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CHAPTER 1

The structure of the nucleus

By the end of this chapter you should be able to:

1. recall that the nuclei of atoms consist of smaller particles called nucleons (protons and neutrons);
2. recall that radioactive decay, nuclear fission and nuclear fusion all involve changes to the nuclei of atoms;
3. interpret and construct nuclear equations using symbols for atomic nuclei and subatomic particles with superscripts/subscripts for nucleon (mass) numbers (*A*), proton (atomic) numbers (*Z*) and electrical charges;
4. sketch and interpret a graph showing how the radius of an atomic nucleus varies with the number of nucleons it contains (*A*);
5. calculate the radius *r* of a nucleus using the relationship $r = r_0 A^{1/3}$, where r_0 is the radius of a hydrogen-1 nucleus;
6. estimate the density of nuclear matter;
7. use Coulomb's law to determine the electrostatic force of repulsion, and Newton's law to determine the gravitational force of attraction between two adjacent protons, and hence appreciate the need for a short-range attractive force between nucleons holding atomic nuclei together;
8. recall the nature of the strong force (interaction) between nucleons, and sketch and interpret a graphical representation of how this varies with the distance between nucleons;
9. appreciate that the stability of nuclei depends on the balance between attractive and repulsive forces between nucleons;
10. understand the concept of binding energy and appreciate the relationship between binding energy and stability.

Nuclear structure and nuclear reactions: a review of basic ideas

You have already looked at the basic structure of atomic nuclei and at various types of nuclear reaction – nuclear fission, nuclear fusion and radioactive decay – in an earlier module. Before looking in more detail at how and why these nuclear reactions occur, you may find it helpful to be briefly reminded about some basic ideas from that earlier module.

Atomic structure

An atom consists of a small, dense **nucleus** that makes up almost all of the mass of the atom.

Surrounding the nucleus is a much larger volume of space. **Electrons**, which have a negative electrical charge −*e* but very little mass, move about in this space.

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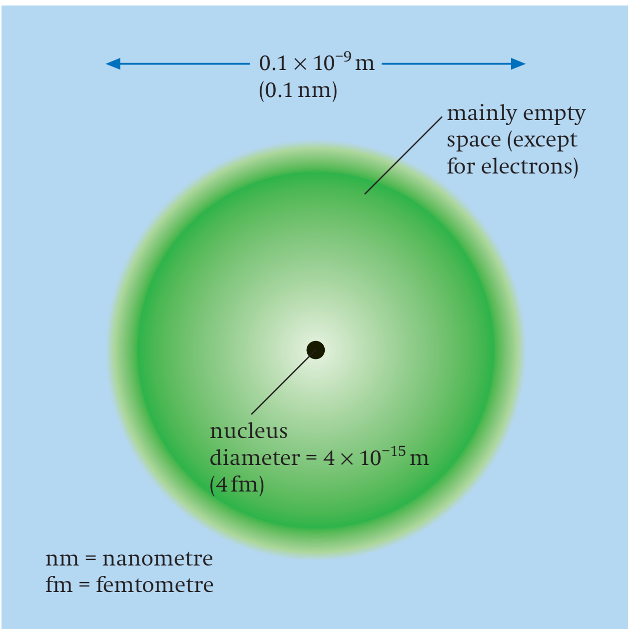


A typical atom.

SAQ 1.1

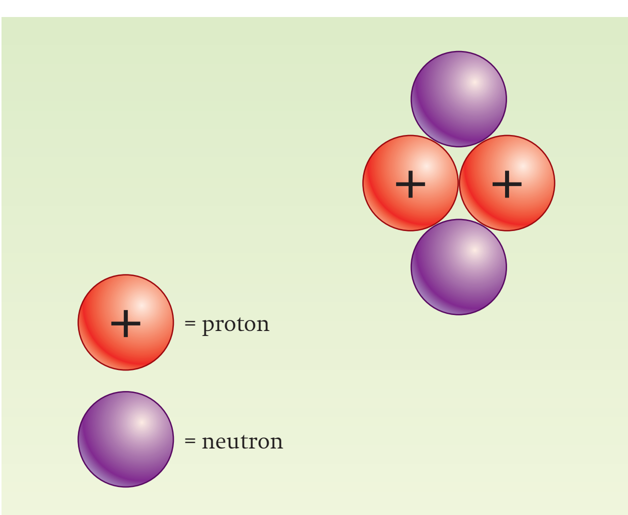
What fraction of the diameter of the atom shown in the above diagram is the diameter of its nucleus?

Atomic nuclei are themselves made up of smaller particles called **nucleons**. There are two types of nucleon:

	Mass (u)*	Charge
proton	about 1	+*e*
neutron	about 1	0

[*See *box 1A* on page 4 for an explanation of this unit of mass.]

The diagram below shows the nucleus of a helium atom.



A convenient way of representing the nucleus of this atoms is as follows:

This is the mass or nucleon number (*A*) (the total number of nucleons) → 4
This is the atomic or proton number (*Z*) (the number of protons) → 2

$${}^{4}_{2}\text{He}$$

All the atoms of a particular element have the same number of protons so all the atoms of that element have the same value for *Z*.

