MACROSCOPIC QUANTUM TUNNELING

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Introduction

We begin by surveying what *macroscopic quantum phenomena* are, and what is the significance of searching for such phenomena, thereby locating *macroscopic quantum tunneling* in the broad perspective of physics in the new century.

1.1 The cat and the moon

It should not be necessary to elaborate on a Young-type interference experiment, which has by now been realized not only with electrons or neutrons but also with atoms such as He, Ne and Na. In a typical experiment, a particle of a given kinetic energy is sent through a double slit to a planar array of particle counters. What happens is that one and only one of the counters fires and is marked by a bright spot. As many particles of the same kind and the same kinetic energy as the first particle are sent one by one successively, bright spots accumulate and eventually emerge as an almost smooth interference pattern. This impressive emergence of the pattern may best be appreciated by watching a movie that records such an experiment in real time. In view of recent remarkable advances in experimental technology, one cannot but be curious about the prospect in the not-unforeseeable future: can the Young-type experiment be realized with an even bigger object, and how big an object can one deal with? Here is a dialogue between Weizsäcker[1] and Glauber[2] at a meeting on quantum mechanics in the early 1990s.

W: However far the technology should advance, one would not be able to see an interference pattern with tennis balls.
G: It might be possible with soccer balls, though.[3]

[1] C. F. von Weizsäcker is a theoretician known for his contribution to nuclear physics, etc. He is a brother of the former president of Germany.
[2] R. J. Glauber is a theoretician known for his contribution to quantum optics, etc.
[3] The molecule C_{60} consisting of 60 carbon atoms is often called a soccer ball because of its shape. **Note added in the English edition**: In 1999, only 2 years after the Japanese edition was published, an interference pattern was observed successfully with C_{60} (see Ref. [10] in the bibliography).

The fundamental equation of quantum mechanics[4] is the Schrödinger equation[5] which describes the time evolution of a given system. Its most important property is linearity. Let $|\Psi(t)\rangle$ be the state of the system at time t. (Hereafter a *state* is to be understood as a *quantum state* unless otherwise mentioned.) The Schrödinger equation may be written generally in the following (integrated) form:

$$|\Psi(t)\rangle = \hat{U}(t)|\Psi(0)\rangle. \tag{1.1.1}$$

The symbol $\hat{U}(t)$ denotes a unitary operator determined by the Hamiltonian $\hat{H}$ of the system. For the present purpose, it is sufficient to regard $\hat{U}(t)$ as a sort of a linear black box; given a state at time 0, (i) the state at an arbitrary time t is determined uniquely by the above equation, and (ii) the following equality holds for arbitrary $|\Psi_1\rangle$, $|\Psi_2\rangle$ and t:

$$\hat{U}(t)(|\Psi_1\rangle + |\Psi_2\rangle) = \hat{U}(t)|\Psi_1\rangle + \hat{U}(t)|\Psi_2\rangle. \tag{1.1.2}$$

This relationship embodies the above-mentioned linearity, which is supported by interference effects demonstrated in various experiments, especially ones of the Young-type.

In the microscopic world,[6] a variety of superpositions (namely, linear combinations) of states have been confirmed experimentally. One of the familiar examples is a superposition of spin-up and spin-down states. By virtue of the linearity (1.1.2), a superposition in the microscopic world can in principle be magnified to one in the macroscopic world. A mechanism of such a magnification is Schrödinger's linear theater, of which a simplified version would run as follows. On the stage is a box containing a cat. The play is so designed that a radioactive nucleus is thrown into the box and is swallowed by the cat at time 0, and that there are two possible states $|\phi_\pm\rangle$ for the nucleus at time 0 such that the nucleus has not yet decayed if it is in $|\phi_+\rangle$, but has decayed already if in $|\phi_-\rangle$. The cat will remain intact if the nucleus is in $|\phi_+\rangle$ at time 0, but will eventually die due to radiation hazard if the nucleus is in $|\phi_-\rangle$ at time 0. Let $|\psi\rangle$ be the state of the cat at time 0, and $|\Psi_\pm\rangle$ be the state describing the cat either remaining intact (subscript $+$) or being dead (subscript $-$), at some time T, after swallowing the nucleus which was in $|\phi_\pm\rangle$. Note that $|\Psi_\pm\rangle$ are states of the entire system composed of the cat and the nucleus:

$$|\Psi_\pm\rangle := \hat{U}(T)|\psi, \phi_\pm\rangle, \quad |\psi, \phi_\pm\rangle \equiv |\psi\rangle|\phi_\pm\rangle. \tag{1.1.3}$$

The curtain is to be closed at the time T. So much for the setting of the stage. Now, immediately before the curtain is opened at time 0, the nucleus is to be prepared

[4] Quantum mechanics here includes quantum field theory as well.

[5] Some of the readers might associate the Schrödinger equation with the equation "$\hat{H}|\Psi\rangle = E|\Psi\rangle$". The latter, however, being a special case of the former, is appropriate only if "$|\Psi\rangle$ is a stationary state".

