

# The Measurement of Time

Time, Frequency and the Atomic Clock

Claude Audoin

*CNRS, Orsay, France*

Bernard Guinot

*Paris Observatory, France*

Translated by Stephen Lyle



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# 1

## Introduction to the book

The use of atomic properties for time measurement was born in 1955 when the first caesium beam frequency standard began regular operation in the United Kingdom. Of course, other types of atomic frequency standard had already been attempted. However, they had proved unable to provide a unit of time and uniform time scales superior to those based on celestial motions, in the way that the caesium standard could. Caesium standards soon proliferated, whilst their accuracy advanced by leaps and bounds. The good agreement between them inspired confidence and, in 1967, it was possible to make an atomic definition of the second to replace the previously used astronomical definition. Several years later, the existence of an adequate atomic time scale for the management of world affairs was officially recognised by the 14th General Conference on Weights and Measures (1971).

Despite these achievements, over a quarter of a century later, the atomic measurement of time has still not really become familiar. Could this be due to a deeply ingrained habit of measuring out our everyday existence by the movements of the celestial bodies? Or could it be a reaction to the lack of poetry in atomic clocks, or their inscrutable accuracy? This may be so, but there are also more objective reasons that can sometimes make it difficult to accept atomic time. One of these is the need to formulate the measurement of time within a relativistic framework, and another is the complex relationships between atomic time and the time of the dynamical theories used by astronomers. The latter problem was a subject of controversy over twenty years or so, and even the International Astronomical Union was not completely able to lay the ghost with a resolution adopted in 1991. Recently, some have thought that the discovery of rapid pulsars, the so-called millisecond pulsars, might bring the measurement of time back within the jurisdiction of astronomy.

Meanwhile, in the peace of their laboratories, physicists are barely susceptible to such interpellations. Following a period of relative stagnation which



lasted about twenty years, a sudden gain in the accuracy of frequency standards was achieved in 1994. The new techniques that made this improvement possible are also full of promise for further progress in the near future. On the other hand, the world of industry has not limited its involvement. Manufactured quartz clocks and atomic clocks of various types have shown continual improvement. The instrument favoured by fundamental metrological research has always been the caesium clock and thousands of these have already been produced. In 1994, a new manufactured version came out which scored a ten-fold enhancement over the whole range of relevant qualities, compared with previous versions.

Through the industrial manufacture of timing equipment, important applications of time–frequency techniques have seen the light of day. An example is the Global Positioning System (GPS), in full operation by 1995. This interests a whole range of potential users, from geodesists seeking to represent the whole globe to the nearest centimetre, to rambles, not forgetting the military applications for which the system was originally devised.

Considering these recent advances in the already well-tested atomic techniques of time measurement and the new problems posed by such a gain in quality, it seemed an opportune moment to present something of their foundations: the basic principles, construction of the best standards, ways of comparing frequencies and times with the ultimate accuracy, and ways of accessing the primary standards, that is, the best realisations of the second and the International Atomic Time at the highest level of accuracy. It also seemed useful to recall the role of astronomical time scales, which remain important for various reasons.

As we have already mentioned, the fundamental measurement of time can no longer be conceived of outside a relativistic framework. Up to the present time, Einstein's general relativity, the simplest of these theories, has always remained in agreement with experimental results. We shall therefore adopt it as our working hypothesis. However, we must express some reservations in this regard. To begin with, we had hoped to treat the measurement of time within the framework of classical physics and then indicate, in small print as it were, what would be required to take relativistic effects into account. But then it seemed to us that this method ran the risk of promoting a widely held view that general relativity is in some sense a mere addition to classical theory, that its contribution to science consists in making a set of relativistic corrections. In reality, general relativity is a complete model for the structure of space and time, as well as for gravitational effects. We have therefore directly established the useful developments within the context of this theory. The reader who is barely familiar with such questions should nevertheless persevere, for we shall

go no further than recalling the postulates of the theory and its mathematical consequences in the greatest simplicity.

The inevitable linearity of a written account is poorly suited to the many interactions between the varied aspects of time measurement. The brief description given here of the various chapters in the book is intended to orient the reader with regard to this problem.

Chapter 2 recalls the fundamental principles of time measurement for macroscopic phenomena, the only ones to be considered here. In truth, this is essentially a review of H. Poincaré's analysis, whose relevance extends to the relativistic context.