Atoms of the same element may, however, have different numbers of neutrons and so have different values for *A*. Different kinds of atom of the same element are called **nuclides** (or isotopes) of that element. For example, the commonest type of carbon atom is ${}^{12}_{6}\text{C}$, but another type of carbon atom is ${}^{14}_{6}\text{C}$.

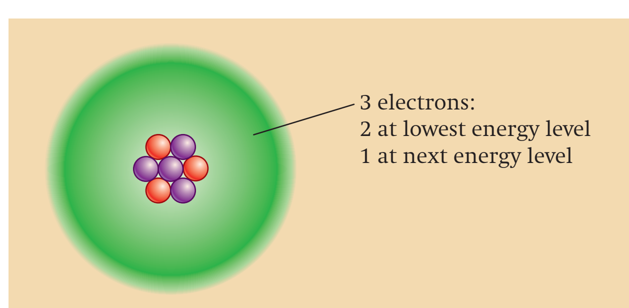
Since the nucleon number of an atom tells you the total number of nucleons in its nucleus, the number of neutrons in an atom can be worked out using the relationship:

neutrons = nucleons − protons
i.e. $N = A - Z$

SAQ 1.2

a In what way are the nuclei of ${}^{12}_{6}\text{C}$ and ${}^{14}_{6}\text{C}$ the same?
b In what way are they different?

Atoms have no overall electrical charge. The positive charges of the protons in the nucleus are exactly balanced by an equal number of electrons in the space surrounding the nucleus. The number of electrons in atoms and the way that these electrons are arranged in energy levels, or shells, determines the way that they combine with different atoms in **chemical reactions**.



A ${}^{7}_{3}\text{Li}$ atom. *Note*: the nucleons are not to the same scale as the electron cloud.

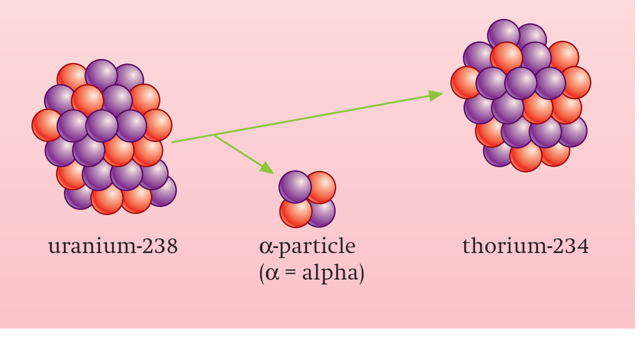
The nuclei of atoms can change, or be made to change, in various ways. Changes to the nuclei of atoms are called **nuclear reactions**. Three important types of nuclear reaction are:

- **radioactive decay**;
- **nuclear fission**;
- **nuclear fusion**.

Radioactive decay

Radioactive decay occurs when the unstable nucleus of an atom *spontaneously* changes into a different nuclide, emitting radiation as it does so. Because it emits radiation, the original unstable nucleus is called a **radionuclide**.

For example:

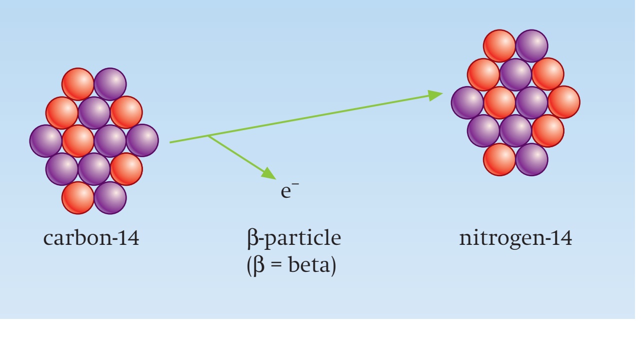


This decay can be described using a nuclear equation:

$$^{238}_{92}\text{U} \rightarrow {}^{4}_{2}\text{He} + {}^{234}_{90}\text{Th}$$

Notice that the totals of the *A* numbers and of the *Z* numbers for the products of this reaction are exactly the same as the *A* and *Z* numbers of the original radionuclide. In other words, the *A* and *Z* numbers on each side of the nuclear equation *balance*.

$^{14}_{6}$C is a radionuclide. It decays as shown below:



The nuclear equation for this decay is:

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N}$$

Notice that because the electron (the β-particle) has hardly any mass compared to a nucleon, it is given a mass (nucleon) number of zero. Also because the electron has the opposite electrical charge to a proton it is given a proton number of −1. This means that the *A* and *Z* numbers on both sides of the nuclear equation still balance.

SAQ 1.3

The chemical reactions of $^{12}_{6}$C and $^{14}_{6}$C are the same but only one isotope is radioactive. Suggest why.

