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PART I

INTRODUCTION

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Recognizing, Defining, and Representing Problems

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What are the problems that you are currently trying to solve in your life? Most of us have problems that have been posed to us (e.g., assignments from our supervisors). But we also recognize problems on our own (e.g., you might have noticed the need for additional parking space in the city where you work). After identifying the existence of a problem, we must define its scope and goals. The problem of parking space is often seen as a need for more parking lots or parking garages. However, in order to solve this problem creatively, it may be useful to turn it around and redefine it as a problem of too many vehicles requiring a space in which to sit during the workday. In that case, you may be prompted to redefine the problem: You decide to organize a carpool among people who use downtown parking lots and institute a daytime local taxi service using these privately owned vehicles. Thus, you solve the problem not as you originally posed it but as you later reconceived it.

Problem solving does not usually begin with a clear statement of the problem; rather, most problems must be identified in the environment; then they must be defined and represented mentally. The focus of this chapter is on these early stages of problem solving: problem recognition, problem definition, and problem representation.

THE PROBLEM-SOLVING CYCLE

Psychologists have described the problem-solving process in terms of a cycle (Bransford & Stein, 1993; Hayes, 1989; Sternberg, 1986). The cycle consists of the following stages in which the problem solver must:

1. Recognize or identify the problem.
2. Define and represent the problem mentally.
3. Develop a solution strategy.
4. Organize his or her knowledge about the problem.

5. Allocate mental and physical resources for solving the problem.
6. Monitor his or her progress toward the goal.
7. Evaluate the solution for accuracy.

The cycle is descriptive, and does not imply that all problem solving proceeds sequentially through all stages in this order. Rather, successful problem solvers are those who are flexible. The steps are referred to as forming a cycle because, once they are completed, they usually give rise to a new problem, and then the steps need to be repeated. For example, if you solve the parking-space problem by carpooling, then you may find that you are facing the problem of a work schedule that diverges from that of the person or people with whom you carpool. In other words, the solution to one problem gave rise to another problem, which then again needs to be solved through the problem-solving cycle.

classes of problems

There are two classes of problems: those that are considered well defined and others that are considered ill defined. Well-defined problems are those problems whose goals, path to solution, and obstacles to solution are clear based on the information given. For example, the problem of how to calculate the price of a sale item is well defined. You see the original price on the tag, calculate the discount percentage, and subtract this amount from the original price. The solution is a straightforward calculation. In contrast, ill-defined problems are characterized by their lack of a clear path to solution. Such problems often lack a clear problem statement as well, making the task of problem definition and problem representation quite challenging. For example, the problem of how to find a life partner is an ill-defined problem. How do you define "life partner"? What traits should that individual have? Where do you look to find such a person? Only after considerable work has been done to formulate the problem can an ill-defined problem become tractable. Even at this stage, however, the path to solution may remain fuzzy. Multiple revisions of the problem representation may be necessary in order to find a path to a solution. In contrast to well-defined problems, ill-defined problems can lead to more than one "correct" solution.

The solution process for well-defined problems has been studied extensively, often using algorithms to describe how each step of a problem is solved (e.g., Newell & Simon, 1972). A well-defined problem can be broken down into a series of smaller problems. The problem may then be solved using a set of recursive operations or algorithms. In contrast, algorithms cannot be used to solve ill-defined problems precisely because the problem cannot be easily defined as a set of smaller components. Before a path to solution is found, ill-defined problems often require a radical change in representation. For example, consider the following problem:

> You have a jug full of lemonade and a jug full of iced tea. You simultaneously empty both jugs into one large vat, yet the lemonade remains separate from the iced tea. How could this happen?

At first, this puzzle is difficult. You imagine two pitchers of refreshing drinks being poured into a common vessel and wonder how they could not mix. (It is safe to assume that the lemonade and iced tea have similar densities). However, if you change your mental representation of the lemonade and iced tea, you see that *frozen* drinks could be easily poured into the same vat without mixing. Though the problem itself does not specify the state of the drinks, most people assume that they are liquid, as is usually the case. But this constraint is simply an assumption. Of course, this puzzle is a fairly trivial one. But in life, we often make unwarranted assumptions in our everyday problem solving. Such assumptions can interfere with our ability to discover a novel solution to an ordinary problem.