[6] The word "world" here is meant to represent vaguely the whole collection of various physical phenomena.

in neither of the two states $|\phi_\pm\rangle$, but in the superposition $|\phi_+\rangle + |\phi_-\rangle$.[7] Hence, the initial state of the entire system is of the following form:

$$|\Psi(0)\rangle = |\psi\rangle(|\phi_+\rangle + |\phi_-\rangle) = |\psi, \phi_+\rangle + |\psi, \phi_-\rangle. \tag{1.1.4}$$

Combining this with (1.1.1), (1.1.2) and (1.1.3), one finds the state at the time T:

$$|\Psi(T)\rangle = \hat{U}(T)(|\psi, \phi_+\rangle + |\psi, \phi_-\rangle) = |\Psi_+\rangle + |\Psi_-\rangle. \tag{1.1.5}$$

This equation shows that the superposition in the microscopic world ($|\phi_+\rangle + |\phi_-\rangle$) can be magnified to that in the macroscopic world ($|\Psi_+\rangle + |\Psi_-\rangle$):

MMM: Magnification {microscopic $\longrightarrow$ macroscopic}

Given the state $|\Psi(T)\rangle$, which is a superposition of $|\Psi_+\rangle$ and $|\Psi_-\rangle$, the cat can neither be said to be alive ($|\Psi_+\rangle$) nor dead ($|\Psi_-\rangle$); it may only be said to be in the state of $\underline{|\Psi_+\rangle \text{ AND } |\Psi_-\rangle}$.

In view of the fact that superposition of distinct states (e.g. $|\phi_\pm\rangle$ in the above example) in the microscopic world has been confirmed experimentally, the appearance of *macroscopic superposition* (or, equivalently, *macroscopic linear combination*) such as (1.1.5) cannot be avoided so long as linearity of the Schrödinger equation is taken for granted. (Here, "macroscopic superposition" is meant to imply "superposition of *macroscopically distinct* states".) However, this sort of strange state is incompatible with the *macrorealism*,[8] according to which the cat, exposed to the radiation, must necessarily be either in the state $|\Psi_+\rangle$ or in $|\Psi_-\rangle$ ($\underline{|\Psi_+\rangle \text{ OR } |\Psi_-\rangle}$); a cat in the state $\underline{|\Psi_+\rangle \text{ AND } |\Psi_-\rangle}$ is totally incomprehensible.[9]

During one of his walks, Einstein is said to have asked his colleague[10] "Do you really believe that the moon is there only when you look at it?" What lies at the core of the discussion is the problem of the transition[11] from <u>AND of quantum mechanics</u> to <u>OR of macrorealism</u>,

TAO: Transition {AND $\longrightarrow$ OR}.

[7] A radioactive nucleus evolves into a state of this type even if it was originally in $|\phi_+\rangle$.

[8] Loosely speaking, a naive everyday-life realism. See Chapter 9 for details.

[9] According to Schrödinger as translated into English, it is 'ridiculous'; he ridiculed it by creating his linear theater but without forgetting to mention '... the living and the dead cat (pardon the expression)...'. Would Schrödinger have said 'Scat!' to the <u>S-Cat</u> ($\equiv$ Schrödinger's cat)? Note that the audience are not allowed to enter the theater before the closing time T! They are invited to examine the cat only somewhat later. Then, some of them will find the cat alive and the others will find it dead.

[10] A. Pais, *Rev. Mod. Phys.* **51** (1979), 863–914, Section X. Although this question may not have addressed specifically to the problem concerning the macroscopic world, it is undoubtedly an eloquent representative of the macrorealism.

[11] Also called Collapse, Objectification or REalisation (put together as CORE).

If a measuring apparatus replaces the cat, this problem reduces to the "problem of measurement in quantum mechanics", which has been debated since the birth of quantum mechanics.

It should be noted that the state (1.1.5) is not a simple product of a state of the nucleus and a state of the cat but a sum (linear combination) of such products. In this state, the nucleus and the cat cannot be separated; rather they are as it were inseparably entangled. In general such a state is called an *entangled state* (see Chapter 4 for details).

1.2 Leggett program

Let us take a look at the "quantum measurement problem" or the "*S-Cat* (Schrödinger's cat) paradox" from a laboratory-rooted point of view. This paradox presupposes the universal validity of quantum mechanics even in the macroscopic world. This premise, however, lacks in experimental evidence. If it were not valid, the paradox would either disappear or change its character. If, on the other hand, the premise is valid and a macroscopic superposition is realized, one should expect *QIMDS*,[12] namely, *quantum interference of macroscopically distinct states*. The question then is this: how macroscopic can an object be for a laboratory experiment to be able to detect QIMDS, thereby confirming a macroscopic superposition with the object? Note here a traditional opinion against QIMDS:

> Even if quantum mechanics was valid in the macroscopic world, it would be impossible in practice to detect QIMDS.

The argument runs as follows:[13]

> A macroscopic system has a large number of degrees of freedom. Accordingly, QIMDS must result as a sum of a large number of interference effects. Even if each of the effects separately produces such a clear-cut interference pattern as that in the Young-type experiment, the net result of summing these patterns would be the disappearance of any interference effect, because in general they are slightly out of phase with each other; that is, the peaks in one of the patterns are slightly displaced compared to those in another. In the example of Schrödinger's linear theater, the entire system in fact consists of the nucleus, the cat and the whole environment surrounding them, although the environment was disregarded for brevity in the preceding section. Thus, the number of degrees of freedom is infinite in effect, and interference will completely disappear.

[12] An acronym invented by A. J. Leggett.

[13] This argument is often followed by the statement "Therefore, AND is synonymous with OR for all practical purposes". As emphasized in the ensuing paragraph, however, this sort of *for all practical purposes argument* does not resolve the TAO problem.