SAQ 1.4

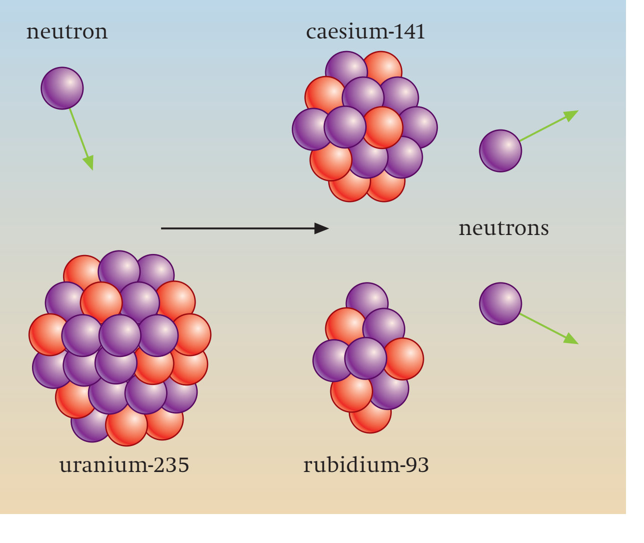
a Give another name for (i) an α-particle, and (ii) a β-particle.

b Describe, in terms of protons, neutrons and electrons, what happens to the nucleus of a carbon-14 atom when it decays.

Nuclear fission

Nuclear fission occurs when a large atomic nucleus splits into two smaller nuclei. This can happen spontaneously but is usually **induced** by bombarding atomic nuclei with neutrons.

For example:



This fission can be shown as a nuclear equation:

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{55}\text{Cs} + {}^{?}_{?}\text{Rb} + 2\,{}^{1}_{0}\text{n}$$

Notice that since neutrons are not protons and, unlike electrons, have no electrical charge, they have a *Z* number of zero.

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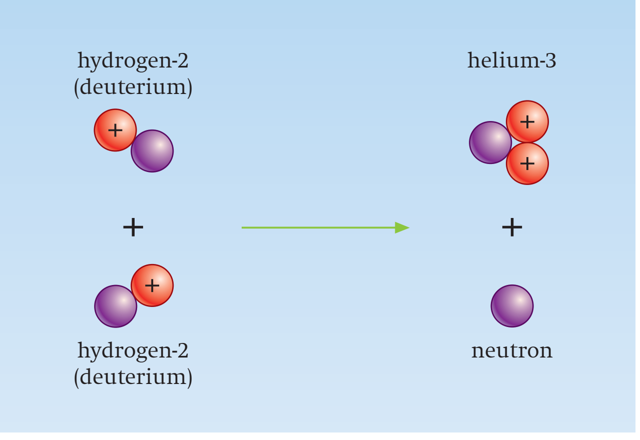
SAQ 1.5

Calculate, from the previous nuclear equation, the nucleon number (A) and the proton number (Z) of the rubidium atom.

Nuclear fusion

Nuclear fusion occurs when two smaller atomic nuclei join together (fuse) to make a single, larger nucleus. This can occur only when the nuclei which are to fuse have very high energies (i.e. are at a very high temperature).

For example:



This fusion can be shown as a nuclear equation:

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n}$$

As always, the *A* numbers and *Z* numbers on both sides of the nuclear equation balance.

SAQ 1.6

Two deuterium nuclei can also fuse to produce a hydrogen-3 nucleus and a proton. Write a nuclear equation for this reaction.

In all three types of nuclear reaction there is a net transfer of energy to the surroundings. Per atom, this energy transfer is very large compared to the energy transferred to the surroundings in an exothermic *chemical* reaction such as burning hydrogen. The transfer of energy to the surroundings during a nuclear reaction involves a loss of mass by the atomic nuclei themselves (see *box 1A*).

The fact that the nuclei produced by nuclear reactions have less mass/energy than the reacting nuclei makes them more stable. The situation is similar to that of a book which has fallen from a shelf to the floor: it then has less gravitational potential energy but is in a more stable position. In order to reverse a nuclear reaction we would need to supply energy, just as we would to put the book back on its shelf. Because the energy that is

Box 1A Units and formulae

You will have met all the following pieces of information in earlier modules. You will, however, find them useful (e.g. for calculations) at many points in this book.

For convenience, the very small masses of atoms, atomic nuclei and nucleons are often given in **unified atomic mass units** (u). This unit is based on the mass of an atom of $^{12}_{6}\mathrm{C}$, which is defined as having a mass of exactly 12 u.

The mass of each proton and neutron is therefore approximately 1 u. It should, however, be noted that:

- the mass of an isolated neutron or proton is slightly greater than 1 u;
- the mass of an isolated neutron is slightly greater than the mass of an isolated proton;
- the masses of neutrons and protons within atomic nuclei are less than the total of the separate masses of the protons and neutrons they contain. (The difference in mass is a measure of the binding energy of the nucleus.)

	Mass (u)	Mass (kg × 10^{-27})
proton	1.0073	1.673
neutron	1.0086	1.675

The electrical charge on an electron ($-e$) is -1.602×10^{-19} C (coulomb).

When this charge moves through a potential difference of 1 V (volt) the amount of work done (energy transferred) is 1.602×10^{-19} J (joules).

This very small amount of energy is also called an **electron-volt** (**eV**). It is a convenient unit of energy to use when considering nuclear reactions:

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J} \approx 1.6 \times 10^{-19}\ \mathrm{J}$$

Occasionally, slightly larger units are more convenient, such as:

$$1\ \mathrm{keV} = 1 \times 10^{3}\ \mathrm{eV}$$
$$1\ \mathrm{MeV} = 1 \times 10^{6}\ \mathrm{eV}$$

A transfer of energy, ΔE, is always accompanied by a change in mass, Δm.

