Air–Sea Interaction

Laws and Mechanisms

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Chapter 1

The Transfer Laws of the Air-Sea Interface

1.1 Introduction

Hurricane Edouard had just passed by Cape Cod when I wrote these lines, after giving us a good scare, and keeping meteorologists of local TV stations out of bed all night. Approaching on a track along the East Coast, Edouard remained a category 3 hurricane, with 180 km/h winds, from the tropics to latitude 38°N. This is where it left the warm waters of the Gulf Stream behind, quickly to lose its punch over the much cooler Mid Atlantic Bight, and to be degraded to category 1, with 130 km/h winds, still enough to uproot a few trees on the Cape.

Edouard's fury came from water vapor, as it ascends the "eye-walls" (Figure 1.1) that surround a hurricane's core, condensing and releasing its latent heat of evaporation. The heat makes the moist air buoyant, turning the eyewalls into a giant chimney with an incredibly strong draft. The draft sucks in sea-level air, causing it to spiral toward the core in destructive winds and to drive waters against nearby coasts in storm surges. The fast air flow over warm water also ensures intense heat and vapor transfer to the air, sustaining the hurricane's strength. Over colder water, where not enough water evaporates, the hurricane dies: The lifeblood of a hurricane is intense sea to air transfer of heat and water vapor. On the other hand, as hurricane winds whip the waters along, they transfer some of their momentum downward. The loss of momentum acts as a brake on the hurricane circulation, keeping the winds from completely getting out of hand.

A hurricane also mimics on a small-scale the global atmospheric circulation, which is similarly "fueled" by latent heat released from condensing water vapor. This happens in "hot towers," concentrated updrafts of the InterTropical Convergence Zone (ITCZ), and also in somewhat less vigorous updrafts within extratropical storms. Many of the