Problem Recognition, Definition, and Representation

Problem recognition, definition, and representation are metalevel executive processes, called metacomponents in Sternberg's (1985) triarchic theory of human intelligence. This theory proposes that metacomponents guide problem solving by planning, monitoring, and evaluating the problem-solving process. The metacomponents include such processes as (1) recognizing the existence of a problem, (2) defining the nature of the problem, (3) allocating mental and physical resources to solving the problem, (4) deciding how to represent information about the problem, (5) generating the set of steps needed to solve the problem, (6) combining these steps into a workable strategy for problem solution, (7) monitoring the problem-solving process while it is ongoing, and (8) evaluating the solution to the problem after problem solving is completed. In this theoretical context, the processes of problem recognition, definition, and representation correspond to the first, second, and fourth metacomponents, which are used in the planning phase of problem solving.

Problem recognition, also referred to as problem finding, is one of the earliest stages of problem solving. Getzels (1982) classified problems based on how they were "found." According to Getzels, there are three kinds of problems: those that are presented, those that are discovered, and those that are created. A presented problem is one that is given to the solver directly. In this case, there is no need to recognize or find the problem; it is stated clearly and awaits solution. A discovered problem, however, is one that must be recognized. Such a problem already exists, but it has not been clearly stated to the problem solver. In this case, the problem solver must put together the pieces of the puzzle that currently exist and seek out a gap in current understanding in order to "discover" what the problem is. In contrast to presented and discovered problems, the third class of problems comprises those that are created. Created problems are those in which the problem solver invents a problem that does not already exist in the field. For this reason, one can argue that a created problem will, in some sense, always produce a creative solution, simply because its problem statement deviated from the usual way of thinking about the problem. Getzels and Csikszentmihalyi (1976) found that artists who spent more time in the problem-finding stage while creating an artwork were judged to have more creative products than did artists who spent less time in problem finding. In fact, the artists who spent more time also remained highly creative seven years later. For the purposes of this chapter, problem recognition refers to both discovered and created problems.

Problem definition is the aspect of problem solving in which the scope and goals of the problem are clearly stated. For example, a presented problem may be easy to define if the problem statement has been prepared for the solver. However, some presented problems are not clearly stated, requiring the problem solver to clarify the precise definition of the problem. Discovered problems usually require definition because the problem solver has identified the problem in his or her field. Defining a created problem is likely to be a challenge, given that the problem solver has gone beyond the current field in inventing the need for a solution in the first place.

Problem representation refers to the manner in which the information known about a problem is mentally organized. Mental representations are composed of four parts: a description of the initial state of the problem, a description of the goal state, a set of allowable operators, and a set of constraints. By holding this information in memory in the form of a mental representation, the problem solver is able to remember more of the problem by chunking the information, in order to organize the conditions and rules of a problem to determine which strategies are useful, and to assess progress toward the goal state (Ellis & Siegler, 1994; Kotovsky, Hayes, & Simon, 1985; Newell & Simon, 1972). A problem may be represented in a variety of ways, for example, verbally or visually. Even a presented problem may require the generation of a new representation in order to be solved. For example, given the problem of finding your way to a new location, you may find it much easier to follow a map than to read a set of directions. If you have trouble following the map, then it may be worthwhile to write out a description of the route in words, re-representing the information in a way that makes it easier to get to your destination.

It is important to note that these three aspects of problem solving are not discrete, sequential stages in the solution process, but rather are interactive and often difficult to tease apart in a real problem-solving situation. When a problem is represented in a new way, the problem solver may decide to redefine the goal accordingly. Similarly, a redefinition may lead to a new representation.