The change in mass which accompanies a transfer of energy is given by the relationship:

$$\Delta E = \Delta m\, c^2$$

where c is the speed of light ($2.998 \times 10^{8}\ \mathrm{m\,s^{-1}}$).

Using this relationship, it can be shown that:

$$1\ \mathrm{u} = 931.5\ \mathrm{MeV}$$

Box 1A continued

SAQ 1.7

a Calculate the energy equivalent, in J, of 1 kg of mass.

b How does your answer to **a** compare with the 5×10^7 J of chemical energy released by burning 1 kg of petrol?

c Use your result from **a** to calculate the energy equivalent, in J, of 1 u of mass.

d Convert your answer to **c** into eV.

e Express your answer to **d** in MeV.

f Express your answer to **e** as a round figure.

SAQ 1.8

A thorium-232 nucleus decays to a radium-228 nucleus by emitting an α-particle. The energy released by the decay is 4.08 MeV.

The mass of a thorium-232 nucleus is 232.038 u and the mass of an α-particle is 4.003 u.

Calculate the mass of a radium-228 atom.

lost by nucleons in a nuclear reaction would have to be supplied to make the nucleons move back to their original, less stable position, we can regard the lost energy as *holding* the nucleons in the more stable position. This is why physicists say that the **binding energy** of the nucleons is greater at the end of a nuclear reaction than it was at the start. This idea is explained in more detail on pages 11–13.

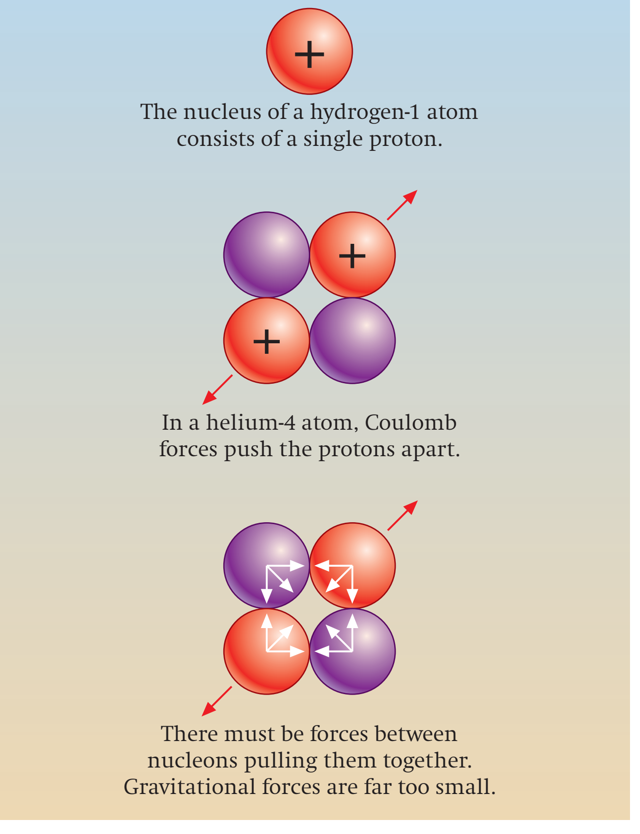
The review of basic ideas about nuclear structure and nuclear reactions is now complete. We next need to develop these ideas further so that we can explain more fully what is happening in nuclear reactions and why such reactions happen (or don't happen). In order to do this we first need to consider the forces that act between the nucleons from which atomic nuclei are made.

What force holds atomic nuclei together?

There is no problem in understanding why the electrons in an atom do not escape from the space surrounding the nucleus. There is an **electrostatic** (**Coulomb**) **force** of attraction between the positively charged nucleus and the negatively charged electrons (see page 27 of *Physics 2*). (What *does* need to be explained is why the electrons do not simply spiral into the nucleus.)

There is, however, a very serious problem with the simple model of an atomic nucleus. The nucleons inside an atomic nucleus are either positively charged protons or electrically neutral neutrons. Electrical forces cannot, therefore, explain how the nucleons in a nucleus are held together. Indeed, because the Coulomb force between protons is a *repulsive* force, we need to explain why such forces don't blow atomic nuclei apart.

This isn't a problem at all, of course, for the nucleus of an ordinary hydrogen atom because this consists of just a single proton (*figure 1.1*). The nuclei of all other elements, however, contain two or more protons and the same number of neutrons or slightly more neutrons than protons. Since there are forces of *repulsion* between each of the protons in these nuclei, there must be some other force acting between nucleons and *attracting* them to each other.



● **Figure 1.1** What force holds atomic nuclei together?

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SAQ 1.9

As well as hydrogen-1 atoms, there are the isotopes hydrogen-2 (also called deuterium) and hydrogen-3 (also called tritium).

a What nucleons do the nuclei of these isotopes contain?

b Why do the nucleons inside the nuclei of these atoms have no tendency to fly apart?