Chapter 3 shows how time measurement relates to general relativity. In an extended space, time is one element of a four-dimensional coordinate system and cannot be dissociated from the spatial coordinates. We shall define the coordinate systems used and establish some equations relating local physics to spacetime coordinates. We shall also discuss the conventions involved in synchronisation and comparison of distant clocks.

Chapter 4 has a historical objective. It shows how time measurement has evolved, in particular, during the twentieth century. We shall discuss the development of ideas stimulated by development of techniques, as well as the problems raised by transition from astronomical measurement of time to its atomic measurement.

Chapter 5 describes the tools involved in using atomic time standards, and in particular, methods for characterising their metrological qualities and comparing one with another. The characterisation of frequency stability has led to new developments in statistics. These refer to time series of measurements and the definition of variances which allow manufacturers and users of frequency standards to speak a common language. The characterisation of the accuracy of a frequency is also a problem specific to time measurement, although it could be extended to any physical quantity whose unit is directly defined by a natural phenomenon that is considered to be reproducible. Concerning the comparison of frequencies and time scales at a distance, we shall see that it is a crucial question in the sense that it often gives rise to greater uncertainties than those involved in the quantities to be compared.

Chapter 6 treats atomic frequency and time standards, also called atomic clocks. We shall begin by reviewing, on an elementary level, the main ideas of atomic physics and spectroscopy required to understand the workings of these standards and their principal features. There are several sorts of atomic clock, all making use of some transition between atomic levels of the same type, but in different atoms, such as caesium, hydrogen and rubidium. Even ions are used, like the mercury ion. It is the caesium clocks built and used in metrolog-

ical laboratories that best achieve the definition of the second. These are the primary frequency and time standards. Their main feature is accuracy. Industrial versions of the caesium clock allow a relatively inexpensive dissemination of the unit of time, and hence of atomic time itself. Other types of atomic clock will also be described. They have been built or developed because they are specially suited to certain fields of application. Some are bulky but exceptionally stable, whilst others are less stable but rather compact. In this chapter, we shall go into the principles by which atoms are manipulated using the radiation from lasers. This experimental technique, which appeared recently, makes it possible to slow down and trap atoms before launching them in a highly controlled manner at very low speeds. Since 1996, this technique has led to an accuracy that excels that of all other primary frequency and time standards. The potential in these methods for manipulating atoms by laser beams, combined with the extraordinary international competition that they arouse, are a firm guarantee of future improvement in the performance of atomic clocks.

Chapter 7 is mainly devoted to the construction of the unique atomic time scale taken by convention as the international standard, namely, the International Atomic Time (denoted in all languages by TAI, from the French name *Temps atomique international*). We shall see how worldwide cooperation has been able to establish and maintain such a scale, and we shall investigate its qualities and the way it is disseminated. But we shall also see how it has been necessary to compromise, accepting as the practical standard a time scale based on the TAI but ingeniously adapted to maintain connections with the Universal Time (UT) as specified by Earth's rotation. This hybrid is known as Coordinated Universal Time (officially denoted by the acronym UTC). We shall show how the so-called proper second required for laboratory work can be obtained from the TAI or UTC.

Chapter 8 gives definitions of the various astronomical times in common use. These time scales have been gradually elaborated by observation of celestial bodies throughout a long period of history. Their usual definitions carry the traces of such historical developments and are thereby somewhat obscured for those unfamiliar with astronomy. We have thus made every effort to bring out the basic concepts upon which the operational definitions given by astronomers are founded. We shall investigate the purpose of these time scales. In particular, the Universal Time, witness to Earth's rotation, is a very important physical quantity requiring continuous measurement. Indeed, the efforts devoted to measuring Universal Time remain on a par with those required to maintain atomic time. Concerning pulsars, although they cannot provide a good measure for time, their study is nevertheless hampered by the uncertainties involved in atomic time. This is one of the most demanding applications for atomic clocks.

Finally, Chapter 9 presents some applications selected for the high level of quality in the time and frequency standards they require. Naturally, these applications belong to the area of fundamental research. The use of time standards in more utilitarian programmes is concomitant with a certain degree of sacrifice as far as their ultimate qualities are concerned, in favour of reliability and reduced costs. However, the more practical instruments sometimes lag only slightly behind those destined for laboratory use when it comes to accuracy and stability. The Global Positioning System provides a good illustration for this. As we are all either direct or indirect users of this system, we shall go into some degree of detail. We shall also show, as an example, how the measurement of time is relevant to a space-based oceanographic mission, TOPEX/POSEIDON. Indeed, we shall see that it underlies many aspects of the mission, with a high requisite level of quality.