The disappearance of interference just argued is called the *decoherence* due to the *environment*.[14] This argument will certainly apply to a majority of situations including a real cat. However, since there is no definite boundary between the microscopic and the macroscopic worlds, possibilities should remain for QIMDS to be detectable with a fairly macroscopic object, so long as quantum mechanics is valid. Even if the number of degrees of freedom is formally infinite, it might not be impossible to reduce the number of those which are harmful to QIMDS; an appropriate control (e.g. cooling to a sufficiently low temperature) over the environment could achieve the desired reduction.

These considerations led Leggett to propose the following program around 1980 (in what follows, "QM $\equiv$ quantum mechanics" and "MR $\equiv$ macrorealism"):

(0) Search both experimentally and theoretically for a macroscopic system which is expected, provided that QM remains valid, to show evidence of QIMDS under an appropriately controlled environment.

(1-0) If the experimental result can be interpreted, on the basis of QM, to show evidence of QIMDS, proceed to the step (2) below.

(1-1) If the experimental result unambiguously denies QIMDS against the quantum-mechanical prediction, then QM may be concluded to be invalid for a system as macroscopic as the one in question. Proceed to modify QM in the light of the negative result.

(2) Scrutinize, without invoking QM, whether or not the experimental result is compatible with MR.

(2-0) If it is, the experiment in question can not decide between QM and MR. Go back to the step (0) and refine the experiment.

(2-1) If it is not, one may conclude that MR is not valid but QM remains to be valid for a system as macroscopic as the one in question. Go back to the step (0) to continue the search for still more macroscopic candidates of QIMDS.

A comment is in order on the step (2). However uncomfortable one might feel with macroscopic superposition predicted by QM, it is illegitimate to reject QM on the basis of one's subjective feeling. A way to quantify this discomfort is to adopt MR, on the basis of which one may derive certain inequalities (Leggett–Garg inequalities[15]) to be satisfied by some measurable quantities (time-correlation functions). Furthermore, it can be shown that there are circumstances where QM

[14] The environment here includes many internal degrees of freedom of the macroscopic system in question (e.g. the cat) apart from those which are reserved to distinguish the macroscopically distinct states (e.g. to distinguish whether the cat is alive or dead). Of course, interference between microscopically different states can also be affected and often washed out by environment. The thesis of the above traditional argument is that the decoherence is inevitable and fatal in the case of QIMDS.

[15] They correspond and have the same mathematical structure as Bell's inequalities which are appropriate in the Einstein–Podolsky–Rosen problem, namely, testing QM against the local realism.

violates these inequalities. A sufficiently skilful experiment should be able to reveal this discrepancy, thereby deciding between QM and MR. See Chapter 9 for details.

Hidden in this program is the expectation that quantum mechanics will cease to be valid for a sufficiently macroscopic system.[16] The program itself, however, is independent both of this expectation and of a belief that QM is absolutely valid; it is a down-to-earth research program to enlarge the range of applicability of QM step-by-step from the microscopic world to the more macroscopic one. This program is to be called the *Leggett program*.[17]

1.3 What is meant by "macroscopic"?

1.3.1 Intuitive consideration

We have frequently used the word *macroscopic*. In order to avoid confusion, it is necessary to agree on its meaning as it is used in this book. As a starting point let us consider a Young-type experiment, where a pair of distinct states is involved; they represent a particle passing through either the upper or the lower slit. If the distance between the two slits is macroscopic (say 0.1 mm), one might be inclined to regard the pair of states as macroscopically distinct from each other, even if the system in question is a microscopic particle such as an electron or a neutron. What is macroscopic here, however, is a mere distance; the number of particles involved is only one. By contrast, the word "macroscopic" in this book refers to those situations where a large number of particles are involved, or to be more precise, the number N of the *dynamical degrees of freedom* is large.

The phrase "dynamical degrees of freedom" (hereafter to be abbreviated as degrees of freedom) should also be used with caution. Imagine a tennis ball passing through a wall without being squeezed. There can be no objection to calling this phenomenon a macroscopic tunneling; if the ball is regarded as a collection of atoms, the number of degrees of freedom involved in this phenomenon is comparable to the number of atoms. However, the same phenomenon can also be described by a single degree of freedom, namely, the center of mass. These two descriptions are related to each other by a transformation of variables and are mathematically equivalent. On the basis of this example, it could be argued that the number of degrees of freedom involved, which is not invariant under transformations of variables, cannot quantify the word "macroscopic"; a physical conclusion should not depend on the choice

[16] See A. J. Leggett, *The Problems of Physics*, Oxford University Press (1987), Chapter 5, Skeletons in the cupboard.

[17] The program announced by Felix Klein (1849–1925) on the occasion of his appointment to a professorship at the University of Erlangen, well known as Erlangen Programm, was so insightful that it played a long-lasting leading role in the synthesis of geometry. The Leggett program, which is still under development, will be regarded by the late twenty-first century physicists to have played a role in physics comparable to the Klein program in twentieth century mathematics.

of a mathematical description. This objection, which might look reasonable at first sight, may be disposed of as follows. Our intuition which regards the above phenomenon as a macroscopic tunneling does not rely on the number of *collective degrees of freedom* such as the center of mass but on the number of *microscopic degrees of freedom* such as the positions of constituent atoms. Collective degrees of freedom are the elite degrees of freedom which are singled out by rearranging the microscopic ones, all of which our intuition leads us to treat on an equal footing. In accordance to our intuition, we adopt the democratic way of counting the number of degrees of freedom, which in general is of the same order as the number of particles composing the system in question. Thus, the above objection is irrelevant. Of course, the number of constituent particles depends on what we count as fundamental particles; N neutrons may be counted as $3N$ quarks, for instance. However, the difference between N and $3N$ is irrelevant as well; the word "macroscopic" may be quantified only by orders of magnitude.