It is useful to consider the roles of problem recognition, definition, and representation in the solution of well-defined versus ill-defined problems. Recall that a well-defined problem is one whose path to solution is straightforward, whereas an ill-defined problem is one that does not lend itself to a readily apparent solution strategy. Consider the following well-defined problem, referred to as the Tower of Hanoi problem:

> There are three discs of unequal sizes, positioned on the leftmost of three pegs, such that the largest disc is at the bottom, the middle-sized disc is in the middle, and the smallest disc is on the top. Your task is to transfer all three discs to the rightmost peg, using the middle peg as a stationing area, as needed. You may move only one disc at a time, and you may never move a larger disc on top of a smaller disc. (Sternberg, 1999)

The problem here is easy to recognize: One needs to move the discs onto the rightmost peg. The problem is also defined clearly; the relative sizes of the discs as well as their locations are easy to distinguish. Also, the solution path is straightforward based on this representation. Working backward, one realizes that the largest disc must be placed onto the rightmost peg, and in order to do so, the other two discs must be removed. So that the medium-sized disc does not end up on the rightmost peg, the smallest disc must first be moved to the far right. Then the medium disc is placed on the middle peg; the small disc is placed on top of the medium disc. The large disc is then free to be placed on the rightmost peg. Finally, the small disc is moved to the left so that the medium disc is free to move to the rightmost peg. The last step is then to move the small disc atop the other two and the problem is solved. Note that this well-defined problem can be expanded to include many pegs and many discs of varying sizes, but its solution will always proceed according to the algorithm described in this, the simplest case.

For the most part, well-defined problems are relatively easy to recognize, define, and represent. However, a well-defined problem may entail some degree of "problem finding," in the sense that a problem exists but must first be discovered. For example, a scientist may struggle to identify a gap in the existing literature on a problem, but the actual process of filling that gap may come easily once the problem itself has been identified. The solution to the discovered problem may follow a path similar to that of other problems in the field (e.g., experimental methods). For example, much early psychological research was conducted using male participants. When a researcher questioned the validity of the results for females, a new problem had been discovered. Given this new problem, the path to solution was well defined: Simply use the same experimental method but include female participants in the study. In this sense, this well-defined problem was somewhat difficult to recognize, yet once identified, it was easily defined and represented in familiar terms.

The representation of well-defined problems is not necessarily easy, however. Consider another problem:

> Three five-handed extraterrestrial monsters were holding three crystal globes. Because of the quantum-mechanical peculiarities of their neighborhood, both monsters and globes come in exactly three sizes, with no others permitted: small, medium, and large. The small monster was holding the large globe; the medium-sized monster was holding the small globe; and the large monster was holding the medium-sized globe. Since this situation offended their keenly developed sense of symmetry, they proceeded to transfer globes from one monster to another so that each monster would have a globe proportionate to its own size. Monster etiquette complicated the solution of the problem since it requires that: 1. only one globe may be transferred at a time; 2. if a monster is holding two globes, only the larger of the two may be transferred; and, 3. a globe may not be transferred to a monster who is holding a larger globe. By what sequence of transfers could the monsters have solved this problem? (See Kotovsky et al., 1985)

Most people find this problem to be more difficult than the Tower of Hanoi problem (Newell & Simon, 1972). However, it is actually directly isomorphic to (i.e., its structure is exactly the same as that of) the Tower of Hanoi problem. In this case, it is the difficulty of representing the problem correctly that increases the level of difficulty of the problem as a whole. After you are told of the isomorphism between the two problems, the solution is simply a matter of mapping relationships from one problem to the other. In summary, problem definition is usually easy for the class of well-defined problems; however, accurate problem recognition and representation are not necessarily straightforward, even when the scope and goals of the problem are clear.

In the case of ill-defined problems, however, it is often the case that all aspects of problem formulation are relatively challenging. Perhaps the easiest stage in attempting to solve an ill-defined problem is that of problem recognition. It is often relatively simple to identify a fuzzy problem. For example, it is easy to identify the problem of developing a test of creativity. It is hard, however, to define the exact contents of such a measure.

The real difficulty in solving an ill-defined problem is in clarifying the nature of the problem: how broad it is, what the goal is, and so on. Although well-defined problems have a clear path to solution, the solution strategy for an ill-defined problem must be determined by the problem solver. To develop a problem-solving strategy, it is first necessary to specify the goals of the task. For example, if we take on the task of designing a creativity test, we must decide whether the goal is (a) to estimate the creativity of undergraduate psychology majors or (b) to measure creative potential among people of all ages and educational and cultural backgrounds. Before the path to solution can be constructed, the goal must be clear.

