This book provides an answer to the question ‘what does the finance and economics literature say about the determination and estimation of a project’s cost of capital?’ Uniquely, it reviews both the theory of asset pricing in discrete time and a range of more applied topics that relate to project valuation, including the effects of corporate and personal taxes, the international dimension, estimation of the cost of equity in practice, and the cost of capital for regulated utilities. It seeks to explain models and arguments in a way that does justice to the reasoning whilst minimising the prior knowledge of finance and maths expected of the reader. It acts as a bridge between a general undergraduate or MBA text in finance, accounting or economics and the modern theoretical literature on the cost of capital.

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The Cost of Capital

Intermediate Theory

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Preface

All companies and investment projects need capital (money tied up in the business), and capital is not a free resource. A project’s cost of capital is the minimum expected rate of return the project needs to offer to attract the money required. But where does the minimum expected rate come from? This book provides an answer. It sets out to explain what the finance and economics literature says about how a project’s cost of capital is determined and estimated. It is written primarily for people who have already done some academic finance, accounting or economics but who are not necessarily well versed in the topics covered. In other words, it acts as a bridge between a general undergraduate or MBA text in finance, accounting or economics and the body of knowledge that relates to the cost of capital. It is suitable for students at honours, M.Sc. or MBA level who will have encountered the concept and who wish to know more.

Ease of comprehension is a high priority. Much of the book consists of explanations of theories, and many of the theories are demanding. I have done my best to present each model or argument in a way in which it can most easily be understood, whilst giving enough detail for the reader to see how a conclusion has been arrived at. The text includes plenty of discussion, supported by numerical examples. The exposition is in discrete time and uses (almost) no matrix notation. The level of mathematics is only slightly higher than that found in a typical undergraduate text in finance; the techniques most frequently used are algebra and differentiation. A basic knowledge of statistics and accounting is also assumed.

The flow of the book is from abstract models of expected returns, to less abstract models, concerned with particular features of the world that affect the cost of capital, and – finally – to methods of estimation. This seems to me to be the natural way to ‘tell the story’ about the cost of capital. I start by assuming a relatively simple world, and then make it more realistic. Part I, *Expected Returns on Financial Assets*, is an introduction to the theory of how expected rates of return on assets are determined. This is also known as the theory of asset pricing. There are no real-world complications in Part I, such as taxes or inflation. The theory is relevant because a project’s cost of capital is the same thing as the expected rate of return on the project’s capital at its market value – i.e. the capital viewed as a financial asset. So the theory of expected returns on financial assets constitutes the fundamental theory of the cost of capital.
It would have been possible to write the book, and it is more or less possible to read it, starting at Part II, which begins with project valuation and moves on to the weighted average cost of capital and tax. But this would have been to start the story halfway through. The theories discussed in Part II cannot themselves provide a satisfactory answer to the question of how the cost of capital is determined. This is because they do not attempt to explain how the expected rates of return on financial assets are determined. They start with an assumed required rate of return without tax, or with a model for such a rate, and then proceed.

The other motive for writing Part I was that there seemed to be a need for a treatment of asset pricing that was slightly above the level of a general undergraduate text in finance. Asset pricing is a huge and difficult area. There are now several very impressive books on the subject, but they are more advanced and detailed than Part I of this book. There have been few recent attempts to present asset pricing theory at a genuinely introductory or intermediate level, a valuable exception being by Danthine and Donaldson (2002). In fact, the same point can be made regarding most of the other topics in the book. A great deal of thinking has been done by researchers, and many absorbing lines of enquiry have been pursued. But there is surprisingly little to help the student take the steps to the research literature.

Part II is entitled A Project's Cost of Capital. It reviews a number of issues in the determination of a discount rate to value a project, often using the capital asset pricing model (CAPM) as the theory that explains the cost of equity. It starts with the framework for project valuation and then considers, in turn, the effects of corporation tax, personal taxes, inflation, and the existence of more than one country and currency. By the end of Part II the reader will have covered the central elements of the theory pertaining to a project’s cost of equity and the weighted average cost of capital.

Part III, Estimating the Cost of Capital, is a discussion of the main methods of estimation used in practice, and of research on these methods. It first considers estimation of the cost of equity of a listed company – i.e. estimation of the expected rate of return on the company’s shares. It then turns to what can be done to estimate the weighted average cost of capital of a project or unlisted company. The final chapter is devoted to the important special case of the cost of equity for regulated utility companies.

The book is about theory and estimation. There are summaries of the empirical evidence on the theories at various points, but there is little discussion of the empirical methods of testing them, or of debated findings, except as regards the equity risk premium. In general, I have paid more attention to the cost of equity than to the cost of debt. The models in the research reviewed are models constructed in discrete time, with a handful of exceptions. There is nothing on monetary policy, nor on models of the term structure of interest rates. Throughout, given the scale and complexity of the relevant literature, and given my own limitations, I have had to be selective both in what to review and in how to review it.

The most similar recent books specifically on the cost of capital are those by Ehrhardt (1994) and Patterson (1995). Ehrhardt provides a clear introductory account, written primarily for practitioners. Patterson offers a comprehensive review of a wide range of applied theory and evidence. The present book covers some of the same ground but, nevertheless, differs considerably from these predecessors in terms both of exposition and of content.
It is much more concerned with the explanation of theoretical arguments, and it contains more, in particular, on asset pricing theory, the equity premium and the effects of taxes. It is similar in approach and style to the book by Copeland and Weston (1988).

I hope that this book gives the reader some perspective on how the diverse topics of research it discusses relate to each other. However, many readers will not be interested in all of these topics, so individual chapters are designed, as far as possible, to be read on a stand-alone basis. They can serve as supplementary references for a variety of courses. Subjects in which the cost of capital features prominently include project and company valuation, capital budgeting, asset pricing, the assessment of company performance and the effects of taxes on corporate debt policy and dividend policy.

In summary, the book is a mixture of a monograph and a textbook. Its orientation around a single topic is in the nature of a monograph; its style of exposition is more like that of a textbook.

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