1.3.2 S-Cattiness

Until the 1970s, the phrase macroscopic quantum phenomena represented collectively superfluidity and superconductivity. For example, the phenomenon of liquid He creeping up along the wall of a glass and flowing out of it is both undoubtedly macroscopic and explicable only in terms of quantum mechanics. In this kind of phenomena, however, microscopic interference at the level of one (or two) particles is enhanced by virtue of cooperation of many particles (or, many pairs of particles), resulting in an effect of macroscopic scale; QIMDS is not involved. Today they are often called *macroscopic quantum phenomena of the first kind* and are distinguished from *macroscopic quantum phenomena of the second kind* in which QIMDS is involved.

Let us elaborate on the difference between the two kinds. Consider, as a typical example of the first kind, superfluid ^{4}He flowing out of a glass. The wavefunction representing its state is conceptually of the form:[18]

$$\prod_{k=1}^{N}\{\psi(\mathbf{r}_k-\mathbf{d}/2)+\psi(\mathbf{r}_k+\mathbf{d}/2)\}, \tag{1.3.1}$$

where $\mathbf{r}_k$ denotes the position of the k-th of the N atoms composing the liquid ^{4}He, $\mathbf{d}/2$ that of the center of the glass, and $-\mathbf{d}/2$ a position outside the glass (Fig. 1.1). Equation (1.3.1) implies that each of the atoms is in a state of superposition $\psi(\mathbf{r}-\mathbf{d}/2)+\psi(\mathbf{r}+\mathbf{d}/2)$ and that the state of the entire liquid is their product.

[18] It should be noted that the following expression is merely schematic. Bose–Einstein condensation by itself is not enough to give rise to superfluidity; interaction among atoms is necessary for a state roughly of the following form to be kept stable.

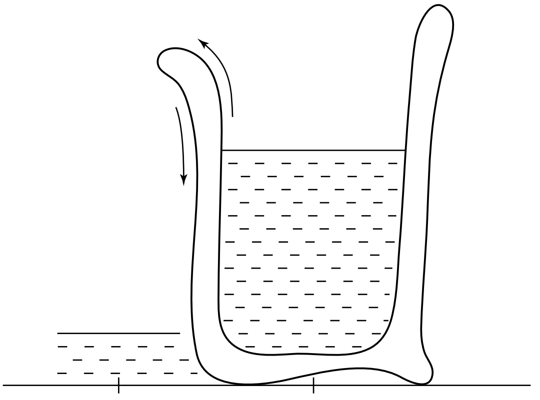

Fig. 1.1. Superfluid ^{4}He flowing out of a glass (macroscopic quantum phenomena of the first kind).

It shows that all the atoms are in one and the same "one-particle state". This is nothing but the situation called the *Bose–Einstein condensation*, which realizes the cooperation of many particles as mentioned above.

The representative of macroscopic quantum phenomena of the second kind is, of course, the S-Cat.[19] Unfortunately, however, quantum states representing a real cat are hard to write down. Consider instead a ball passing through double slits. If the ball is assumed to behave as a rigid body, its state is represented conceptually by a wavefunction of the form

$$\prod_{k=1}^{N} \psi_k(\mathbf{r}_k - \mathbf{d}/2) + \prod_{k=1}^{N} \psi_k(\mathbf{r}_k + \mathbf{d}/2), \tag{1.3.2}$$

where $\mathbf{r}_k$ denotes the position of the k-th of the N atoms composing the ball, and $\pm\mathbf{d}/2$ are the positions of the upper and the lower slit, respectively. In this wavefunction the first and the second terms represent the state in which the center of the ball is located near $\pm\mathbf{d}/2$, respectively (Fig. 1.2).

Each ψ in Eq. (1.3.2) carries the subscript k, which takes account of the fact that each atom occupies a different position within the ball. This is not important, however, in the comparison of (1.3.1) and (1.3.2). What is essential is the difference in the order of the sum (i.e. linear combination) and the product; the sum is the first to be taken in (1.3.1), whereas the product is the first in (1.3.2).

In what follows, a macroscopic quantum phenomenon of the second kind is to be simply called a macroscopic quantum phenomenon and abbreviated as MQP.

[19] The present author has not found out why Schrödinger invoked a cat instead of a dog for instance. Perhaps, a cat is more suitable for germinating a sense of strange uneasiness. Also, S-Cat is reminiscent of Carroll's cat in *Alice in Wonderland*.

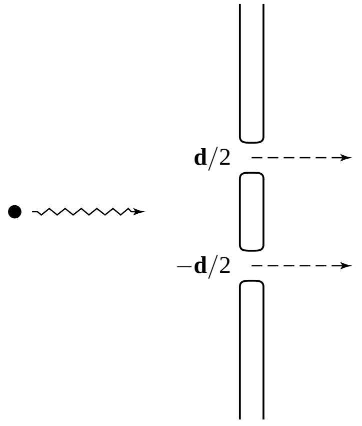

Fig. 1.2. A ball passing through double slits (MQP).