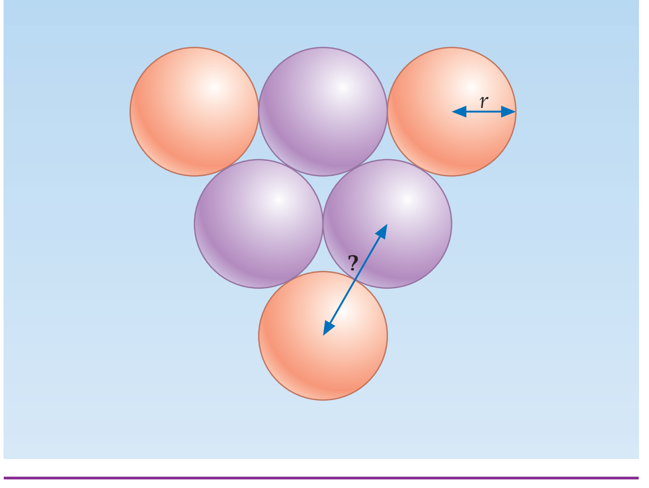
There will, of course, be a **gravitational force** of attraction between nucleons just as there is between any bodies that have mass. The size of such gravitational forces is, however, far too small to balance the electrostatic forces of repulsion. In fact, as we shall see later, gravitational forces are 10^{36} (i.e. a million million million million million million) times too small!

So some other force of attraction between nucleons must also be acting. This force, which acts between two protons, between two neutrons or between a proton and a neutron, is called the **strong force**.

To understand more about this strong force and to be able to calculate the sizes of the various forces that act between the nucleons inside an atomic nucleus, we need information about the sizes of nucleons and the distances between them. We can obtain this information by measuring the radii of atomic nuclei.

SAQ 1.10

Assuming that nucleons are all the same size, with radius *r*, and are tightly packed, what distance apart will they be (measured between their centres)?



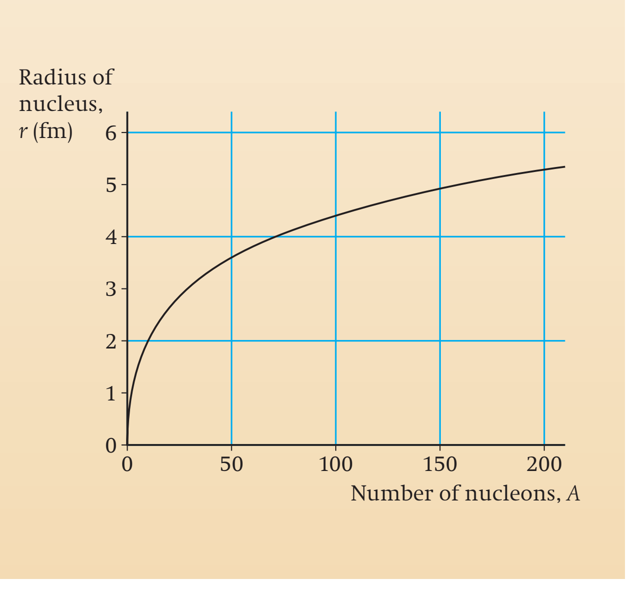
The radii of atomic nuclei

We can't measure the radius of an atomic nucleus directly in the way that we can measure the radius of, for example, a tennis ball (see *box 1B*). There are, however, ways in which it can be done.

Box 1B Problems in measuring the radius of an atomic nucleus

- An atomic nucleus doesn't have definite edges. Rather like the Earth's atmosphere, it doesn't suddenly stop but just very gradually fades away.
- An atomic nucleus is far too small to affect light waves. We can, however, explore its edges by using other methods, such as the scattering (diffraction) of high-energy beams of electrons (see chapter 12 of *Physics 2*).
- Different methods of measuring the radii of atomic nuclei give results which can differ by up to 50%.

If we measure the nuclear radii *r* of various different atoms we can then plot these values against the number of nucleons *A* in each nucleus (*figure 1.2*).



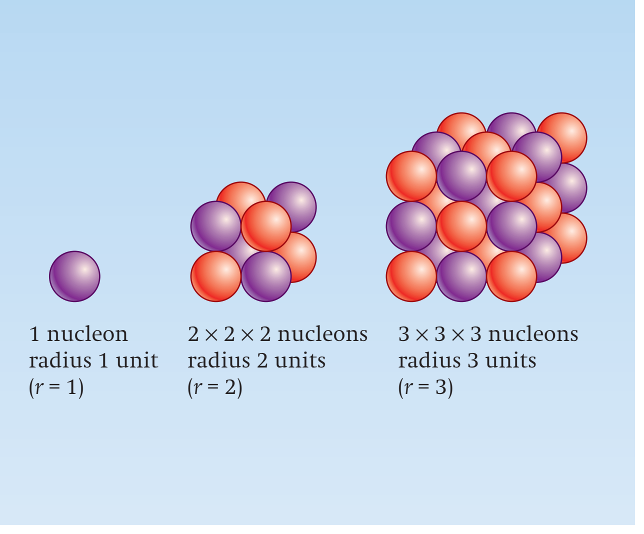
● **Figure 1.2** How the radius of an atomic nucleus varies with the number of nucleons it contains. Note: 1 fm (femtometre) = 10^{-15} m.

SAQ 1.11

a Use the graph in *figure 1.2* to find the value of *r* when *A* is 50, 100 and 200.

b Describe the effect on *r* of doubling the value of *A*.

As we would expect, the nuclear radius increases with the number of nucleons. The relationship is not proportional, however: the radius increases much more slowly than the number of nucleons. The following diagram shows why.



