

Quantitative Models in Marketing Research

Philip Hans Franses
and
Richard Paap



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1 Introduction and outline of the book

Recent advances in data collection and data storage techniques enable marketing researchers to study the characteristics of many actual transactions and purchases, that is, revealed preference data. Owing to the large number of observations and the wealth of available information, a detailed analysis of these data is possible. This analysis usually addresses the effects of marketing instruments and the effects of household-specific characteristics on the transaction. Quantitative models are useful tools for this analysis. In this book we review several such models for revealed preference data. In this chapter we give a general introduction and provide brief introductions to the various chapters.

1.1 Introduction

It is the aim of this book to present various important and practically relevant quantitative models, which can be used in present-day marketing research. The reader of this book should become able to apply these methods in practice, as we provide the data which we use in the various illustrations and we also add the relevant computer code for EViews if it is not already included in version 3.1. Other statistical packages that include estimation routines for some of the reviewed models are, for example, LIMDEP, SPSS and SAS. Next, the reader should come to understand (the flavor of) the latest methodological developments as these are put forward in articles in, for example, *Marketing Science*, the *Journal of Marketing Research*, the *Journal of Consumer Research* and the *International Journal of Research in Marketing*. For that matter, we also discuss interesting new developments in the relevant sections.

The contents of this book originate from lecture notes prepared for undergraduate and graduate students in Marketing Research and in Econometrics. Indeed, it is our intention that this book can be used at different teaching levels. With that aim, all chapters have the same format, and we indicate

which sections correspond with which teaching level. In section 1.2, we will provide more details. For all readers, however, it is necessary to have a basic knowledge of elementary regression techniques and of some matrix algebra. Most introductory texts on quantitative methods include such material, but as a courtesy we bring together some important topics in an Appendix at the end of this book.

There are a few other books dealing with sets of quantitative models similar to the ones we consider. Examples are Maddala (1983), Ben-Akiva and Lerman (1985), Cramer (1991) and Long (1997). The present book differs from these textbooks in at least three respects. The first is that we discuss the models and their specific application in marketing research concerning revealed preference data. Hence, we pay substantial attention to the interpretation and evaluation of the models in the light of the specific applications. The second difference is that we incorporate recent important developments, such as modeling unobserved heterogeneity and sample selection, which have already become quite standard in academic marketing research studies (as may be noticed from many relevant articles in, for example, the *Journal of Marketing Research* and *Marketing Science*). The third difference concerns the presentation of the material, as will become clear in section 1.2 below. At times the technical level is high, but we believe it is needed in order to make the book reasonably self-contained.

1.1.1 On marketing research

A useful definition of marketing research, given in the excellent introductory textbook by Lehmann et al. (1998, p. 1), is that “[m]arketing research is the collection, processing, and analysis of information on topics relevant to marketing. It begins with problem definition and ends with a report and action recommendations.” In the present book we focus only on the part that concerns the analysis of information. Additionally, we address only the type of analysis that requires the application of statistical and econometric methods, which we summarize under the heading of quantitative models. The data concern revealed preference data such as sales and brand choice. In other words, we consider models for quantitative data, where we pay specific attention to those models that are useful for marketing research. We do not consider models for stated preference data or other types of survey data, and hence we abstain from, for example, LISREL-type models and various multivariate techniques. For a recent treatment of combining revealed and stated preference data, see Hensher et al. (1999). Finally, we assume that the data have already been collected and that the research question has been clearly defined.

The reasons we focus on revealed preference data, instead of on stated preference data, are as follows. First, there are already several textbooks on LISREL-type models (see, for example, Jöreskog and Sörbom, 1993) and on multivariate statistical techniques (see, for example, Johnson and Wichern, 1998). Second, even though marketing research often involves the collection and analysis of stated preference data, we observe an increasing availability of revealed preference data.

Typical research questions in marketing research concern the effects of marketing instruments and household-specific characteristics on various marketing performance measures. Examples of these measures are sales, market shares, brand choice and interpurchase times. Given knowledge of these effects, one can decide to use marketing instruments in a selective manner and to address only apparently relevant subsamples of the available population of households. The latter is usually called segmentation.