It is desirable to have a measure to quantify the extent to which a given MQP is macroscopic. Such a measure is to be called *S-Cattiness* and denoted by **D**. It may be defined[20] roughly as:

> the maximum number of those democratically-counted degrees of freedom which are involved in an irreducible linear combination,

where an irreducible linear combination is a superposition that can not be factorized into linear combinations involving fewer degrees of freedom. According to this definition, $\mathbf{D} \sim 1$ in the state (1.3.1), $\mathbf{D} \sim N$ in the state (1.3.2), and so on. The larger **D** becomes, the closer the given QIMDS will be to the full-fledged S-Cat and the harder to detect experimentally.

1.4 Macroscopic quantum tunneling

1.4.1 Leggett program and macroscopic quantum tunneling

In the previous section, we have briefly described the traditional argument against QIMDS. Let us examine it in more detail. Consider for simplicity a system of N spins $\{\hat{\mathbf{s}}^{(1)}, \hat{\mathbf{s}}^{(2)}, \ldots, \hat{\mathbf{s}}^{(N)}\}$ with each of the spins being of magnitude $1/2$, and suppose that it has been prepared in the following state $|\Psi\rangle$ (see Appendix D for the notation concerning spin):

$$|\Psi\rangle := c_+|\Psi_+\rangle + c_-|\Psi_-\rangle, \tag{1.4.1}$$

$$|\Psi_\pm\rangle \equiv \prod_{k=1}^{N} \left| \pm \frac{1}{2} \right\rangle^{(k)}, \quad |c_+|^2 + |c_-|^2 = 1, \tag{1.4.2}$$

[20] A more precise mathematical definition has been given by Leggett in Ref. [1] in the bibliography, where the measure was called the *disconnectivity*. Later it was also informally called cattiness.

where

$$\hat{s}_3^{(k)}\left| \pm \frac{1}{2}\right\rangle^{(k)} = \pm\frac{1}{2}\left| \pm \frac{1}{2}\right\rangle^{(k)}. \tag{1.4.3}$$

This state $|\Psi\rangle$ has the structure in common with the state (1.3.2). Accordingly its S-Cattiness is of order N. How can one detect an interference effect predicted by the superposition (1.4.1)? To answer this question, it is helpful to review the Young-type experiment. Pay attention to any one of the counters and suppose that its center is situated at $\mathbf{x}_0$ and that it occupies a spatial region $C(\mathbf{x}_0)$. Then the probability $P(\mathbf{x}_0)$ for the counter to click at time T is given roughly as

$$P(\mathbf{x}_0) = \int_{C(\mathbf{x}_0)} \mathrm{d}\mathbf{x}\, |\psi(\mathbf{x})|^2, \quad \psi(\mathbf{x}) \equiv \langle \mathbf{x}|\psi\rangle, \tag{1.4.4}$$

where $|\psi\rangle$, being the state of the particle at the time T, is a superposition of $|\psi_\pm\rangle$ which describe the particle having passed through the upper and the lower slit, respectively:

$$|\psi\rangle = |\psi_+\rangle + |\psi_-\rangle. \tag{1.4.5}$$

By use of the *projector*[21] onto the region $C(\mathbf{x}_0)$ defined by

$$\hat{\Pi}_{\mathbf{x}_0} := \int_{C(\mathbf{x}_0)} \mathrm{d}\mathbf{x}\, |\mathbf{x}\rangle\langle\mathbf{x}|, \tag{1.4.6}$$

the above probability may be rewritten as

$$\begin{aligned} P(\mathbf{x}_0) &= \langle\psi|\hat{\Pi}_{\mathbf{x}_0}|\psi\rangle \\ &= \langle\psi_+|\hat{\Pi}_{\mathbf{x}_0}|\psi_+\rangle + \langle\psi_-|\hat{\Pi}_{\mathbf{x}_0}|\psi_-\rangle + 2\Re\langle\psi_+|\hat{\Pi}_{\mathbf{x}_0}|\psi_-\rangle. \end{aligned} \tag{1.4.7}$$

This is just a roundabout rephrasing of the elementary result; the last term is what is called the interference term:

$$\langle\psi_+|\hat{\Pi}_{\mathbf{x}_0}|\psi_-\rangle = \int_{C(\mathbf{x}_0)} \mathrm{d}\mathbf{x}\, \psi_+^*(\mathbf{x})\psi_-(\mathbf{x}), \quad \psi_\pm(\mathbf{x}) \equiv \langle\mathbf{x}|\psi_\pm\rangle. \tag{1.4.8}$$

The lesson to be learnt from this example is the following:

> There exists an operator (i.e. $\hat{\Pi}_{\mathbf{x}_0}$) such that (a) it corresponds to the experimental procedure of counting a particle, and (b) its off-diagonal element (i.e. (1.4.8)) does not vanish.

It is this condition that renders the Young-type interference effect detectable.