Representing information about the problem is also difficult in the formulation of an ill-defined problem. Consider again the problem of parking mentioned at the beginning of the chapter. The representation of the problem affects the solution. If we think of the parking problem in terms of parking spaces, we are likely to seek additional spaces when there are too many cars to park. However, if we think of parking in terms of too many idle vehicles, we are more likely to consider new ways of making use of the cars that have remained idle during the workday (e.g., driving other people who need transportation around the city). This latter perspective will guide us to seek solutions that maximize efficiency rather than maximizing the amount of concrete and asphalt in the downtown area. To solve a problem, it often is necessary or, at least, desirable to try out several representations of the problem in order to hit upon one that leads to an acceptable solution.

Problem-solving research has not revealed a great deal about the processes involved in problem recognition, problem definition, and problem representation. Indeed, the emphasis in research has been on the latter rather than the earlier phases of problem solving. Yet these earlier phases are critical to accurate and efficient problem solving, especially in the solution of ill-defined problems. The study of ill-defined problems generally has been less fruitful than the study of well-defined problems. Well-defined problems are well described by current theories of problem solving; however, ill-defined problems are ill understood by psychologists. Yet arguably most of the problems in the real world are not well defined. Most are fuzzy problems, often difficult to delineate and sometimes even harder to represent in a way that makes them solvable. Our current educational system better prepares children to answer questions that are well defined and presented to them in the classroom than it does to formulate the nature of problems in the first place. Often the skills involved in solving well-defined problems are not the same as those involved in recognizing a nonobvious problem or creating a problem. The skills needed clearly to state a problem and to represent information about it in a way that permits solution are also often not emphasized in current classrooms. In this chapter we consider what factors influence the metacognitive processes involved in recognizing, defining, and representing problems.

Research on problem solving has identified several variables that influence problem-solving performance. Among these are knowledge, cognitive processes and strategies, individual differences in ability and dispositions, as well as external factors such as social context. Those variables known to influence general problem solving will be examined with respect to the three particular aspects of problem solving that are the focus of this chapter: problem recognition, problem definition, and problem representation.

KNOWLEDGE

Everyone approaches a problem situation with a unique knowledge base. That knowledge base is essentially a set of expectations about the way the world works. As you began to read this chapter, your experience with reading chapters in similar books led you to expect a certain structure and content. Similarly, when you identify, define, and represent a problem, it is in terms of what you already know. For example, consider how the parking problem mentioned in the beginning of the chapter would be approached differently by individuals with different knowledge bases. An urban planner is more likely to identify or notice that problem as one of primary importance than is a person who does not live in an urban area. The urban planner is also more likely to consider different variables in defining the problem than someone from a small town. For example, the urban planner defines the problem in terms of how it may affect the city's income (e.g., parking meters or garages) and use the city's resources (e.g., administrative factors associated with employees and regulation of parking). In contrast, the small town resident may define the problem in terms of the esthetics of housing many vehicles (e.g., parking garages are often not welcome sights in small towns) because the solution of this problem is less likely to generate funds for the town than it would in an urban setting. According to the definition of the problem, the problem would be represented differently depending on the knowledge of the problem solver, be it an urban planner or small town parking supervisor. Problem-solving research has accumulated a tremendous amount of information regarding the relationship between knowledge and problem definition and representation and, to a lesser extent, regarding problem recognition.

It is important to keep in mind that knowledge may help or hinder problem solving. For example, knowledge plays an important role in the solution of analogies. In such problems, your task is to map the relationship between two items onto two other items. For example, *apple* is to *apple tree* as *pear* is to *pear tree*. The relationship here should be clear: You are pairing fruits with their respective trees of origin. Consider the following analogy problem.

Nefarious is to Dromedary as Eggs are to:
A: Chapel
B: Yellow
C: Bees
D: Friend (Concept Mastery Test; Terman, 1950)