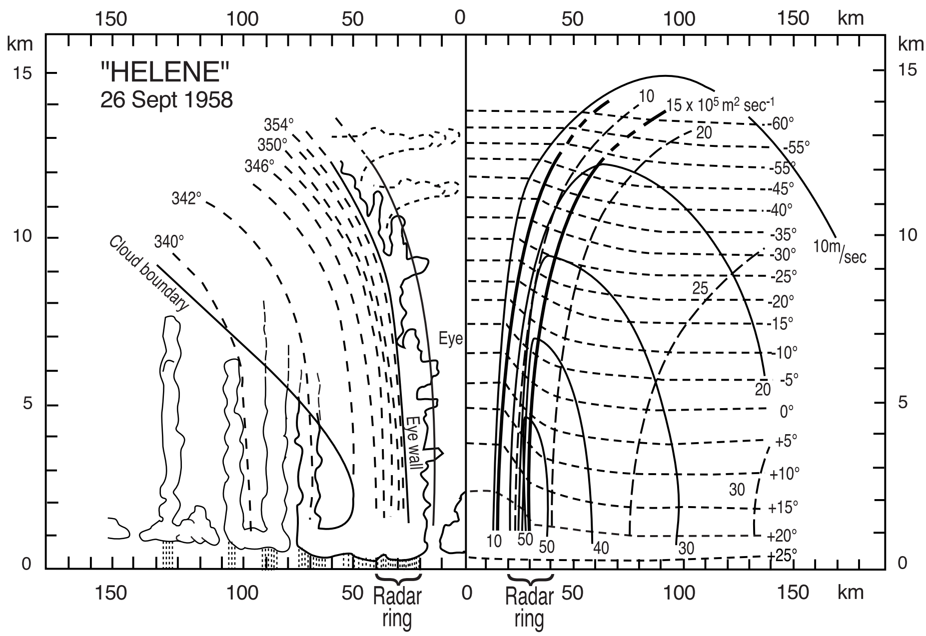

Figure 1.1 Mean structure of a mature hurricane ("Helene," 26 Sept. 1958) in cross section, supposing axial symmetry. The left-hand half shows the boundaries of the eye-wall (solid lines, bending outward with height) and illustrates the cloud structure. The broken lines are contours of constant "equivalent potential temperature," the absolute temperature in degrees Kelvin that the air would have with all of the latent heat in its vapor content released, and the pressure brought down to sea level pressure. In the right-hand half section, thin full lines are contours of constant wind speed in m s^{-1}(the thick lines repeat the eye wall boundaries), the broken lines are angular momentum contours, the dotted lines contours of temperature in $^\circ$C. The maximum wind speed is in excess of 180 km/h. Note the stratiform cloud (dashed lines in the left half) extending to 13 km height, to the top of the troposphere, where the temperature is -55°C, $= 218$ K. Satellites see this "cloud-top" temperature. From Palmén and Newton (1969).

latter draw their vapor supply from the warm Gulf Stream and its Pacific counterpart, the Kuroshio, ocean currents transporting massive amounts of heat from warm to cold regions. Hot towers make their presence known to travelers crossing the equator, and wake them from their slumber when updrafts toss around their jetliner, as high as 10 or 12 km above sea level. Heat release in the updrafts, and compensating cooling and subsidence, are part of a thermodynamic cycle that energizes various atmospheric circulation systems, including the easterly winds of the tropics and subtropics, and the westerlies of mid-latitudes. The winds in turn sustain sea to air heat and vapor transfer, supplying the fuel, moist air, for the updrafts. The associated air to sea transfer of momentum from the winds is again the control on the strength of the atmospheric circulation.

Important to the operation of hurricanes and to large-scale atmospheric and ocean circulation systems is therefore in what amount, and by what mechanisms, momentum, heat and vapor pass from one medium to the other. The rates of transfer, per unit time and unit surface area, depend on a variety of conditions and processes; relationships between the rates and the variables influencing them are the "transfer laws" of the air-sea interface that we seek in this chapter. As all laws of physics, these too are distilled from observation, and, as most such laws, they are more or less accurate approximations. Their establishment requires painstaking work, hampered by difficulties of observation at sea. After nearly a century of research by many scientists from a variety of nations, there are still many uncertainties affecting the transfer laws.

1.2 Flux and Resistance

Transfers of momentum, heat and mass, are all *irreversible* processes. A number of texts deal with irreversible molecular processes of transfer, viscosity, heat conduction or diffusion, but their common thermodynamic characteristics have only engaged the interests of scientists relatively recently. De Groot's seminal synthesis (1963) bears the title "Thermodynamics of Irreversible Processes," while a later development (De Groot and Mazur 1984) is called "Non-equilibrium thermodynamics." These monographs develop the subject for molecular transfer processes, and show that their laws have the general form:

$$Flux = Force/Resistance \tag{1.1}$$

where the "Force" has the character of a potential gradient, the "Resistance" of inverse conductivity.

Irreversible processes change the entropy of the system in which they occur. Entropy changes because it flows in and out of the system, and also because internal irreversible processes generate it. The rate of entropy generation, the internal entropy "source" term in the entropy balance, is always positive, according to the Second Law of thermodynamics. "To relate the entropy source explicitly to the various irreversible processes that occur in the system" is the main preoccupation of nonequilibrium thermodynamics (De Groot and Mazur, 1984). When only one Force is acting, the entropy production rate is proportional to the product of Flux and Force. Absorbing the proportionality factor in the Force, entropy production can be made equal to the Flux-Force product. With several Fluxes and their conjugate Forces present, a similar standardization of the Forces yields the entropy production rate as the sum of the Flux-Force products, a result known as "Onsager's theorem."

