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Introduction

The laws of geometrical optics were known from experiments long before the electromagnetic theory of light was established [1]. Today we recognize that they constitute an approximate solution for Maxwell's field equations. This solution describes the propagation of light and radio waves in media that change gradually with position [2]. The wavelength is taken to be zero in this approximation and diffraction effects are completely ignored. The field is represented by signals that travel along ray paths connecting the transmitter and receiver. In most applications these rays can be approximated by straight lines. These trajectories are uniquely determined by the dielectric constant of the medium and by the antenna pattern of the transmitter. In this approach energy flows along these ray paths and the signal acts locally like a plane wave. Geometrical optics provides a convenient description for a wide class of propagation problems when certain conditions are met.

The assumption that the medium changes gradually means that geometrical optics cannot describe the scattering by objects of dimensions comparable to a wavelength. Similarly, it cannot describe the boundary region of the shadows cast by sharp edges. A further condition is that rays launched by the transmitter must not converge too sharply – as they do for focused beams. These conditions must be refined when ray theory is used to describe propagation in random media.

Geometrical optics is widely used to describe electromagnetic propagation in the nominal atmosphere of the earth, other planets and the interstellar medium. Refractive bending of starlight and microwave signals in the troposphere is accurately described by this approximation. Standard atmospheric-profile models are used to calculate ray paths, radio horizons and angles of arrival for various elevation angles and surface conditions [3]. Ray theory is also the primary tool for describing the reflection of radio signals in the ionosphere [4]. The maximum usable frequency can be estimated for shortwave broadcast and communication services if the electron-density profile of the ionosphere is known from vertical sounding measurements or modeling. The same techniques are used to describe the transmission of acoustic waves in the ocean.

It was initially thought that geometrical optics would provide a valid description for propagation through random media and early studies all relied on this approach [5][6][7][8]. The concept assumed that the signal fluctuations are induced by small dielectric variations located close to the nominal ray trajectory. It was hoped that perturbation solutions of the ray equations would yield valid expressions for phase and amplitude variations. Only the first half of that expectation was realized.

Geometrical optics provides a good description for the phase fluctuations imposed by random media. These are caused by the random *speeding up and slowing down* of the signal as it travels along the nominal ray trajectory. Phase fluctuations computed in this way agree with experiment, even when the path is long and the fluctuations are large. For line-of-sight propagation the predicted phase variance is proportional to the path length and the first moment of the spectrum of turbulent irregularities. This means that phase fluctuations depend primarily on the largest eddies and diffraction effects can be ignored.

Geometrical optics also describes angle-of-arrival fluctuations over a wide range of propagation conditions. Angular errors at the receiver are the result of many small *random refractive bendings* along the ray path. This sets the threshold for astronomical seeing with ground-based telescopes. The angular variance is proportional to distance traveled and to the third moment of the spectrum. Angular errors depend primarily on small eddies. As a practical matter, aperture smoothing suppresses the contributions of eddies smaller than the receiver and such measurements depend primarily on the inertial range of the turbulent spectrum. Again, diffraction effects are relatively unimportant.

By contrast, this method cannot describe amplitude and intensity fluctuations in most situations of practical interest. These scintillations are due to the random *bunching and diverging* of energy-bearing rays in this approximation. The resulting expression for the logarithmic variance of the amplitude is proportional to the third power of distance and the fifth moment of the spectrum. Intensity scintillation therefore depends primarily on the smallest eddies for which diffraction effects play a dominant role. To use this approximation the influential eddies must be larger than the Fresnel length. That condition is seldom met and one cannot use this method to define scintillation levels – unless large receivers and/or transmitters are employed. The geometrical optics description of amplitude fluctuations is primarily of historical interest and one is referred to standard texts for expressions for the variance and correlation [9][10]. Amplitude and intensity fluctuations will be analyzed with diffraction theory in the next volume.

The goal of the second chapter is to describe random media. In the following chapter we adapt geometrical optics to describe propagation through random media and to establish the validity conditions for its application. The single-path phase variance is estimated in Chapter 4. In the following chapter we calculate the phase structure function as a function of the separation between receivers and compare it with results from phase-difference experiments. The temporal correlation of phase and the corresponding power spectrum are addressed in Chapter 6. In the next chapter we describe the angle-of-arrival errors induced by a random medium. We show that the random phase and phase difference are distributed as Gaussian random variables in Chapter 8. Moments of the electric field strength calculated with geometrical optics are presented in the last chapter. Problems are included at the end of each chapter to develop additional insights and to explore related topics. Helpful mathematical relations are summarized in the appendices.