Recent advances in data collection and data storage techniques, which result in large data bases with a substantial amount of information, seem to have changed the nature of marketing research. Using loyalty cards and scanners, supermarket chains can track all purchases by individual households (and even collect information on the brands and products that were not purchased). Insurance companies, investment firms and charity institutions keep track of all observable activities by their clients or donors. These developments have made it possible to analyze not only what individuals themselves state they do or would do (that is, stated preference), but also what individuals actually do (revealed preference). This paves the way for greater insights into what really drives loyalty to an insurance company or into the optimal design for a supermarket, to mention just a few possible issues. In the end, this could strengthen the relationship between firms and customers.

The large amount of accurately measured marketing research data implies that simple graphical tools and elementary modeling techniques in most cases simply do not suffice for dealing with present-day problems in marketing. In general, if one wants to get the most out of the available data bases, one most likely needs to resort to more advanced techniques. An additional reason is that more detailed data allow more detailed questions to be answered. In many cases, more advanced techniques involve quantitative models, which enable the marketing researcher to examine various correlations between marketing response variables and explanatory variables measuring, for example, household-specific characteristics, demographic variables and marketing-mix variables.

In sum, in this book we focus on quantitative models for revealed preference data in marketing research. For conciseness, we do not discuss the various issues related to solving business problems, as this would require an

entirely different book. The models we consider are to be viewed as helpful practical tools when analyzing marketing data, and this analysis can be part of a more comprehensive approach to solving business problems.

1.1.2 Data

Marketing performance measures can appear in a variety of formats. And, as we will demonstrate in this book, these differing formats often need different models in order to perform a useful analysis of these measures.

To illustrate varying formats, consider “sales” to be an obvious marketing performance measure. If “sales” concerns the number of items purchased, the resultant observations can amount to a limited range of count data, such as 1, 2, 3, However, if “sales” refers to the monetary value in dollars (or cents) of the total number of items purchased, we may consider it as a continuous variable. Because the evaluation of a company’s sales may depend on all other sales, one may instead want to consider market shares. These variables are bounded between 0 and 1 by construction.

Sales and market shares concern variables which are observed over time. Typically one analyzes weekly sales and market shares. Many other marketing research data, however, take only discrete (categorical) values or are only partially observed. The individual response to a direct mailing can take a value of 1 if there is a response, and 0 if the individual does not respond. In that case one has encountered a binomial dependent variable. If households can choose between three or more brands, say brands A, B, C and D, one has encountered a multinomial dependent variable. It may then be of interest to examine whether or not marketing-mix instruments have an effect on brand choice. If the brands have a known quality or preference ranking that is common to all households, the multinomial dependent variable is said to be ordered; if not, it is unordered. Another example of an ordered categorical variable concerns questionnaire items, for which individuals indicate to what extent they disagree, are indifferent, or agree with a certain statement.

Marketing research data can also be only partially observed. An example concerns donations to charity, for which individuals have received a direct mailing. Several of these individuals do not respond, and hence donate nothing, while others do respond and at the same time donate some amount. The interest usually lies in investigating the distinguishing characteristics of the individuals who donate a lot and those who donate a lesser amount, while taking into account that individuals with perhaps similar characteristics donate nothing. These data are called censored data. If one knows the amount donated by an individual only if it exceeds, say, \$10, the corresponding data are called truncated data.

Censoring is also a property of duration data. This type of observation usually concerns the time that elapses between two events. Examples in marketing research are interpurchase times and the duration of a relationship between a firm and its customers. These observations are usually collected for panels of individuals, observed over a span of time. At the time of the first observations, it is unlikely that all households buy a product or brand at the same time, and hence it is likely that some durations (or relationships) are already ongoing. Such interpurchase times can be useful in order to understand, for example, whether or not promotions accelerate purchasing behavior. For direct marketing, one might model the time between sending out the direct mailing and the response, which perhaps can be reduced by additional nationwide advertising. In addition, insurance companies may benefit from lengthy relationships with their customers.