The transfer laws of the air-sea interface are also relationships between Fluxes and Forces in the sense of nonequilibrium thermodynamics. They are, however, the result of an interaction between turbulent flows in air and water, and wind waves on the sea surface, and are more complex than linear relationships between a Flux and a Force with a constant Resistance. They are empirical laws of physics depending on material properties, properties of the turbulent flows in air and water, and of wind waves. Their usual form is an implicit Flux-Force relationship:

$$func\left(Flux, Force, \sum_{i=1}^{n} X_i\right) = 0 \tag{1.2}$$

where the X_i are n variables having measurable influence on the transfer law.

An important requirement of a physical law is that it must be independent of units of measurement. This dictates the use of a consistent system of units, and leads to Buckingham's theorem, according to which all physical laws are expressible as relationships between nondimensional combinations of variables, in appropriate products and quotients. Therefore, yet another way to state the transfer law of Equation 1.2 is:

$$func\left(\sum_{i=1}^{m} N_i\right) = 0 \tag{1.3}$$

where N_i are nondimensional combinations of the variables, including the Flux and the Force. Their number, m, is less than the $n + 2$ of Equation 1.2, usually by the number of measurement units in the dimensional relationship 1.2. Such formulations of the transfer laws are most useful if either the Flux or the Force appears in only one of the N_i; that variable can then be treated as the dependent one, the others deemed independent.

De Groot and Mazur (1984) discuss Flux-Force relationships valid locally, between heat flux and temperature gradient, and analogous quantities in other irreversible processes, while the most useful formulation of the air-sea transfer laws is between a property *difference* across a layer of air above the interface, and the flux across the interface, in what we might call a "bulk" relationship. To illustrate the difference between local and bulk relationships, and also to give a taste of the classical results of nonequilibrium thermodynamics, next we discuss viscous momentum transfer in a simple situation.

1.2.1 Momentum Transfer in Laminar Flow

Suppose that air and water are two semi-infinite viscous fluids in contact at the $z = 0$ plane, with the upper fluid impulsively accelerated to a velocity $u = U = const.$ at time $t = 0$. In the absence of other forces, and as long as the flow remains laminar and unidirectional, shear stress between layers accelerates the lower fluid while retarding the upper one. The shear stress τ, force per unit area, equals viscosity times velocity gradient (see e.g., Schlichting 1960):

$$\tau = \rho\nu\frac{\partial u}{\partial z} \tag{1.4}$$

where ρ is density and ν is kinematic viscosity. A layer of fluid between two levels δz apart experiences a net force equal to the difference in shear, which then accelerates the fluid:

$$\frac{\partial(\rho u)}{\partial t} = \frac{\partial \tau}{\partial z} \tag{1.5}$$

where the left-hand side is mass times acceleration or rate of change of horizontal momentum ρu. A legitimate interpretation of this relationship is that the shear stress is equivalent to vertical flux of horizontal momentum, the difference of which across the layer increases the local momentum.

In this light, the previous relationship, Equation 1.4, is now seen as one between a Flux (of momentum) and a Force, the gradient of the velocity $\partial u/\partial z$, a local law, valid at any level z. The dynamics is contained in Equation 1.5. Multiplying that equation by u, we arrive at the energy balance:

$$\frac{\partial(\rho u^2/2)}{\partial t} = u\frac{\partial \tau}{\partial z} \tag{1.6}$$

which, after rearrangement and substitution from Equation 1.4, transforms into:

$$\frac{\partial(\rho u^2/2)}{\partial t} = \frac{\partial}{\partial z}\left(\nu\frac{\partial}{\partial z}(\rho u^2/2)\right) - \rho\nu\left(\frac{\partial u}{\partial z}\right)^2. \tag{1.7}$$

The first term on the right is the divergence of viscosity times the gradient of kinetic energy, legitimately interpreted as energy flux. The divergence of this quantity signifies vertical energy transfer from one location to another, leaving the total energy unchanged. The second term, however, is always negative, and signifies loss of mechanical energy, its transformation into heat through viscosity. The heat added to the air or water increases its entropy at the rate of heat generation divided by absolute temperature. This then is the entropy source term, locally, level by level, equal to the product of the Force $\partial u/\partial z$ and (by Equation 1.5) the Flux τ, conforming to Onsager's theorem.