This consideration may be generalized to conclude that the following condition is necessary for the interference between $|\Psi_\pm\rangle$ in the superposition (1.4.1) to be detectable:

[21] Abbreviation of *projection operator.*

There exists an operator $\hat{O}$ such that (a) it corresponds to the actual measurement, and (b)

$$\langle \Psi_+ | \hat{O} | \Psi_- \rangle \neq 0. \tag{1.4.9}$$

If it were not for an operator with this property, the superposition (1.4.1) would be indistinguishable from the *mixture*[22] represented by the following density operator $\hat{\rho}$:

$$\hat{\rho} = |c_+|^2 |\Psi_+\rangle\langle\Psi_+| + |c_-|^2 |\Psi_-\rangle\langle\Psi_-|. \tag{1.4.10}$$

For example, the operator $\hat{O} \equiv \hat{s}_1^{(1)}\,\hat{s}_1^{(2)}$ does not satisfy the above condition:

$$\langle \Psi_+ | \hat{O} | \Psi_- \rangle = \prod_{k=1}^{2} {}^{(k)}\!\left\langle \frac{1}{2} \middle| \hat{s}_1^{(k)} \middle| -\frac{1}{2} \right\rangle^{(k)} \quad \prod_{k=3}^{N} {}^{(k)}\!\left\langle \frac{1}{2} \middle| -\frac{1}{2} \right\rangle^{(k)} = 0. \tag{1.4.11}$$

Hence, no measurement of the physical quantity represented by this operator can reveal the desired interference effect. Any operator of the form of a product of $N - 1$ spins or less will not do either. A product involving all of the N spins is needed. However, an experimental realization of such an operator would be impossible for a system with a large N (say, $N \sim 10^{10}$).

It might seem hopeless to dispute this argument. Fortunately, however, any physical system is endowed with a time-evolution operator $\hat{U}(t)$. This operator contains all the powers (i.e. $\hat{H}$, $\hat{H}^2$, $\hat{H}^3, \ldots$) of the Hamiltonian $\hat{H}$, which in turn contains all the degrees of freedom relevant to the system. Therefore, even if an operator $\hat{O}$ contains only a few degrees of freedom, its time evolution

$$\hat{O}(t) \equiv \hat{U}^\dagger(t)\hat{O}\hat{U}(t) \tag{1.4.12}$$

can have the property

$$\langle \Psi_+ | \hat{O}(t) | \Psi_- \rangle = \langle \Psi_+(t) | \hat{O} | \Psi_-(t) \rangle \neq 0\,, \quad |\Psi_\pm(t)\rangle \equiv \hat{U}(t)|\Psi_\pm\rangle. \tag{1.4.13}$$

Thus, although an experimenter can not prepare a desired operator, nature can; it is sufficient for an experimenter to measure $\hat{O}$ for the system in the state $|\Psi_\pm(t)\rangle$ instead of the original state $|\Psi_\pm\rangle$. Detection of QIMDS will be possible only if this naturally endowed property is successfully exploited. To start with, of course, $|\Psi_+\rangle$ and $|\Psi_-\rangle$ should be macroscopically distinct from each other. This condition may be guaranteed if a sufficiently large potential barrier separates these two states. In order to exploit (1.4.13), however, $|\Psi_+\rangle$ should be able to evolve in time to $|\Psi_-\rangle$

[22] It is often the case that a mixture is called a "mixed state" in contrast to a "pure state" which is synonymous with the phrase "quantum state" as used in this book. Readers are warned not to confuse the word "mixed" in "mixed state" with the word "mixing" used in "mixing angle", "s–p mixing" and so on. In these phrases, "mixing" implies a linear combination; for instance, an "s–p-mixed state" ($\equiv$ "state with s–p mixing") is a "pure state".

and vice versa. Both of these requirements may be met by a situation where the macroscopic system is able to undergo quantum tunneling.

For this reason, macroscopic quantum tunneling, the title of this book, occupies the central position in the Leggett program. To recapitulate, its significance lies in the following features[23] which are obviously interrelated:

- It is genuinely quantum mechanical. Although quantum-mechanical effects are involved in one way or another in any physical phenomena, they may be identified unambiguously in a phenomenon without classical-mechanical analogue. Quantum tunneling is a typical case.
- It guarantees macroscopic distinctness. Two states, which can evolve into each other only via quantum tunneling, are prohibited to do so classically. Hence, an experimenter can unambiguously distinguish them with a macroscopic parameter by preparing them in a nearly classical situation.

Before closing this subsection, we should dispose of an opinion, which insists that it is impossible to detect quantum tunneling in a macroscopic system. The argument goes as follows:[24]

> In general the probability of tunneling is proportional to $\exp(-2S/\hbar)$, where S is the action characteristic of the tunneling in question.[25] Let N be the number of degrees of freedom involved in the tunneling, then
>
> $$S/\hbar \sim N \gg 1. \tag{1.4.14}$$
>
> Thus, the probability of tunneling, which decreases exponentially as N increases, practically vanishes for a macroscopic system. Furthermore, a system may be said to be macroscopic with rigor only in the limit
>
> $$N \to \infty. \tag{1.4.15}$$
>
> In this limit, the probability of tunneling vanishes rigorously.

This opinion would certainly be right if the word “macroscopic” is defined by the condition (1.4.14). Although this condition is expected to hold in many cases, it does not necessarily agree with what we mean intuitively by “macroscopic”. Even if many particles are involved and the two states in question are macroscopically distinct, the potential barrier separating the two states might not necessarily be large. Such a situation as

$$S/\hbar = \mathcal{O}(1) \tag{1.4.16}$$

[23] These are necessary for MQP in general, but quantum tunneling is not. A candidate of MQP without tunneling is a superposition of macroscopically distinct coherent states of light (i.e. electromagnetic field), which, however, is outside the scope of this book.