From the diagram, you can see that the number of nucleons (A) increases in proportion to the *cube* of the radius:

$$A \propto r^3$$

Putting this the other way round, we can say that the radius of a nucleus is proportional to the *cube root* of the number of nucleons:

$$r \propto A^{1/3}$$

So:

$$r = kA^{1/3}$$

where k is a constant.

If we call the radius of a single nucleon r_0, then a hydrogen atom ($A = 1$, a single proton) has this radius. So we can write:

$$r_0 = k1^{1/3}$$

So:

$$r_0 = k$$

This means that we can rewrite the general equation as:

$$r = r_0 A^{1/3}$$

Plotting a graph of r against $A^{1/3}$ should therefore give a straight line through the origin (*figure 1.3*).

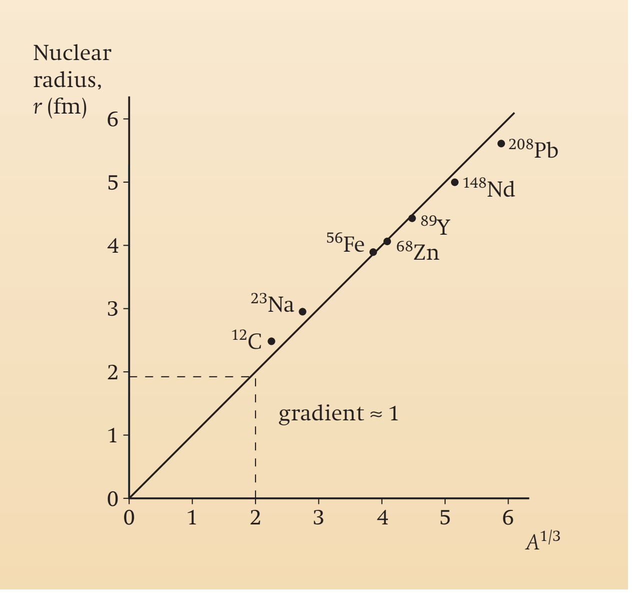


Figure 1.3 Plotting r against $A^{1/3}$.

Furthermore, the gradient of this line will be r_0, and will give a value for the radius of a proton:

gradient of graph = $r_0 \approx 1$

$\therefore$ radius of hydrogen nucleus (proton) ≈ 1 fm

SAQ 1.12

The line drawn on the graph in *figure 1.3* is *not* the line of best fit through the *plotted* values. Explain why.

The density of nuclear matter

The relationship between the number of nucleons and their volume is very close to being proportional. This strongly suggests that the size and the spacing of the nucleons in an atomic nucleus is – as we had earlier assumed – hardly affected at all by the number of nucleons in the nucleus. It also means that the **density** of all atomic nuclei will be very similar and will, in fact, be similar to the density of the individual nucleons themselves.

We know the mass m of nucleons, so we can use their radius r to calculate their density ρ using the relationship:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

[since $\frac{4}{3}\pi r^3$ is the volume of a sphere, radius r].

Because nucleons in atomic nuclei are tightly packed this will also give a good indication of the density of atomic nuclei.

SAQ 1.13

a Estimate the density of nuclear matter using the following data:
mass of a nucleon ≅ 1.67×10^{-27} kg
radius of a nucleon ≅ 1 fm (1×10^{-15} m)

b Compare the density of nuclear matter with:
(i) the density of solids on Earth (10^3–10^4 kg m^{-3});
(ii) the density of a neutron star (10^{17} kg m^{-3}).

c Suggest how the similarities and differences can be explained.

Looking in detail at the forces between nucleons

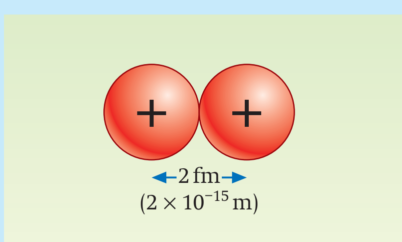
Box 1C Calculating electrostatic and gravitational forces

The electrostatic or Coulomb force F, measured in newtons, between two bodies whose centres are separated by distance r, measured in metres, and which have charges of Q_1 and Q_2, measured in coulombs, is given by:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

where ε_0 is a constant called the permittivity of free space with the value 8.85×10^{-12} F m^{-1} (farad per metre).

The gravitational force F, measured in newtons, between two bodies with centres separated by distance r, measured in metres, and which have masses of m_1 and m_2, measured in kilograms, is given by:



$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant with the value 6.673×10^{-11} N m^2 kg^{-2}.

SAQ 1.14

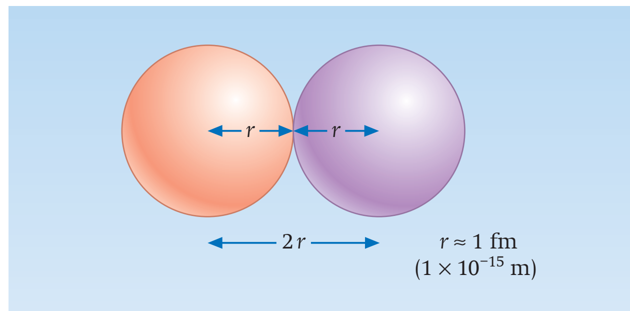
Use the above formulae, together with the data in *box 1A*, to calculate:

a the Coulomb force of repulsion between two adjacent protons;

b the gravitational force of attraction between two adjacent nucleons;

c the ratio of your answers to **a** and **b**.