The fluid properties, viscosity and density, are constant in either medium, but change at the interface: They will bear indices a, w, for air above, water below. Writing down Equations 1.4 and 1.5 separately for air and water, and eliminating τ, we have two second order differential equations for u to solve. The boundary conditions are as follows: Far above the interface the velocity is the undisturbed U, far below it is zero. At the interface, the velocity and the shear stress are continuous. The solution follows the standard approach to such problems, see e.g., Carslaw and Jaeger (1959). The results are:

$$u_a = u_0 erfc\left(\frac{z}{2\sqrt{\nu_a t}}\right) + U erf\left(\frac{z}{2\sqrt{\nu_a t}}\right) \tag{1.8}$$

$$u_w = u_0 erfc\left(\frac{-z}{2\sqrt{\nu_w t}}\right)$$

where u_0 is the common interface velocity. The boundary condition of continuous interface stress yields a relationship for u_0:

$$\frac{u_0}{U} = \left(1 + \frac{\rho_w\sqrt{\nu_w}}{\rho_a\sqrt{\nu_a}}\right)^{-1}. \tag{1.9}$$

The solution represented by Equations 1.8 and 1.9 reveals the velocity distributions to be error functions and complementary error functions of the distance from the interface, portraying air-side and water-side boundary layers of thickness $2\sqrt{\nu t}$, which grow with the square root of time. The water-side velocities are much slower than the air-side ones: The typical value of u_0/U is $1/200$. This can be anticipated from Equation 1.5, which shows accelerations to be inversely proportional to density. The density of water is about 800 times greater than the density of air, balanced somewhat in Equation 1.9 by the kinematic viscosity of the air being some 16 times greater than that of water.

From the solution we find the value of the interface stress, alias momentum flux from air to water:

$$\tau_i = \frac{U}{R} \tag{1.10}$$

with

$$R = \frac{1}{\rho_a}\sqrt{\frac{\pi t}{\nu_a}} + \frac{1}{\rho_w}\sqrt{\frac{\pi t}{\nu_w}}.$$

The result is clearly of the form of Equation 1.1, constituting a bulk relationship between the interface momentum flux and the undisturbed velocity difference between air and water, which plays the role of the conjugate Force. The Resistance R consists of two additive components, identifiable as air-side and water-side resistance, respectively. Each component is proportional to the boundary layer thickness on that side, and inversely proportional to dynamic viscosity $\rho\nu$. With the values of material properties substituted, the air-side resistance turns out to be some 200 times greater than water-side resistance, so that the latter is for all practical purposes negligible.

The momentum transfer law must be reducible to a nondimensional form, containing fewer variables. One such form is:

$$C_D = \frac{2}{\sqrt{\pi}}\mathrm{Re}^{-1}(1 + [\rho_a/\rho_w\sqrt{\nu_a/\nu_w}])^{-1} \tag{1.11}$$

with $C_D = \tau_i/\rho_a U^2$ a drag coefficient or nondimensional interface momentum flux, and $\mathrm{Re} = 2U\sqrt{\nu_a t}/\nu_a$ a Reynolds number based on air-side boundary layer thickness. Counting the density ratio and the viscosity ratio as two separate parameters, the nondimensional version of the transfer law contains four variables, versus seven in the dimensional formulation. The reduction by three corresponds to the three units of measurement – mass, time and length – quantifying the dimensional variables.

In Equation 1.11, the density-viscosity ratio term is small compared to unity, so that a sufficiently accurate form of the transfer law is the much simpler: $C_D = \frac{2}{\sqrt{\pi}}\mathrm{Re}^{-1}$. A lesson to be learned here is that not all variables playing a role in momentum transfer necessarily have a significant impact on the interface transfer law: Nobody could argue that the density or viscosity of water is irrelevant to momentum flux, yet neither significantly affects it in this example.