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Waves in Random Media

The first step in studying electromagnetic scintillation is to establish a firm physical foundation. This chapter attempts to do so for the entire work and it will not be repeated in subsequent volumes. We proceed cautiously because the issues are complex and the measured effects are often quite subtle. Section 2.1 explores the way in which Maxwell's equations for the electromagnetic field are modified when the dielectric constant experiences small changes. Because atmospheric fluctuations are much slower than the electromagnetic frequencies employed, their influence can be condensed into a single relationship: the *wave equation for random media.* This equation is the starting point for all developments in this field.

To proceed further one must characterize the dielectric fluctuations. We want to do so in ways that accurately reflect atmospheric conditions. Because we are dealing with a random medium, we must use statistical methods to describe them and their influence on electromagnetic signals. For instance, we want to know how dielectric fluctuations measured at a single point vary with time. Even more important, we need to describe the way in which fluctuations at separated points in the medium are correlated. There are several ways to do so and they are developed in Section 2.2. These descriptions assume that the random medium is isotropic and homogeneous. Those convenient assumptions are seldom realized in nature and we show how to remove them at the end of this section. Turbulence theory now gives an important but incomplete physi cal description of these fluctuations. Its results in the primary region of interest are expressed as a power-law scaling of the spectrum of turbulent irregularities. This approach depends on a few physical parameters, which must be found by experiment.

In Section 2.3 we describe direct measurements of these turbulence parameters in the troposphere. This region is nondispersive over broad frequency bands. Fluctuations of the refractive index are related to those of temperature and humidity. We present measurements both of surface values and of height profiles for three parameters: (a) the level of turbulence, (b) the inner scale length and (c) the outer scale length. We also examine what is known about anisotropy and how it changes with height. Scintillation experiments now provide the most accurate way to measure these parameters.

Electromagnetic propagation through the ionized layers above 100 km is quite different. Dielectric variations there depend on the electromagnetic frequency and on the electron density in the plasma created by solar radiation. This dispersion made early exploration of the ionosphere by using reflected radio signals possible. Microwave signals from artificial satellites and radio-astronomy sources now provide a more flexible and accurate way to probe the ionosphere. What we know about electron-density fluctuations in its elevated layers is summarized in Section 2.4. The picture is necessarily less complete than it is for the troposphere because we can seldom make direct measurements of the important parameters. *In situ* measurements of ion density are possible with scientific earth satellites and infrequent rocket flights. From microwave-transmission experiments we know that the irregularities are significantly elongated in the direction of the terrestrial magnetic field. The spectrum of electron-density fluctuations is described by a power law, although it may be different than the tropospheric form.

2.1 Maxwell's Equations in Random Media

Our first task is to establish the equations which describe electromagnetic propagation in a random medium. We start with Maxwell's equations for the various components of the electromagnetic field. The electric, magnetic, displacement and induction fields are governed by four vector equations:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.1}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J} \tag{2.2}$$

$$\nabla \cdot \mathbf{D} = 4\pi\rho_{\mathrm{e}} \tag{2.3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.4}$$

Here $\mathbf{J}$ is the current density and ρ_{e} is the net charge density. These equations are verified by careful laboratory experiments. In combination they describe the generation and propagation of electromagnetic waves. We have used mixed Gaussian units for notational efficiency but this choice will have no effect on the final result.

The divergence of the induction field **B** vanishes because there is no magnetic charge. By taking the divergence of (2.2) and using (2.3) one establishes the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J} + \frac{\partial \rho_{\mathrm{e}}}{\partial t} = 0. \tag{2.5}$$

In our applications, this balance relates to the current and charge on the transmitting antenna or laser source. Elsewhere in the transmission region, one can ignore these quantities and the divergence of the displacement field **D** also vanishes:

$$\boldsymbol{\nabla} \cdot \mathbf{D} = 0 \tag{2.6}$$

To this set of Maxwell's equations we must add the *constitutive equations* which characterize the propagation medium. These equations connect similar field components by employing atmospheric properties. The magnetic permeability relates the magnetic field to the induction field in a linear manner:

$$\mathbf{B} = \mu_{\mathrm{m}} \mathbf{H} \tag{2.7}$$

In our units $\mu_{\mathrm{m}} = 1$ in the earth's atmosphere so that **B** and **H** are virtually the same vector. The more important relation for our work connects the electric and displacement fields:

$$\mathbf{D} = \varepsilon(\mathbf{r}, t)\mathbf{E} \tag{2.8}$$

The dielectric constant $\varepsilon(\mathbf{r}, t)$ contains all the information we need to describe the propagation of electromagnetic waves in random media and therefore all of our attention will be focused on its consequences. It is customary to decompose this quantity into its average value and a small component that is a stochastic function of position and time:

$$\varepsilon(\mathbf{r}, t) = \varepsilon_0(\mathbf{r}) + \Delta\varepsilon(\mathbf{r}, t) \tag{2.9}$$

