Surveying Instruments of Greece and Rome

The Greeks and, especially, the Romans are famous for the heroic engineering of their aqueducts, tunnels and roads. They also measured the circumference of the earth and the heights of mountains with fair precision. This book presents new translations (from Greek, Latin, Arabic, Hebrew and Syriac) of all the ancient texts concerning surveying instruments, including major sources hitherto untapped. It explores the history of surveying instruments, notably the Greek dioptra and the Roman libra, and with the help of tests with reconstructions explains how they were used in practice. This is a subject which has never been tackled before in anything like this depth. The Greeks emerge as the pioneers of instrumental surveying and, though their equipment and methods were simple by modern standards, they and the Romans can be credited with a level of technical sophistication which must count as one of the greatest achievements of the ancient world.

M. J. T. Lewis is Senior Lecturer in Industrial Archaeology at the University of Hull. His publications include Temples in Roman Britain (1966), Early Wooden Railways (1970) and Millstone and Hammer: the Origins of Water Power (1997), and many articles in such journals as History of Technology, Technology and Culture and Papers of the British School at Rome.
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First, a few definitions. Surveying is the science based on mathematics which involves measuring any part of the earth's surface and any artificial features on it, and plotting the result on a map or plan drawn to a suitable scale. Often, though by no means always, it also involves levelling or some similar process to record relative heights. Setting out is effectively the converse process, namely locating intended boundaries, structures or engineering works on the ground, in the correct position in all three dimensions. The surveyor will often have to carry out both procedures, especially when linear features such as aqueducts or railways are to be built: first to record the existing shape of the terrain and then, in the light of this information, to decide the best route and to mark it on the ground.

Almost without exception, surveying with instruments that rose above the level of low technology began with the Greeks and Romans, and a proper understanding of their achievements entails straddling two very different disciplines. The present-day surveyor who is curious about the origins of his profession may not be deeply informed on ancient history or engineering, while the classical historian may not have a detailed command of the principles of surveying. The resulting challenge, constantly encountered by historians of technology, is to try to put across the background, the material and the arguments at such a level that no reader feels neglected or patronised. I have done my best to strike a happy mean. My credentials, such as they are, for accepting this challenge are an upbringing as a classicist and classical archaeologist and a lifetime spent on the history of technology. I am not a trained surveyor, but through fieldwork I have acquired a working knowledge of surveying techniques. Since the techniques and instruments of ancient surveying were essentially similar to, if simpler than, those that I have experienced, I hope that this is qualification enough.

I am indebted to Denis Hopkin for constructing a dioptra for me, to David Palmer for making a libra, to Dr Guy Stiebel of the Hebrew University, Jerusalem, for help over Talmudic references, and particu-
PREFACE

I am particularly grateful to Dr Youcef Bouandel for translating al-Karaji’s Arabic. I am grateful too to Pauline Hire of Cambridge University Press for suggesting improvements to the layout of this book and for seeing it through the press with such care. But I owe most to my family. The staffman’s job is, at the best of times, tedious. To act as staffman for a surveyor who is struggling with the idiosyncrasies of totally strange instruments is more tedious still. This is what my son Hywel did for me, with exemplary patience, over the many days when I was testing the reconstructed dioptra and libra. His understanding of the principles and his sound common sense, moreover, helped me through many a difficulty. My debt to him is very great. So it is too to my wife, who has also held the staff on occasion and who has commented on my drafts with her usual perception.
EXPLANATORY NOTES

CROSS-REFERENCES

Part III contains translations both of the four major treatises and of extracts from other sources arranged in the same order and under the same headings as the chapters and sections of Parts I and II. References to these translations are in bold type: in the form *Dioptra* 22, *Africanus* 4, *Anonymus* 10, *Al-Karaji* 2 to the treatises, in the form *Source* 33 to the other sources. Thus the cross-reference *Source* 33 in Chapter 3.0 on the astrolabe should guide the reader to the extract in Part III from Severus Sebokht. Occasionally a source deals with more than one instrument, in which case a note at the end of one section in Part III draws attention to relevant material in another section.

TRANSLATION

Few of the sources have hitherto been translated into English. Where they have been, the results vary from the excellent to the downright misleading. All the translations from Greek and Latin used here are therefore my own, done for the purpose of this book. The major treatises are written in a bewildering jumble of tenses and persons, sometimes in the same sentence: *I turn the alidade*, for example, *one will turn the alidade, the alidade was turned*, and *let the alidade be turned*. All these, and comparable phrases, have generally (but not always) been standardised into the imperative, *turn the alidade*. Greek reference letters are retained. Otherwise all Greek, Latin, Hebrew, Syriac and Arabic is translated, except for occasional words which, because of their untranslatable connotations, are simply transliterated. Semitic words are transliterated without diacritical signs (may purists forgive me), except that Arabic H· and T· are differentiated from H and T when used as reference letters.