Since the nucleons inside an atomic nucleus appear to be tightly packed, the distance between the centres of adjacent nucleons will be the sum of their radii, i.e. about 2 fm.



We can use this distance to calculate the electrostatic force of repulsion between two adjacent protons and the gravitational force of attraction between any two adjacent nucleons (*see box 1C*).

We can discount the very small gravitational force of attraction between two adjacent protons. So there must be some other force of attraction between them acting in the opposite direction to the electrostatic force of repulsion between them. As indicated earlier, this force is called the strong force.

We also know that the radii of nucleons and the distance between adjacent nucleons are not affected by the size of the nucleus that they are in. This means that the strong force must become a *repulsive* force as soon as the nucleons begin to 'overlap', i.e. when the centres of adjacent nucleons become closer than the sum of their radii by even a small amount.

Furthermore, the strong force is different from both the electrostatic and gravitational forces in that it seems to have hardly any effect outside of the nucleus itself. In other words, it is a very *short-range* force. The influence of the strong force extends very little further than the diameter of a single nucleon.

The strong force between two nucleons varies with the distance between the centres of the two nucleons as shown on the graph (*figure 1.4*).

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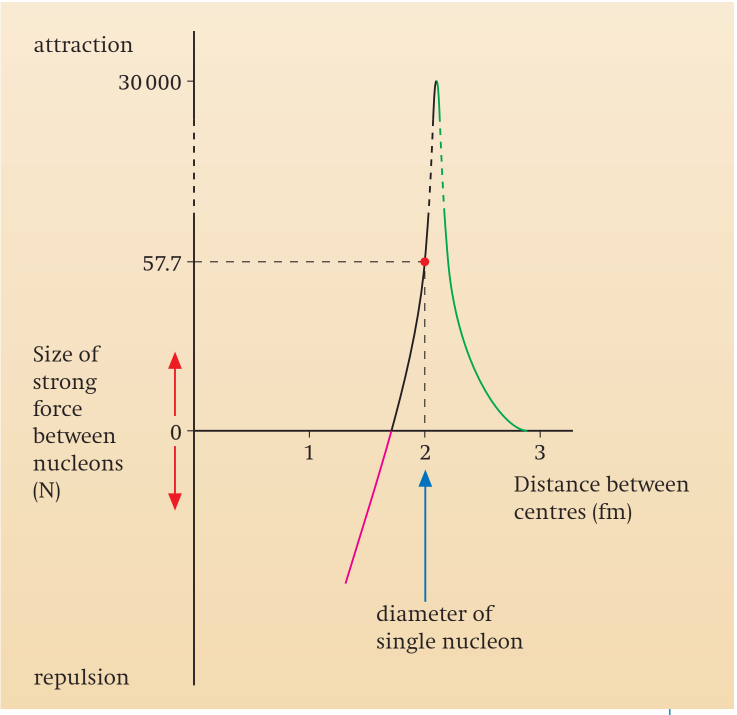
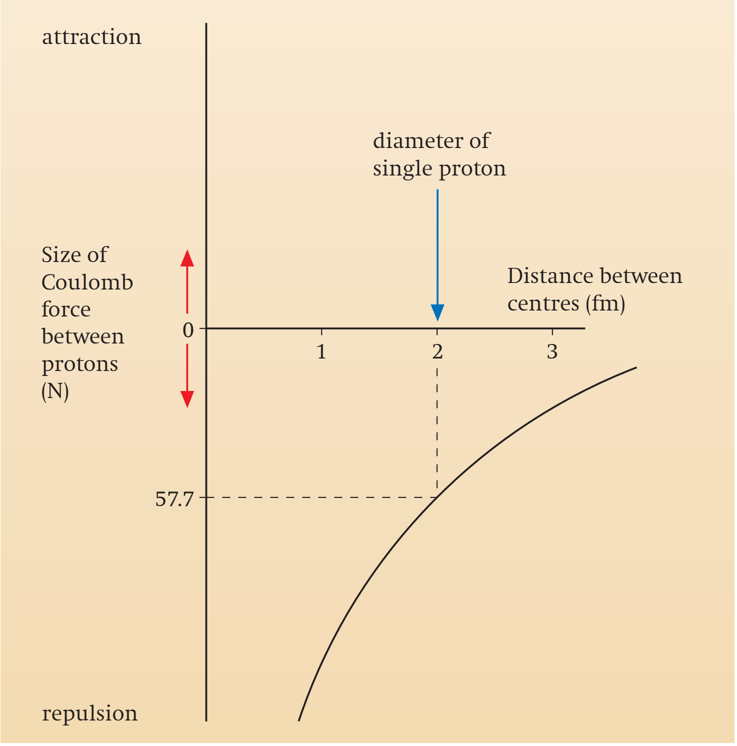


• **Figure 1.4** The strong force between two nucleons.

SAQ 1.15

Explain the significance of the red point and the regions marked green and yellow on the graph in *figure 1.4*.



• **Figure 1.5** The electrostatic (Coulomb) force between two protons.

The graph in *figure 1.5* shows how the electrostatic (Coulomb) force of repulsion between two protons varies with the distance between them.

