms. These solutions can be expressed as algebraic combinations of comparable terms from the Born series, which now appear as building blocks in the more powerful theory.

It was initially hoped that this approach would provide a description of multiple scattering and thus describe electromagnetic propagation for optical and infrared frequencies at large distances. Considerable debate regarding the applicability of the Rytov approximation ensued. One group maintained that it is no better than the Born method for describing transmission through random media. Others felt sure that it described a much wider class of problems. This debate was remarkable both for its intensity and for its duration. It took almost two decades to answer the following simple question: What are the limits of applicability for the Rytov method? Using qualitative arguments, Pisareva demonstrated that different restrictions are placed on the phase and amplitude of the propagating field [5]. She showed that the variance of logarithmic amplitude variations must be less than unity in all situations:

\[
\text{Rytov condition: } \left\langle \chi^2 \right\rangle < 1
\]

Notice that this is the same condition as that placed on the logarithmic amplitude by geometrical optics by Equation (3.45) in Volume 1. Tatarskii confirmed this with explicit calculations [3]. Pisareva also showed that the phase is unbounded for the usual case of Fresnel scattering. These conditions give us the flexibility required to characterize weak scattering.

In the hierarchy of propagation theories we are developing, the Rytov approximation represents a natural stopping point between geometrical optics and modern theories of strong fluctuations. It describes some features of multiple scattering, just as geometrical optics does. On the other hand, it can describe weak fluctuations
in amplitude and intensity – which geometrical optics cannot. We shall show that its results reduce to those of geometrical optics when the influential scatterers are concentrated near the ray path of the unperturbed field. Like Born theory, it captures the influence of diffraction phenomena. The log-normal distribution emerges as a natural consequence of the new method. An important difference with Born theory is that phase fluctuations appear naturally in the exponent of the Rytov solutions and can be very large for most applications. This amounts to a significant improvement over Born theory and means that one can treat both microwave and optical propagation in an even-handed way.

1.3 The Plan for this Volume

This volume emphasizes fluctuations in amplitude and intensity imposed on electromagnetic signals that propagate through random media. It does so because geometrical optics provides a valid description for phase and angle-of-arrival variations over a wide range of applications.

In the second chapter we develop the Rytov approximation as the method of choice for describing scintillation phenomena. We will rely primarily on the basic or first-order Rytov solution. That solution reduces to the geometrical-optics description when diffraction effects can be ignored. Moreover, it reduces to the Born approximation when the phase and amplitude variations are both small. One needs the second-order solution in some applications and two equivalent versions are established for it.

Expressions for the variance of fluctuations in amplitude and intensity are established in Chapter 3. The results for spherical and beam waves are developed for horizontal transmission paths. These predictions compare favorably with experimental data. Since amplitude fluctuations are determined primarily by small eddies, the dissipation region of the spectrum is important for interpreting optical measurements made on short paths. Aperture averaging is always a central consideration for terrestrial and astronomical measurements. This feature is readily included in our descriptions. Plane waves arriving from stellar sources are analyzed in order to understand the scintillation imposed on astronomical signals. The predicted scaling of scintillation level with zenith angle and telescope size agrees with astronomical observations. Source averaging can also influence scintillation levels and this coupling provides an important way to explore distant galaxies. Microwave signals from spacecraft and radio-astronomical sources are examined last, recognizing that the ionosphere often plays an influential role for frequencies below 10 GHz.

The correlation of amplitude fluctuations measured at adjacent receivers is addressed in Chapter 4. From the first observations of galactic radio sources, spatial correlations of radio-astronomical signals have provided important information
about the ionosphere. The spatial correlation depends only on the ratio of the inter-receiver separation and the Fresnel length. That general conclusion is applied to spherical and beam waves traveling close to the surface. Aperture-averaging and inner-scale corrections are sometimes important for such links. Diffraction of plane waves passing through the atmosphere generates shadow patterns at the surface. These shadow patterns are readily observed at the focal plane of a telescope and techniques for inverting them in order to reconstruct the profile of atmospheric irregularities are reviewed.

The time correlation and power spectrum provide equivalent descriptions of the rapid fluctuations in amplitude imposed by atmospheric irregularities. In Chapter 5 we emphasize the power spectrum because it can be combined more easily with other system characteristics to estimate performance in the presence of the time-varying scintillations. We use Taylor’s frozen-random-medium hypothesis to calculate the amplitude power spectrum for plane and spherical waves. Those predictions agree with atmospheric measurements in the weak-scattering regime.

