

Contents

<i>Preface</i>	xiii
1 Introductory overview	1
1.1 Brief motivations	1
1.2 Brief summary	4
1.3 Spaces and superspaces	4
1.4 Chirality as a kind of Grassmann analyticity	6
1.5 $N = 1$ chiral superfields	6
1.6 Auxiliary fields	7
1.7 Why standard superspace is not adequate for $N = 2$ supersymmetry	8
1.8 Search for conceivable superspaces (spaces)	9
1.9 $N = 2$ harmonic superspace	10
1.10 Dealing with the sphere S^2	10
1.10.1 Comparison with the standard harmonic analysis	11
1.11 Why harmonic superspace helps	13
1.12 $N = 2$ supersymmetric theories	15
1.12.1 $N = 2$ matter hypermultiplet	15
1.12.2 $N = 2$ Yang–Mills theory	16
1.12.3 $N = 2$ supergravity	18
1.13 $N = 3$ Yang–Mills theory	19
1.14 Harmonics and twistors. Self-duality equations	20
1.15 Chapters of the book and their abstracts	23
2 Elements of supersymmetry	27
2.1 Poincaré and conformal symmetries	27
2.1.1 Poincaré group	27
2.1.2 Conformal group	28
2.1.3 Two-component spinor notation	29
2.2 Poincaré and conformal superalgebras	29

viii	<i>Contents</i>	
	2.2.1 $N = 1$ Poincaré superalgebra	29
	2.2.2 Extended supersymmetry	30
	2.2.3 Conformal supersymmetry	31
	2.2.4 Central charges from higher dimensions	32
2.3	Representations of Poincaré supersymmetry	33
	2.3.1 Representations of the Poincaré group	33
	2.3.2 Poincaré superalgebra representations. Massive case	34
	2.3.3 Poincaré superalgebra representations. Massless case	36
	2.3.4 Representations with central charge	37
2.4	Realizations of supersymmetry on fields. Auxiliary fields	38
	2.4.1 $N = 1$ matter multiplet	38
	2.4.2 $N = 1$ gauge multiplet	41
	2.4.3 Auxiliary fields and extended supersymmetry	41
3	Superspace	44
3.1	Coset space generalities	44
3.2	Coset spaces for the Poincaré and super Poincaré groups	46
3.3	$N = 2$ harmonic superspace	50
3.4	Harmonic variables	54
3.5	Harmonic covariant derivatives	58
3.6	$N = 2$ superspace with central charge coordinates	60
3.7	Reality properties	61
3.8	Harmonics as square roots of quaternions	63
4	Harmonic analysis	66
4.1	Harmonic expansion on the two-sphere	66
4.2	Harmonic integrals	67
4.3	Differential equations on S^2	69
4.4	Harmonic distributions	70
5	$N = 2$ matter with infinite sets of auxiliary fields	74
5.1	Introduction	74
	5.1.1 $N = 1$ matter	74
	5.1.2 $N = 2$ matter multiplets on shell	76
	5.1.3 Relationship between q^+ and ω hypermultiplets	77
	5.1.4 Off-shell $N = 2$ matter before harmonic superspace	78
5.2	Free off-shell hypermultiplet	79
	5.2.1 The Fayet–Sohnius hypermultiplet constraints as analyticity conditions	79
	5.2.2 Free off-shell q^+ action	82
	5.2.3 Relationship between q^+ and ω hypermultiplets off shell	85
	5.2.4 Massive q^+ hypermultiplet	86
	5.2.5 Invariances of the free hypermultiplet actions	87

	<i>Contents</i>	ix
5.3	Hypermultiplet self-couplings	90
5.3.1	General action for q^+ hypermultiplets	90
5.3.2	An example of a q^+ self-coupling: The Taub–NUT sigma model	91
5.3.3	Symmetries of the general hypermultiplet action	95
5.3.4	Analogy with Hamiltonian mechanics	98
5.3.5	More examples of q^+ self-couplings: The Eguchi–Hanson sigma model and all that	99
6	$N = 2$ matter multiplets with a finite number of auxiliary fields. $N = 2$ duality transformations	107
6.1	Introductory remarks	107
6.2	$N = 2$ tensor multiplet	109
6.3	The relaxed hypermultiplet	111
6.4	Further relaxed hypermultiplets	112
6.5	Non-linear multiplet	114
6.6	$N = 2$ duality transformations	117
6.6.1	Transforming the tensor multiplet	118
6.6.2	Transforming the relaxed hypermultiplet	121
6.6.3	Transforming the non-linear multiplet	122
6.6.4	General criterion for equivalence between hypermultiplet and tensor multiplet actions	123
6.7	Conclusions	126
7	Supersymmetric Yang–Mills theories	128
7.1	Gauge fields from matter couplings	128
7.1.1	$N = 0$ gauge fields	128
7.1.2	$N = 1$ SYM gauge prepotential	129
7.1.3	$N = 2$ SYM gauge prepotential	131
7.2	Superspace differential geometry	134
7.2.1	General framework	135
7.2.2	$N = 1$ SYM theory	136
7.2.3	$N = 2$ SYM theory	138
7.2.4	V^{++} versus Mezincescu’s prepotential	143
7.3	$N = 2$ SYM action	144
8	Harmonic supergraphs	148
8.1	Analytic delta functions	148
8.2	Green’s functions for hypermultiplets	150
8.3	$N = 2$ SYM: Gauge fixing, Green’s functions and ghosts	152
8.4	Feynman rules	156
8.5	Examples of supergraph calculations. Absence of harmonic divergences	160