1.1.3 Models

As might be expected from the above summary, it is highly unlikely that all these different types of data could be squeezed into one single model framework. Sales can perhaps be modeled by single-equation linear regression models and market shares by multiple-equation regression models (because market shares are interconnected), whereas binomial and multinomial data require models that take into account that the dependent variable is not continuous. In fact, the models for these choice data usually consider, for example, the probability of a response to a direct mailing and the probability that a brand is selected out of a set of possible brands. Censored data require models that take into account the probability that, for example, households do not donate to charity. Finally, models for duration data take into account that the time that has elapsed since the last event has an effect on the probability that the next event will happen.

It is the purpose of this book to review quantitative models for various typical marketing research data. The standard Linear Regression model is one example, while the Multinomial Logit model, the Binomial Logit model, the Nested Logit model, the Censored Regression model and the Proportional Hazard model are other examples. Even though these models have different names and take account of the properties of the variable to be explained, the underlying econometric principles are the same. One can summarize these principles under the heading of an econometric modeling cycle. This cycle involves an understanding of the representation of the model (what does the model actually do? what can the model predict? how can one interpret the parameters?), estimation of the unknown parameters, evaluation of the model (does the model summarize the data in an adequate

way? are there ways to improve the model?), and the extension of the model, if required.

We follow this rather schematic approach, because it is our impression that studies in the academic marketing research literature are sometimes not very explicit about the decision to use a particular model, how the parameters were estimated, and how the model results should be interpreted. Additionally, there are now various statistical packages which include estimation routines for such models as the Nested Logit model and the Ordered Probit model (to name just a few of the more exotic ones), and it frequently turns out that it is not easy to interpret the output of these statistical packages and to verify the adequacy of the procedures followed. In many cases this output contains a wealth of statistical information, and it is not always clear what this all means and what one should do if statistics take particular values. By making explicit several of the modeling steps, we aim to bridge this potential gap between theory and practice.

1.2 Outline of the book

This book aims to describe some of the main features of various potentially useful quantitative models for marketing research data. Following a chapter on the data used throughout this book, there are six chapters, each dealing with one type of dependent variable. These chapters are subdivided into sections on (1) representation and interpretation, (2) the estimation of the model parameters, (3) model diagnostics and inference, (4) a detailed illustration and (5) advanced topics.

All models and methods are illustrated using actual data sets that are or have been effectively used in empirical marketing research studies in the academic literature. The data are available through relevant websites. In chapter 2, we discuss the data and also some of the research questions. To sharpen the focus, we will take the data as the main illustration throughout each chapter. This means that, for example, the chapter on a multinomial dependent variable (chapter 5) assumes that such a model is useful for modeling brand choice. Needless to say, such a model may also be useful for other applications. Additionally, to reduce confusion, we will consider the behavior of a household, and assume that it makes the decisions. Of course this can be replaced by individuals, customers or other entities, if needed.

1.2.1 How to use this book

The contents of the book are organized in such a way that it can be used for teaching at various levels or for personal use given different levels of training.

The first of the five sections in each of chapters 3 to 8 contains the representation of the relevant model, the interpretation of the parameters, and sometimes the interpretation of the full model (by focusing, for example, on elasticities). The fourth section contains a detailed illustration, whose content one should be able to grasp given an understanding of the content of the first section. These two sections can be used for undergraduate as well as for graduate teaching at a not too technical level. In fact, we ourselves have tried out these sections on undergraduate students in marketing at Erasmus University Rotterdam (and, so far, we have not lost our jobs).

Sections 2 and 3 usually contain more technical material because they deal with parameter estimation, diagnostics, forecasting and model selection. Section 2 always concerns parameter estimation, and usually we focus on the Maximum Likelihood method. We provide ample details of this method as we believe it is useful for a better understanding of the principles underlying the diagnostic tests in section 3. Furthermore, many computer packages do not provide diagnostics and, using the formulas in section 2, one can compute them oneself. Finally, if one wants to program the estimation routines oneself, one can readily use the material. In many cases one can replicate our estimation results using the relevant standard routines in EViews (version 3.1). In some cases these routines do not exist, and in that case we give the relevant EViews code at the end of the relevant chapters. In addition to sections 1 and 4, one could consider using sections 2 and 3 to teach more advanced undergraduate students, who have a training in econometrics or advanced quantitative methods, or graduate students in marketing or econometrics.