[24] This is another example of a *for all practical purposes* argument. cf. Footnote 13.

[25] This action is proportional roughly to the square root of the area of the potential barrier.

could be realized; even if S is formally proportional to the large number N, the coefficient of proportionality could be controlled to be small. The situation of interest in this book is not the mathematical limit (1.4.15) but a realistic circumstance where the condition (1.4.16) holds in spite of $N \gg 1$.

1.4.2 Classification of macroscopic quantum tunneling: MQC and MQT

Given a macroscopic system, let R be the set of relevant macroscopic degrees of freedom (i.e. collective degrees of freedom) to describe its quantum tunneling. The number of such degrees of freedom need not be one. For example, in the case of the ball discussed in the preceding section, R is the center-of-mass position which is a three-component vector, that is, R consists of three degrees of freedom. The entire number of degrees of freedom of the given macroscopic system may be rearranged into R and the rest. For simplicity, suppose that R consists of a single degree of freedom and that its quantum-mechanical behavior is governed by the Schrödinger equation of the same form as that for a particle:

$$\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}_{\mathcal{S}}|\psi(t)\rangle, \tag{1.4.17}$$

$$\hat{H}_{\mathcal{S}} := \tfrac{1}{2M}\hat{P}^2 + U(\hat{R}), \tag{1.4.18}$$

$$[\hat{R},\ \hat{P}] = \mathrm{i}\hbar, \tag{1.4.19}$$

where M is a positive constant (effective inertial mass), $\hat{P}$ is the momentum operator conjugate to the position operator $\hat{R}$, and $U(\hat{R})$ is an appropriate potential to be specified later. Hereafter, the macroscopic degrees of freedom R is to be called the *macrosystem* as distinguished from the original macroscopic system as a whole; the set of the remaining degrees of freedom is called the *environment* (i.e. the environment for R). Accordingly the above $\hat{H}_{\mathcal{S}}$, which refers to the macrosystem alone, is to be called the *macrosystem Hamiltonian*.[26] The Hamiltonian for the entire system consists of three parts: the macrosystem Hamiltonian, the part referring to the environment alone, and the part specifying the interaction of the macrosystem with the environment. It is, therefore, not obvious whether the fundamental quantum-mechanical description can be reduced to the form (1.4.17)–(1.4.19) which is closed with respect to R. This issue is to be discussed in detail in the fourth and subsequent chapters. For the moment, we assume that the closed form (1.4.17)–(1.4.19) is valid; the aim of this subsection and the following two chapters is to achieve a conceptual understanding of macroscopic quantum tunneling.

Macroscopic quantum tunneling is grossly classified into MQC (macroscopic quantum coherence) and MQT (macroscopic quantum tunneling in the narrow

[26] The subscript $\mathcal{S}$ stands for the system, namely, not the entire macroscopic system but the macrosystem which is of primary interest.

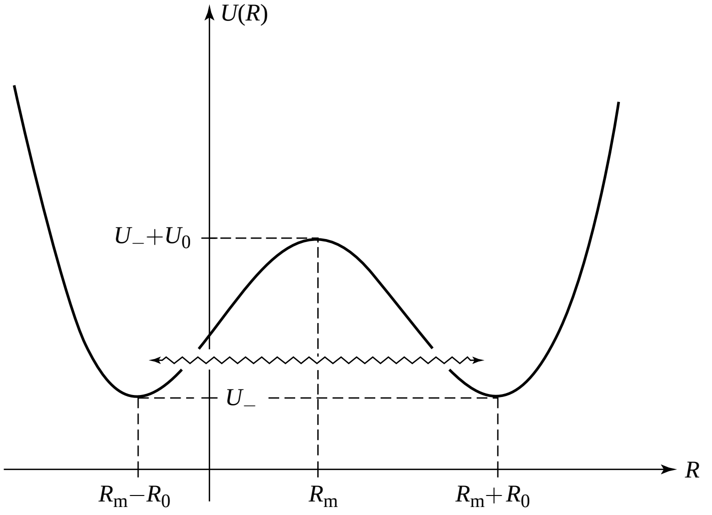

Fig. 1.3. MQC-situation: symmetric double well.

sense). In a phenomenon classified as MQC, a macrosystem R oscillates in time between macroscopically distinct states. Such an oscillation may be said to be a temporal counterpart of the Young-type interference pattern that is spatial (see Section 2.2); QIMDS is involved directly. Thus, MQC is a convenient substitute for Young-type experiments, which are presumably difficult for a macrosystem. As mentioned in the preceding subsection, one need not construct a delicate double-slit apparatus to produce a desired macroscopic superposition, which may be prepared naturally by the time evolution of the macrosystem itself. A typical potential giving rise to a MQC-situation is the symmetric double well (Fig. 1.3). The central barrier is supposed to be so large that the inter-well distance $2R_0$ is a macroscopic quantity and that the macrosystem initially localized in the left well cannot go over to the right well classically but can do so only via quantum tunneling. In this way, the left state (i.e. the state in which the macrosystem is localized in the left well) is guaranteed to be macroscopically distinct from the right state. The oscillation of the macrosystem between the two wells is analogous to the classical-mechanical resonance of a pair of coupled pendulums or tuning forks with a common eigenfrequency. Hence, MQC may as well be called *macroscopic quantum resonant oscillation*.