The average value ε_0 can be a function of position and it is therefore important to retain this slowly varying term in describing short-wave signals reflected by the ionosphere. In the lower atmosphere ε_0 is different than unity by less than 300 parts per million [1]. We concentrate here on the fluctuating component $\Delta\varepsilon(\mathbf{r}, t)$ which gives rise to electromagnetic scintillation.

To establish the wave features of electromagnetic propagation, one must combine Maxwell's equations. We apply the curl operator to the first equation and use the second to express $\boldsymbol{\nabla} \times \mathbf{H}$ in terms of the current density and displacement vectors:

$$\begin{aligned}\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} &= -\frac{1}{c} \boldsymbol{\nabla} \times \frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{c} \frac{\partial}{\partial t} (\boldsymbol{\nabla} \times \mathbf{H}) \\ &= -\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J} \right) \\ &= -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} (\varepsilon \mathbf{E}) - \frac{4\pi}{c} \frac{\partial \mathbf{J}}{\partial t} \end{aligned} \tag{2.10}$$

The double curl operation can be simplified since the following relation holds for any vector:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} = -\nabla^2 \mathbf{E} + \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{E})$$

We use the second constitutive relation to relate the divergence of $\mathbf{E}$ to the gradient of the dielectric constant:

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \boldsymbol{\nabla} \cdot (\varepsilon \mathbf{E}) = \mathbf{E} \cdot \boldsymbol{\nabla} \varepsilon + \varepsilon \, \boldsymbol{\nabla} \cdot \mathbf{E} = 0$$

so that

$$\boldsymbol{\nabla} \cdot \mathbf{E} = -\mathbf{E} \cdot \boldsymbol{\nabla}(\log \varepsilon).$$

By combining these results we establish a general equation for the electric field vector:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} [(1 + \Delta\varepsilon)\mathbf{E}] = \frac{4\pi}{c} \frac{\partial \mathbf{J}}{\partial t} - \boldsymbol{\nabla} \{ \mathbf{E} \cdot \boldsymbol{\nabla} [\log(1 + \Delta\varepsilon)] \} \tag{2.11}$$

The last term describes polarization changes induced by scattering in the random medium. In Volume 2 we shall find that this depolarization is negligible for atmospheric propagation and this term will be carried no further.

The electric field is generated by the transmitter and modified by $\Delta\varepsilon$ as it travels through the medium. The current density and dielectric variations are functions of position and time. On the other hand, their characteristics are quite different. For a microwave system, the current density is confined to the transmitting antenna and oscillates very rapidly at microwave frequencies. By contrast, the dielectric fluctuations pervade the entire region but change very slowly with time. We can exploit this profound difference to separate their effects.

We turn first to the current density $\mathbf{J}(\mathbf{r},t)$. Its variation with time can be a complicated function describing pulse transmissions or modulation formats used to carry information. It can also be a narrow-band signal centered on a single carrier frequency. If the dielectric fluctuations have no time dependence the wave equation (2.11) represents a linear relationship between $\mathbf{E}$ and $\mathbf{J}$. Both functions could then be Fourier analyzed and their spectra related algebraically. It is possible to do so even when $\Delta\varepsilon$ depends on time – as it does in the atmosphere. This occurs because dielectric fluctuations vary quite slowly relative to the field. In fact, their frequency components are trivial compared with the microwave or optical frequency of the transmitted field. In addition, they are usually small with respect to modulation components of the field. The frequency mixing that occurs in the term $\Delta\varepsilon\,\mathbf{E}$ is therefore not important. This means that we can consider a single frequency both for the source current and for the electric field:

$$\mathbf{J}(\mathbf{r},t)=\mathbf{J}(\mathbf{r})\exp(-i\omega t) \qquad \text{and} \qquad \mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})\exp(-i\omega t) \tag{2.12}$$