Finally, section 5 of each chapter contains advanced material, which may not be useful for teaching. These sections may be better suited to advanced graduate students and academics. Academics may want to use the entire book as a reference source.

1.2.2 Outline of chapter contents

The outline of the various chapters is as follows. In chapter 2 we start off with detailed graphical and tabulated summaries of the data. We consider weekly sales, a binomial variable indicating the choice between two brands, an unordered multinomial variable concerning the choice between four brands, an ordered multinomial variable for household-specific risk profiles, a censored variable measuring the amount of money donated to charity and, finally, interpurchase times in relation to liquid detergent.

In chapter 3 we give a concise treatment of the standard Linear Regression model, which can be useful for a continuous dependent variable. We assume some knowledge of basic matrix algebra and of elementary

statistics. We discuss the representation of the model and the interpretation of its parameters. We also discuss Ordinary Least Squares (OLS) and Maximum Likelihood (ML) estimation methods. The latter method is discussed because it will be used in most chapters, although the concepts underlying the OLS method return in chapters 7 and 8. We choose to follow the convention that the standard Linear Regression model assumes that the data are normally distributed with a constant variance but with a mean that obtains different values depending on the explanatory variables. Along similar lines, we will introduce models for binomial, multinomial and duration dependent variables in subsequent chapters. The advanced topics section in chapter 3 deals with the attraction model for market shares. This model ensures that market shares sum to unity and that they lie within the range $[0, 1]$.

The next chapter deals with a binomial dependent variable. We discuss the binomial Logit and Probit models. These models assume a nonlinear relation between the explanatory variables and the variable to be explained. Therefore, we pay considerable attention to parameter interpretation and model interpretation. We discuss the ML estimation method and we provide some relevant model diagnostics and evaluation criteria. As with the standard Linear Regression model, the diagnostics are based on the residuals from the model. Because these residuals can be defined in various ways for these models, we discuss this issue at some length. The advanced topics section is dedicated to the inclusion of unobserved parameter heterogeneity in the model and to the effects of sample selection for the Logit model.

In chapter 5 we expand on the material of chapter 4 by focusing on an unordered multinomial dependent variable. Quite a number of models can be considered, for example the Multinomial Logit model, the Multinomial Probit model, the Nested Logit model and the Conditional Logit model. We pay substantial attention to outlining the key differences between the various models in particular because these are frequently used in empirical marketing research.

In chapter 6 we focus on the Logit model and the Probit model for an ordered multinomial dependent variable. Examples of ordered multinomial data typically appear in questionnaires. The example in chapter 6 concerns customers of a financial investment firm who have been assigned to three categories depending on their risk profiles. It is the aim of the empirical analysis to investigate which customer-specific characteristics can explain this classification. In the advanced topics section, we discuss various other models for ordered categorical data.

Chapter 7 deals with censored and truncated dependent variables, that is, with variables that are partly continuous and partly take some fixed value (such as 0 or 100) or are partly unknown. We mainly focus on the Truncated

Regression model and on the two types of Tobit model, the Type-1 and Type-2 Tobit models. We show what happens if one neglects the fact that the data are only partially observed. We discuss estimation methods in substantial detail. The illustration concerns a model for money donated to charity for a large sample of individuals. In the advanced topics section we discuss other types of models for data censored in some way.

Finally, in chapter 8 we deal with a duration dependent variable. This variable has the specific property that its value can be made dependent on the time that has elapsed since the previous event. For some marketing research applications this seems a natural way to go, because it may become increasingly likely that households will buy, for example, detergent if it is already a while since they purchased it. We provide a discussion of the Accelerated Lifetime model and the Proportional Hazard model, and outline their most important differences. The advanced topics section contains a discussion of unobserved heterogeneity. It should be stressed here that the technical level both of chapter 8 and of chapter 5 is high.

Before we turn to the various models, we first look at some marketing research data.