The net force between two protons at any given distance is the difference between the electrostatic force and the strong force. These two forces are balanced when the centres of the protons are separated by a distance equal to the sum of their radii.

SAQ 1.16

a Sketch on the same axes, but using different colours, graphs of the electrostatic force and the strong force between two protons against the distance apart of their centres.

b Still on the same axes, and using a third colour, sketch a graph of the net force between the two protons.

c Describe what the third graph indicates about the net force between two protons and explain the effect this net force will have on the protons when the distance between their centres is: (i) less than 2 fm; (ii) 2 fm; (iii) just over 2 fm; (iv) well over 2 fm.

When the nucleons in a nucleus are packed tightly together, the electrostatic forces of repulsion between protons which are tending to force the protons apart are balanced by the strong force of attraction between adjacent nucleons.

Nuclear forces and the stability of nuclei

Apart from the nuclei of hydrogen-1 atoms, atomic nuclei comprise mixtures of protons and neutrons. There is a very delicate balance between the forces that act on these nucleons inside atomic nuclei: the subtle interplay between the strong forces that act between adjacent nucleons and the electrostatic forces that act between protons determines how stable (or how unstable) a particular nucleus is.

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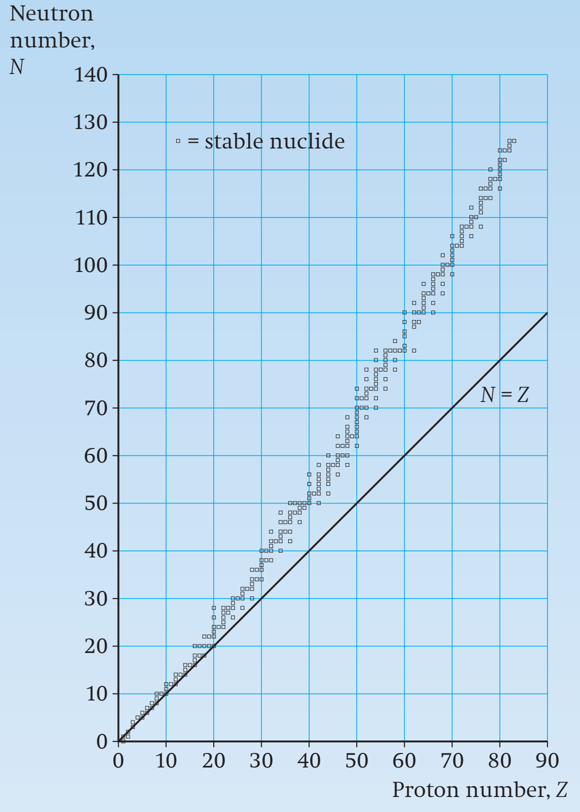
In relatively light nuclei with up to about 40 nucleons (i.e. $A \leq 40$), the strong forces and the electrostatic forces acting on protons balance; the nuclei are stable, provided that the number of protons and neutrons in the nucleus are approximately equal.

Heavy nuclei, however, are stable only if there are more neutrons in the nucleus than there are protons. To understand why, we need to consider the forces that are acting on the protons that are most likely to become detached from a nucleus, i.e. the protons on the outer edge of the nucleus.

As the size of a nucleus increases, the number of protons contributing to the electrostatic force of repulsion on a proton at the edge of the nucleus also increases. This happens because although the electrostatic force decreases in proportion with the square of the distance between protons, it is still quite significant between protons that are at most only a few diameters apart. The number of nucleons contributing to the strong force of attraction acting on the proton, however, remains the same. This is because the strong force falls away very rapidly with distance so that it only acts to any significant extent between adjacent nucleons. To restore the balance between these opposing forces in larger nuclei, additional neutrons are needed. These neutrons increase the distance between protons and so reduce the electrostatic force of repulsion between them.



● **Figure 1.6** A plot of N against Z for stable atomic nuclei.

The progressive increase in the ratio of neutrons to protons in stable atomic nuclei, as the size of the nucleus increases, shows up clearly by plotting the number of neutrons (N) against the number of protons (Z) on a graph (*figure 1.6*).

SAQ 1.17

The following are the commonest isotopes of some elements. All of the isotopes are stable.

$^{23}_{11}\text{Na}$ $^{24}_{12}\text{Mg}$ $^{40}_{20}\text{Ca}$ $^{56}_{26}\text{Fe}$ $^{107}_{47}\text{Ag}$ $^{208}_{82}\text{Pb}$

a Calculate the number of neutrons in each nucleus.

b Calculate the neutron : proton ratio in each nucleus.

c What patterns can you see in the neutron : proton ratio?

SAQ 1.18

a Describe, in terms of elements and isotopes, atoms with the same value of Z but different values of N.

b Up to what value of Z on the N : Z graph is:
(i) $N \approx Z$, on average, for all stable nuclei;
(ii) $N \approx Z$ for at least one isotope of the particular element?

c There are naturally occurring atoms with values of Z up to 92. Why are no atoms shown on the graph with $Z > 83$?

Many unstable nuclei are known to exist. Some of these occur naturally; others are produced artificially, e.g. in nuclear reactors. The N : Z graph in *figure 1.7* also includes these unstable nuclei.