In a phenomenon classified as MQT, on the other hand, a macrosystem R tunnels only once from a state to another with the latter being macroscopically distinct from the former. A typical potential giving rise to a MQT-situation is the *bumpy slope*[27] (Fig. 1.4), where the barrier is supposed to be sufficiently large as in the MQC-situation. If the macrosystem is initially localized in the well, it cannot escape towards the right of the barrier classically but can do so only via quantum tunneling.

[27] A better nickname may be desirable.

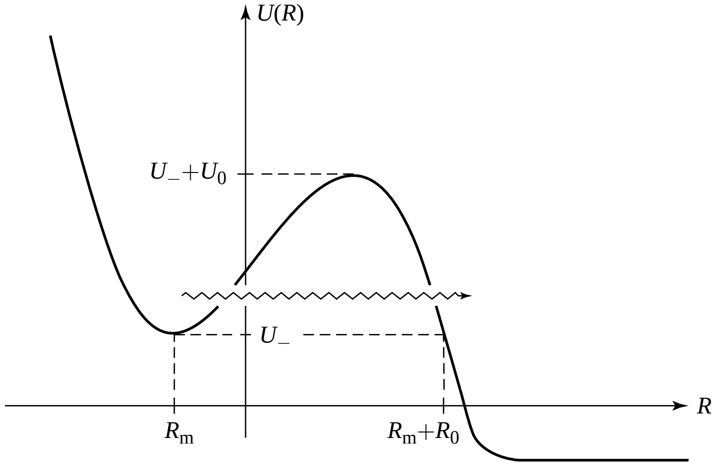

Fig. 1.4. MQT-situation: bumpy slope.

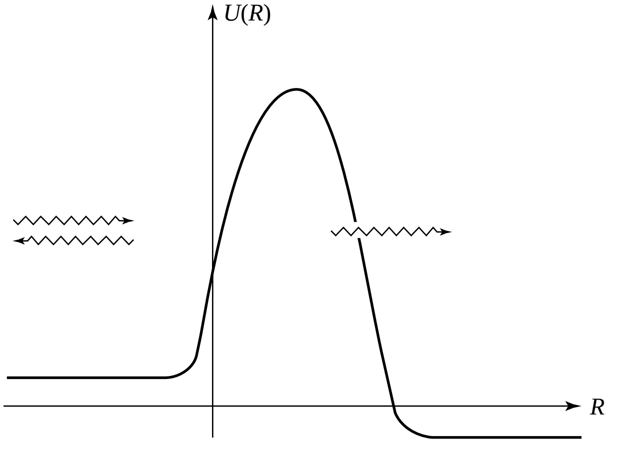

Fig. 1.5. Free-tunneling situation: simple barrier.

This situation is analogous to the α-decay of a nucleus, for instance; the initial state is *metastable*, and the well around $R = R_m$ is called the *metastable well* . Thus, MQT may as well be called *macroscopic quantum decay* of a *metastable state*. As in the case of the MQC-situation, the large barrier guarantees that this metastable state (i.e. the state in which the macrosystem is localized in the metastable well) is macroscopically distinct from the decayed state (i.e. the state in which the macrosystem has escaped to the right of the barrier). Although MQT cannot provide direct evidence for QIMDS (see Chapter 9), its detection would dramatically demonstrate that a macrosystem can exhibit a quantum-mechanical behavior without a classical analogue.

Perhaps the most familiar type of quantum tunneling is the situation depicted in Fig. 1.5, which may be called the free tunneling, since no force acts on the particle either before or after the passage of the barrier.[28] It seems, however, difficult to set up a corresponding situation for a macrosystem.

Incidentally, MQT is often described as a free tunneling followed by several oscillations inside the metastable well. In a situation where this popular account is valid, however, the probability of tunneling would be too small for MQT to be detected. It is therefore advisable to distinguish MQT from free tunneling.

Exercises

Exercise 1.1. Devise a mathematically precise definition of the S-Cattiness (cf. Ref. [1]).

Exercise 1.2. Given a state $|\Psi\rangle$, the density operator $\hat{\rho}_\Psi$ representing it is defined by the property

$$\mathrm{Tr}(\hat{\rho}_\Psi \hat{A}) = \langle\Psi|\hat{A}|\Psi\rangle \quad \text{for an arbitrary operator } \hat{A}, \tag{1.4.20}$$

or equivalently as

$$\hat{\rho}_\Psi := |\Psi\rangle\langle\Psi|. \tag{1.4.21}$$

For $|\Psi\rangle$ given by (1.4.1), show that

$$\hat{\rho}_\Psi = \hat{\rho} + c_+ c_-^* |\Psi_+\rangle\langle\Psi_-| + c_- c_+^* |\Psi_-\rangle\langle\Psi_+|, \tag{1.4.22}$$

where $\hat{\rho}$ represents the mixture as given by (1.4.10).

Exercise 1.3. Design a Young-type experiment with C_{60}; estimate the required orders of magnitude of the inter-slit separation, the distance between the double slits and the array of counters, and so on (cf. the actual experiment in Ref. [10] in the bibliography).

[28] Hence, the particle behaves asymptotically as a free particle.