x	<i>Contents</i>	
8.6	A finite four-point function at two loops	167
8.7	Ultraviolet finiteness of $N = 4, d = 2$ supersymmetric sigma models	171
9	Conformal invariance in $N = 2$ harmonic superspace	175
9.1	Harmonic superspace for $SU(2, 2 2)$	175
9.1.1	Cosets of $SU(2, 2 2)$	175
9.1.2	Structure of the analytic superspace	177
9.1.3	Transformation properties of the analytic superspace coordinates	179
9.1.4	Superfield representations of $SU(2, 2 2)$	182
9.2	Conformal invariance of the basic $N = 2$ multiplets	185
9.2.1	Hypermultiplet	185
9.2.2	Tensor multiplet	186
9.2.3	Non-linear multiplet	186
9.2.4	Relaxed hypermultiplet	187
9.2.5	Yang–Mills multiplet	187
10	Supergravity	189
10.1	From conformal to Einstein gravity	189
10.2	$N = 1$ supergravity	192
10.3	$N = 2$ supergravity	195
10.3.1	$N = 2$ conformal supergravity: Gauge group and prepotentials	195
10.3.2	Central charge vielbeins	200
10.3.3	Covariant harmonic derivative $\mathcal{D}^{--}$	202
10.3.4	Building blocks and superspace densities	203
10.3.5	Abelian gauge invariance of the Maxwell action	206
10.4	Different versions of $N = 2$ supergravity and matter couplings	207
10.4.1	Principal version of $N = 2$ supergravity and general matter couplings	207
10.4.2	Other versions of $N = 2$ supergravity	211
10.5	Geometry of $N = 2$ matter in $N = 2$ supergravity background	214
11	Hyper-Kähler geometry in harmonic space	217
11.1	Introduction	217
11.2	Preliminaries: Self-dual Yang–Mills equations and Kähler geometry	219
11.2.1	Harmonic analyticity and SDYM theory	220
11.2.2	Comparison with the twistor space approach	223
11.2.3	Complex analyticity and Kähler geometry	224
11.2.4	Central charge as the origin of the Kähler potential	229
11.3	Harmonic analyticity and hyper-Kähler potentials	232

<i>Contents</i>	<i>xi</i>
11.3.1 Constraints in harmonic space	234
11.3.2 Harmonic analyticity	236
11.3.3 Harmonic derivatives in the λ world	238
11.3.4 Hyper-Kähler potentials	240
11.3.5 Gauge choices and normal coordinates	244
11.3.6 Summary of hyper-Kähler geometry	245
11.3.7 Central charges as the origin of the hyper-Kähler potentials	248
11.3.8 An explicit construction of hyper-Kähler metrics	253
11.4 Geometry of $N = 2, d = 4$ supersymmetric sigma models	256
11.4.1 The geometric meaning of the general q^+ action	256
11.4.2 The component action of the general $N = 2$ sigma model	258
12 $N = 3$ supersymmetric Yang–Mills theory	263
12.1 $N = 3$ SYM on-shell constraints	263
12.2 $N = 3$ harmonic variables and interpretation of the $N = 3$ SYM constraints	264
12.3 Elements of the harmonic analysis on $SU(3)/U(1) \times U(1)$	266
12.4 $N = 3$ Grassmann analyticity	268
12.5 From covariant to manifest analyticity: An equivalent interpretation of the $N = 3$ SYM constraints	271
12.6 Off-shell action	273
12.7 Components on and off shell	275
12.8 Conformal invariance	277
12.9 Final remarks	279
13 Conclusions	281
Appendix: Notations, conventions and useful formulas	283
A.1 Two-component spinors	283
A.2 Harmonic variables and derivatives	284
A.3 Spinor derivatives	285
A.4 Conjugation rules	286
A.5 Superspace integration measures	287
<i>References</i>	289
<i>Index</i>	304