Does the bulk relationship, Equation 1.10, conform to Onsager's theorem? The total energy dissipation is the integral of the local value $\rho\nu(du/dz)^2$. Using the approximate formula taking into account air-side resistance only, neglecting u_0, and integrating on the air side from zero to infinity, we find the total dissipation to be $U^2/(\sqrt{2}R)$, or momentum Flux U/R times $U/(\sqrt{2})$. The latter is then the conjugate Force in the bulk version of the viscous transfer law.

The laminar flow example treated here is an overidealization of conditions near the sea surface, but its overriding weakness is that hydrodynamic instability causes laminar shear flow to break down into the chaotic motions of turbulence in a very short time. In turbulent flow, different and more complex laws govern momentum transfer. The one important feature of the laminar momentum transfer law that carries over into turbulent flow is that the air-side Resistance still dominates. Perhaps paradoxically, this is because, whichever way momentum gets across the interface, the light air still has a hard time moving the much heavier water around.

1.3 Turbulent Flow Over the Sea

1.3.1 Turbulence, Eddies and Their Statistics

Turbulence consists of a continuous succession of chaotic movements by parcels of fluid, analogous perhaps to molecular agitation, but occurring on a much larger than molecular scale. Moving parcels of fluid displace other fluid that eventually has to fill in the space vacated. This is known as continuity. Irregular and ephemeral closed flow structures arise in this manner, loosely called eddies. The details of eddy motion are complex, yet "stochastic" average properties of the flow (averages over many "realizations" in statistical theory, time-mean properties in practice) obey ascertainable laws, not unlike laws that quantify the macroscopic effects of molecular agitation.

The chaotic motions of turbulence are three-dimensional, so that at a fixed point there are velocity fluctuations along all three coordinate axes, u', v', w', even if the mean velocity has the same "alongwind" direction, $\overline{u} > 0, \overline{v} = 0, \overline{w} = 0$ (primes distinguish fluctuations from mean quantities carrying overbars). The mean square velocity fluctuations are then nonzero and their square roots provide measures of eddy velocity, a velocity "scale," such as $u_m = \sqrt{\overline{u'^2}}$. They also define the important Turbulent Kinetic Energy, TKE per unit mass in J kg^{-1}:

$$E_t = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \tag{1.12}$$

Eddies also stir up the fluid, and if some fluid property is unevenly distributed, they try to equalize it. Thus, when mean flow momentum $\rho\overline{u}$ varies in the vertical, fluctuating vertical eddy motions of velocity w' bring faster fluid from the momentum-rich region, which locally appears as excess velocity, positive u'. Averaged, the effects of these eddy motions add up to eddy transport of momentum, $\rho\overline{u'w'}$, also known as Reynolds flux of momentum or Reynolds stress, after Osborne Reynolds who first formulated equations of motion for a turbulent fluid with Reynolds stresses included.

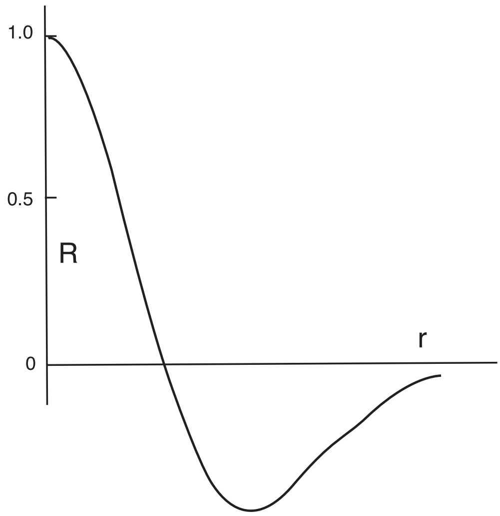

Figure 1.2 Typical correlation function of a turbulent velocity component, $\overline{w'(x)w'(x+r)}$, against distance r in the perpendicular direction. Adapted from Townsend (1956).

