Computation and Complexity in Economic Behavior and Organization

This book presents a model of computing and a measure of computational complexity that are intended to facilitate the analysis of computations performed by people, machines, or a mixed system of people and machines. The model is designed to apply directly to models of economic theory, which typically involve continuous variables and smooth functions, without requiring an analysis of approximations. The model permits an analysis of the feasibility and complexity of the calculations required of economic agents in order for them to arrive at their decisions. The treatment contains applications of the model to game theory and economics, including a comparison of the complexities of different solution concepts in certain bargaining games, to the trade-off between communication and computation in an example of an Edgeworth Box economy, and to problems of economic organization.

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Additional Praise for *Computation and Complexity in Economic Behavior and Organization*

“This book summarizes the research done over the past two decades by these two pioneers in the theory of bounded rationality in organizations. Anyone who is trying to model economic agents in an organization, and especially anyone who is concerned with the processing of information by organization, will find this an important reference. The models in this book, where agents are information processors within a network, are significantly richer than the conventional model of a single boundedly rational agent as a finite automaton. This approach offers a fresh perspective and tools for modeling computational complexity in an organization, tools that will be very valuable in capturing within a model the limited computational capabilities of both individuals and organizations. The treatment is both insightful and rigorous, making the book particularly suitable to advanced graduate students and researchers.”

– In-Koo Cho, University of Illinois

“This book opens a challenging new path in the theory of organization. An organization’s task is to compute a function of certain external variables. A well-designed organization does so quickly. It breaks the task into subtasks, each requiring a unit of time to complete, with the result becoming an input for a higher subtask. Some of the subtasks can be performed simultaneously. The challenge is to arrange the subtasks in a network so as to minimize the total elapsed time until the full task is finished. This is a novel and fruitful way to look at efficient organizations and to compare the difficulty of the tasks they undertake. Some general results are obtained and they are illustrated in a rich assortment of examples, including resource-allocating organizations and games. Contemporary work in the economic theory of organization has many motives and many approaches. Those who seek to move it in new directions ought to make a serious study of this book.”

– Thomas Marschak, University of California, Berkeley

“Mount and Reiter overcome the idiosyncratic, problem specific nature of previous models of computation and complexity by developing an approach based around the most common building blocks of economic models: real numbers and smooth functions. On the technical side this powerful innovation opens the way for the use of classical analysis and algebra in analyzing complexity of decision-making. At the same time the use of real numbers and smooth functions makes Mount and Reiter’s approach immediately applicable to standard models in game theory and organizational economics. The detailed examples in the text allow the applied theorist to see this new approach at work in familiar problems without having to master all the theoretical underpinning of this powerful new theory.”

– Kieron Meagher, University of New South Wales, Australia
To Bertha, Cynthia, John, Lisa and Greg.
K.R.M.

To Nina, Carla, Frank, Carrol and Miles.
S.R.
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