The wavelength correlation for amplitude fluctuations is considered in Chapter 6. It should be the same for plane and spherical waves traveling along terrestrial paths when inner-scale and aperture-averaging effects can be ignored. This simple model agrees with microwave measurements. By contrast, the wavelength correlation of optical signals is confirmed only when the inner-scale region is properly modeled. Astronomical measurements of bichromatic correlations agree with a plane-wave model of atmospheric transmission. The possibility of inverting scintillation data recorded at different wavelengths is evaluated. This chapter also examines the frequency correlation of radio-astronomical and satellite sources, which are strongly influenced by the ionosphere at the VHF frequencies usually employed.

The discussion turns to phase fluctuations in Chapter 7, using the Rytov approximation to incorporate diffraction effects. The phase variance calculated in this way is little different than the geometrical-optics result. However, the Rytov approach is needed for estimating the cross correlation of phase and amplitude fluctuations. Diffraction expressions are also used to evaluate the phase structure function, angle-of-arrival errors and power spectrum of phase-difference measurements. The Rytov method suggests that the probability density function for phase variations is Gaussian, as has been suggested by geometrical optics and confirmed by experiments.

Double scattering of waves by refractive irregularities is important for developing a complete description of the field strength. The average value of the second order Rytov solution is required in order to estimate field-strength moments and is addressed in Chapter 8. This average can be evaluated exactly for spherical waves traveling along horizontal paths and for plane waves passing vertically through the atmosphere. These expressions include wide-angle scattering and therefore
describe microwave propagation with the same fidelity as that normally associated with optical expressions. An important relationship is established between these double-scattering averages and the combined variances of phase and amplitude. Beam waves traveling along terrestrial paths can be analyzed only by assuming that the individual scattering events are described with the paraxial approximation. The results of this chapter are primarily analytical and intended to provide modules needed later. The reader who is more interested in applications may elect to pass over this material.

The description of propagation in random media is significantly enlarged in Chapter 9, where various moments of the electric-field strength are derived. One needs both the first- and the second-order Rytov solution in order to generate accurate descriptions of these moments. The average field strength or mean field decays exponentially with distance for all three types of wave. By contrast, the mean irradiance is everywhere equal to its free-space value for plane and spherical waves. This is not true for beam waves, which are broadened by scattering in the random medium. Conservation of energy is demonstrated using both diagrammatic and analytical methods. The mutual coherence function provides a fundamental description of the electromagnetic field and is calculated for the three types of wave. These expressions agree generally with astronomical and laser-link measurements. Irradiance fluctuations are often measured with logarithmic amplifiers to compress the large dynamic range of scintillations encountered at optical and infrared wavelengths. The mean logarithmic irradiance and its variance are simply related to familiar path and signal parameters when the scattering is weak. These predictions are confirmed by optical experiments over a limited range. The predictions and measured quantities rapidly depart from one another above a certain level. The experimental data saturate as the path length or structure constant increases. This behavior represents the onset of multiple scattering and is addressed in Volume 3. It cannot be explained with the Rytov approximation and such experiments establish an experimental boundary for our approach to weak scattering.

The probability density function for amplitude fluctuations is discussed in Chapter 10. The basic Rytov solution predicts a log-normal distribution for short-term variations, which is confirmed over a wide range of frequencies and propagation conditions. The second order Rytov solution suggests that this distribution should be slightly skewed and that prediction too is confirmed. The bivariate distribution of rapid fluctuations measured with separated receivers or displaced times is examined. Intermittent atmospheric structures exert a strong influence on the measured distribution when the path length is short or the sample size small. The amplitude distribution measured over much longer time scales is important for predicting the performance of a communication system. Diurnal and seasonal variations of signal-level distributions are closely related to atmospheric conditions and
are best described by phenomenological models. Satellite-signal fluctuations are
examined separately and seem to fit a somewhat different pattern.

The polarization of electromagnetic waves is altered very slightly as they travel
through random media. This depolarization is explored in Chapter 11 and found to
be so small that it should not be measurable on line-of-sight paths. That prediction
is confirmed by experiments.

In Chapter 12 we return to the following important question: Under what circum-
stances is the Rytov approximation valid? The answer emerges when one compares
the first- and second-order terms in the Rytov series. This conclusion is confirmed
by the measurements summarized in Chapter 9.

An extensive review of refractive irregularities in the troposphere and ionosphere
was provided in Chapter 2 of Volume 1. That material is not repeated in this volume.
Instead, frequent references are made in the text and in footnotes to the relevant
figures and descriptions in Volume 1.

The appendices are a combination of new material and subjects presented at
the end of the first volume. The glossary is enlarged to include new symbols that
have been introduced in this volume. The previous appendices on probability dis-
tributions and Kummer functions in Volume 1 have been expanded to provide
additional results that are needed here. New appendices have been added to cover
Green’s function, cumulant analysis, diffraction integrals and Feynman formulas.

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