Another important turbulence property is characteristic eddy size. This can again be assigned only from statistical properties of the flow, traditionally from a two-point correlation function, such as $\overline{w'(x)w'(x+r)}$, the mean product of the vertical velocity component at two along-wind locations a distance r apart. See Townsend (1956) for a fuller discussion. Figure 1.2 shows the typical shape of such a correlation function, with a negative correlation loop required by continuity, on the principle of "what goes up must come down." The distance where the correlation function drops to zero is a measure of eddy size, or a "length-scale" of turbulence, say ℓ.

Correlation functions contain more information. According to a well-known theorem of statistics, a two-point velocity correlation function is the Fourier transform of an energy spectrum that assigns portions of kinetic energy to wavenumbers k (radians per unit length), and vice versa, the spectrum function $\phi(k)$ is the Fourier transform of the correlation function. The most useful correlation function in this context is $\overline{u'(x)u'(x+r)}$, between alongwind velocity fluctuations at downwind distances. The corresponding energy spectrum of turbulence peaks at a wavenumber k_p, which is close to ℓ^{-1}derived from the $\overline{w'(x)w'(x+r)}$ correlation. An alternative choice for eddy length scale is then $\ell = k_p^{-1}$. A physical interpretation of the spectrum is that reciprocal wavenumbers are characteristic dimensions of smaller and larger eddies, the values of the spectrum function a measure of their energy.

Apart from length and velocity scales, an important property of turbulence is the rate at which it dissipates energy, conventionally denoted by ε, in W kg^{-1}. Energy dissipation is the work of the sharpest instantaneous velocity gradients that occur in the eddying motion; viscous shear stress times the velocity gradient being the rate at which mechanical energy is converted into heat. Laboratory observations of many different types of turbulent shear flow revealed the general "similarity" principle that the dissipation rate is proportional to u_m^3/ℓ, varying from one part of the flow to another with the velocity and length scales as this product does. The proportionality constant changes, however, with the boundary conditions on the shear flow, as well as with the different possible choices for velocity and length scales of the turbulence.

The same similarity principle applies to other properties or effects of turbulence and constitutes the great simplifying factor in an otherwise almost untreatably complex phenomenon: once we have information on the variation of the velocity and length scales of turbulence in space or time, we are often able to quantify other properties of the flow. This was first recognized by Ludwig Prandtl, who introduced the concept of a "mixing length" for the eddy length scale, and used it to considerable advantage in constructing theories for different species of turbulent shear flow subject to simple boundary conditions, such as the flow in boundary layers, jets and wakes. The empirical finding, that two independent variables characterizing a turbulent shear flow are sufficient to describe other flow properties, is analogous to the thermodynamic principle that two state variables are all that is needed to determine different properties of a pure substance.

Energy dissipation occurs in the sharpest velocity gradients and therefore at the smallest scales (i.e., at the highest wavenumbers). Kolmogorov (1941) hypothesized that the spectrum function well beyond the peak of the spectrum toward the dissipative range (in the "inertial subrange") depends only on the wavenumber k and the energy dissipation ε (instead of separately on ℓ and u_m). This implies by dimensional reasoning:

$$\phi(k) = a\varepsilon^{2/3}k^{-2/3} \tag{1.13}$$

with a a constant, equal to 0.47 according to Lumley and Panofsky (1964). Observations of the spectrum in the intermediate range thus yield the rate of energy dissipation. Recalling that ε is proportional to u_m^3/ℓ, known ℓ allows the velocity scale to be determined: This is the basis of the so-called "dissipation method" of determining wind stress (see below).