TERMINOLOGY

To avoid potential confusion, although the context normally makes the distinction clear, two sets of terms need explaining. In levelling, *back*
EXPLANATORY NOTES

sight and fore sight, each in two words, denote the sightings taken through the instrument looking backwards and forwards at the staff. Backsight and foresight, each in one word, are terms borrowed from the rifle to denote the actual parts of the instrument (holes or slits) through which sightings are taken.

GRADIENTS

Gradients can be given in different ways:

- The vertical reduced to unity relative to the horizontal, e.g. 1 in 200 or 1:200
- The vertical as a percentage of the horizontal, 0.5%
- The vertical as so much per thousand horizontal 5‰
- Metres per kilometre 5m per km
- Vertical divided by horizontal 0.005

All of the above figures mean exactly the same thing. The form most widely used in engineering circles is 0.5%. But (at least in Britain) the most common form found in histories of engineering is 1 in 200; and I feel that by this system the non-engineer can most easily visualise a given gradient: in this case a rise or fall of one unit of length for every 200 units of distance. I have therefore adopted this form throughout, and engineers will have no difficulty in converting it to their own preferred version.

MEASURES

Ancient measures are a minefield for the unwary. For our purposes the precise value of a particular unit is normally of no great importance; as a rule of thumb it is often sufficient to take the cubit as rather under half a metre and the stade as rather under 200 metres. Exactitude is desirable only when comparing ancient estimates of length and height with known modern equivalents; the problem is that it is often impossible to tell which of several different values was in fact being used. The units encountered in this book are listed below.

Greek

The relationships are constant, regardless of the actual length of each unit:

xviii
The value of Greek measures, however, varied from place to place and from time to time. Four values of feet and stades which were widely used in Hellenistic and Roman times deserve mention here.2

|              | Attic–Roman | ‘standard’ | Olympic | Philetan
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<td>1 foot</td>
<td>29.6 cm</td>
<td>30.8 cm</td>
<td>32.0 cm</td>
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<tr>
<td>1 stade</td>
<td>177.6 m</td>
<td>185 m</td>
<td>192 m</td>
<td>197.3 m</td>
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<tr>
<td>stades to Roman mile</td>
<td>8.33</td>
<td>8.00</td>
<td>7.71</td>
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**Roman**

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<th>1 foot = 12 inches</th>
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<td>1 foot</td>
<td>16 digits</td>
<td>1 foot</td>
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<td>1 foot = 12 inches</td>
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<td>7.71</td>
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The values are well established: 1 foot = 29.6 cm, 1 mile = 1480 m.

**Islamic**

<table>
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<tr>
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<th>4 fingers = 1 palm</th>
<th>12 fingers = 1 span</th>
<th>24 fingers = 1 legal cubit</th>
<th>32 fingers = 1 Hasimi cubit</th>
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<tbody>
<tr>
<td>4 fingers</td>
<td>= 1 palm</td>
<td>= 1 span</td>
<td>= 1 legal cubit</td>
<td>= 1 Hasimi cubit</td>
</tr>
<tr>
<td>12 fingers</td>
<td>= 1 span</td>
<td></td>
<td>= 1 legal cubit</td>
<td></td>
</tr>
<tr>
<td>24 fingers</td>
<td>= 1 legal cubit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32 fingers</td>
<td>= 1 Hasimi cubit</td>
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<tr>
<td>60 Hasimi cubits = 1 cord (asf)</td>
<td>= 80 legal cubits</td>
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<tr>
<td>4,000 legal cubits = 1 mil</td>
<td>= 50 cords</td>
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<tr>
<td>3 mil</td>
<td>= 1 farshal</td>
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1 But 9/4 span = 1 fathom for measuring cultivated land.

2 Based on Hultsch 1882. There are useful summaries in *KP* v 336–7 and *OCD* 942–3.

The calculations of Lehmann-Haupt 1929, though seemingly authoritative, need to be treated with caution. See also Dicks 1960, 42–6.
The value of Islamic measures was widely variable. The legal cubit was usually 49.875 cm and the Hasimi cubit 66.5 cm.³