With this assumption we find that the wave equation depends primarily on the carrier frequency $\omega=2\pi f$:

$$\nabla^2\mathbf{E}(\mathbf{r})-\mathbf{E}(\mathbf{r})\,\frac{1}{c^2}\exp(i\omega t)\frac{\partial^2}{\partial t^2}\{[1+\Delta\varepsilon(\mathbf{r},t)]\exp(-i\omega t)\}=-\frac{4\pi i\omega}{c}\mathbf{J}(\mathbf{r})$$

The effects of the random medium are concentrated in the second term. We are concerned with both the spatial and the temporal fluctuations of $\Delta\varepsilon(\mathbf{r},t)$. Let us focus first on its variability with time and write the second derivative as follows:

$$\frac{1}{c^2}e^{i\omega t}\,\frac{\partial^2}{\partial t^2}\Big\{[1+\Delta\varepsilon(\mathbf{r},t)]e^{-i\omega t}\Big\}=k^2[1+\Delta\varepsilon(\mathbf{r},t)]$$
$$-\frac{2ik}{c}\,\frac{\partial}{\partial t}\Delta\varepsilon(\mathbf{r},t)+\frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\Delta\varepsilon(\mathbf{r},t)$$

where $k=2\pi/\lambda$ is the electromagnetic wavenumber. We must estimate each term that is proportional to $\Delta\varepsilon$. We suspect that the first term is the most influential, but we must demonstrate this by estimating the others.

The first derivative of $\Delta\varepsilon$ with respect to time can be estimated with the following qualitative argument. We shall learn later that the fluctuations are induced both by internal rearrangements of the turbulent structure and by its carriage on prevailing winds. These changes are most rapid when a horizontal wind bears the structure past a measuring point, because the prevailing wind speed is considerably greater than the turbulent velocities which it induces. The first derivative is then related to the average speed of the irregularities and the eddy of size ℓ by the following approximate relationship:

$$\frac{\partial}{\partial t}\Delta\varepsilon(\mathbf{r},t) \simeq \frac{v}{\ell}\Delta\varepsilon$$

To this we must add the Doppler shift of the moving irregularities:

$$\frac{\partial}{\partial t}\Delta\varepsilon(\mathbf{r},t) \simeq v\,\Delta\varepsilon\left(\frac{1}{\ell}+\frac{1}{\lambda}\right)$$

We want to compare this expression with the first term involving $\Delta\varepsilon$:

$$k^2\,\Delta\varepsilon \qquad \text{versus} \qquad \Delta\varepsilon\,\frac{2vk}{c}\left(\frac{1}{\ell}+\frac{1}{\lambda}\right)$$

We note that $k^2\,\Delta\varepsilon$ is substantially larger than the term on the right-hand side because the wind speed is trivial relative to the speed of light. We are therefore justified in dropping the first time derivative of $\Delta\varepsilon$. Similar reasoning shows that the second derivative is even smaller [2] and we can write the final wave equation as

$$\nabla^2\mathbf{E}(\mathbf{r}) + k^2[1+\Delta\varepsilon(\mathbf{r},t)]\mathbf{E}(\mathbf{r}) = -4\pi ik\mathbf{J}(\mathbf{r}). \tag{2.13}$$

The vector components of the electric field are not mixed in this equation. If the source current is aligned in the x direction, only the x component of the field is excited. This means that we can drop the vector notation. Each of the components of $\mathbf{E}$ which is excited by the source must satisfy the following scalar wave equation:

$$\nabla^2 E(\mathbf{r}) + k^2[1+\Delta\varepsilon(\mathbf{r},t)]E(\mathbf{r}) = -4\pi ikJ(\mathbf{r}) \tag{2.14}$$

This wave equation for random media will be the starting point for all predictions developed in these volumes. One cannot solve this equation exactly because $\Delta\varepsilon(\mathbf{r},t)$ is a stochastic function of position and time. Our challenge is to find approximate solutions that agree with experimental results.

We shall employ a succession of more capable solutions for the random wave equation. This hierarchy of increasingly sophisticated solutions has a common thread, i.e., they all depend on integrals of $\Delta\varepsilon$. In this first volume we will use the techniques of geometrical optics to express the measured quantities as line integrals of $\Delta\varepsilon$ taken along the nominal ray paths. In the second volume we use the Rytov approximation which expresses the measured quantities as volume integrals of $\Delta\varepsilon$. We describe strong scattering in the last volume using the method of path integrals in which the field strength is represented by functional integrals of $\Delta\varepsilon$. This increasingly capable and complex program means that we shall need to know a good deal about the