According to the similarity principle of turbulence, the Reynolds stresses should be proportional to density times the square of the velocity scale, $-\rho\overline{u'w'} = const.\rho\overline{u'^2}$, choosing u_m for the velocity scale, as suggested above. An alternative legitimate choice for the velocity scale is therefore the "friction velocity" $u^* = \sqrt{-\overline{u'w'}}$, particularly useful where the Reynolds stress is constant in a region of the flow. This is (nearly) true of the airflow at low levels over the sea, where the Reynolds stress differs little from τ_i, the effective shear force on the interface (that includes any pressure forces acting on wind waves), alias momentum flux from air to water.

1.3.2 The Air-side Surface Layer

Air flow above the sea is variable, but changes in atmospheric conditions take place slowly enough to regard the mean wind speed steady at a few tens of meters above the interface, in what we will call a surface layer. Nor does the mean wind direction vary noticeably with height in this layer, only the wind speed: $\overline{u} = U(z)$, $\overline{v} = 0$, $\overline{w} = 0$. The mean velocity is thus a function only of the distance z above the mean position of the

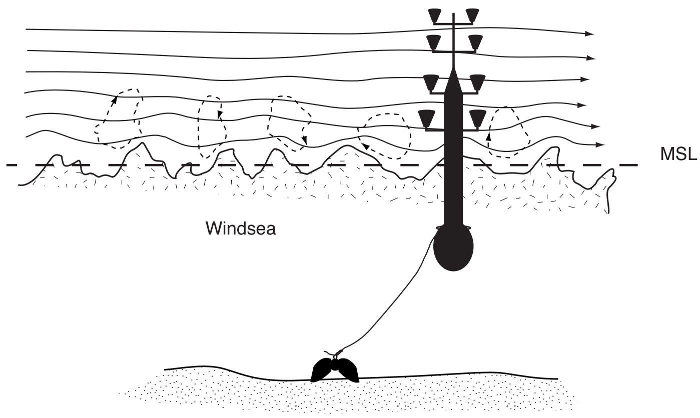

Figure 1.3 The windsea with air flow and eddies over it, and a spar buoy carrying anemometers recording the wind speed at different levels.

interface (known as the Mean Sea Level, MSL). What makes the problem of observing any property in the surface layer very difficult, is that, under wind, the interface is a highly irregular surface that also rapidly changes its shape. The visible structures on the wind-blown interface are wind waves in common parlance, but to avoid even a suggestion of regular parallel-crested water waves we will call them collectively the "windsea." A U.S. Navy Hydrographic publication (Bigelow and Edmondson, 1947) distinguishes between "sea" and "swell," two different wave-like phenomena, "sea" under storms, "swell" what is left over from a storm, more or less regular parallel-crested waves propagating away from the region where the storm generated the "sea." Windsea is a less confusing name than sea, and is certainly descriptive.

Figure 1.3 illustrates the surface layer above the windsea, indicating the air flow, eddies possibly tied to individual irregular waves, and a spar buoy with instruments to observe the mean wind at different levels. Smith (1978) gave details of such a "stable platform"; it was designed to withstand waves of 18 m height crest to trough, albeit protruding only 12.5 m above MSL. In moderate winds, waves are only 2 m height or less, and instruments on platforms similar to Smith's (e.g., fixed towers or ship masts) are able to determine the mean wind at several levels.

Such observations have revealed that, some distance above the windsea, the turbulent air flow has the same character as over a solid boundary, in what is known as a "wall layer." The mean velocity increases with distance above the sea surface, while the Reynolds flux of momentum, $\rho\overline{u'w'}$, that dwarfs viscous stress, is approximately constant with height from just above the waves to 10 m or more, and equal to the effective interface stress τ_i. The latter, the net horizontal force on the interface, includes pressure forces on the inclined surfaces of wind waves, as well as shear stress. The effective interface shear stress defines the friction velocity, $u^* = \sqrt{\tau_i/\rho}$, which then serves as the velocity scale of the turbulent flow in the entire surface layer. Above the waves, where the flow has the character of a wall layer, also described as the constant stress layer, observations have shown the eddy length scale ℓ to be proportional to the distance above the smoothed air-sea interface, z. Other properties of turbulent flow in this region are then expressible in terms of these two scales.

One effect of the eddies is to smooth out mean velocity variations, acting much as viscosity. According to the similarity principle, the vertical gradient of the mean velocity should depend only on the velocity and length scales of the eddies, i.e., by dimensional reasoning:

$$\frac{z}{u^*}\frac{dU}{dz} = \kappa^{-1} = const. \tag{1.14}$$

where the constant κ has the empirically determined value of about 0.4, and is known as Kármán's constant, after one of the great fluid dynamicists of the early twentieth century. Integration from some reference level z_r now results in:

$$\frac{U(z)}{u^*} = \frac{U(z_r)}{u^*} + \kappa^{-1} \ln\left(\frac{z}{z_r}\right). \tag{1.15}$$

The reference level is arbitrary, except that it has to be in the constant stress layer, where the velocity and length scales of the eddies are u^* and *const.* z. The velocity at a given level $U(z)$ must be independent of the choice of z_r, however, implying a relationship between reference level height and velocity:

$$U(z_r) - u^*\kappa^{-1}\ln(z_r) = const. \tag{1.16}$$

For the right-hand side constant not to depend on the unit of length or time, it must have the dimension of a velocity, and contain a constant times the logarithm of a length. Writing r for that length, $Cu^* - u^*\kappa^{-1}\ln(r)$ for the right-hand side with C a dimensionless constant, we arrive at the following form of the velocity distribution:

$$\frac{U(z)}{u^*} = \kappa^{-1} \ln\left(\frac{z}{r}\right) + C. \tag{1.17}$$

We anticipate the length r and the constant C to depend on the interplay of the windsea and the air-side turbulence. For the present, they are two empirical parameters of the velocity distribution over the windsea.

Countless observations support this "logarithmic law" in the atmospheric surface layer over the sea. Roll (1965) lists fourteen sets of field observations that do so over various natural water surfaces. In semi-logarithmic representation, at constant u^*, the velocity distributions, $U(\ln z)$ are straight lines, displaced upward or downward according to how much velocity change occurs between the interface and the top of the waves. That displacement depends on just how vigorously the wave-bound eddies stir up the air: the more stirring, the less velocity change. The stirring is the work of the windsea.

1.3.3 Properties of the Windsea

The waves of the windsea are just as chaotic as turbulence, and under simple conditions their stochastic average properties also obey simple laws. Surface elevation is the windsea analog of velocity in turbulence, a random function of time at a fixed location or of location at a fixed time, that defines a frequency or wavenumber spectrum. Chapter 2 discusses windsea properties in detail; here we only catalog the wave-related variables that might influence air-sea momentum flux, with a view to connecting the two empirical parameters in Equation 1.17 to properties of the windsea.

Under a steady wind, and in the absence of waves originating from a distant storm ("swell"), the phase velocity $C_p = \sqrt{g/k_p}$ of a gravity wave (g is the acceleration of gravity, k_p the wavenumber at the peak of the spectrum) defines the "characteristic wave." To a casual observer, larger waves appear to progress with phase velocity C_p, and to have a dominant wavelength of $2\pi/k_p$, amidst much other complexity.

Everyday observation shows that the height of the characteristic wave grows with distance from an upwind shore. Far enough from such a shore, the wave field becomes saturated, and the wave height stops growing. Here the height of the characteristic wave, $H_{1/3}$, defined as the average height of the 1/3 highest waves, depends only on friction velocity $u^* = \sqrt{\tau_i/\rho}$ and g. Thus, $gH_{1/3}/u^{*2} = const.$, with similar relationships for other wave properties. Notice that u^{*2}/g is a waveheight-scale, u^* a common velocity scale of the windsea and the surface layer turbulent shear flow.

Closer to an upwind shore, under a steady and horizontally uniform wind, and again in the absence of swell, waves grow from small to large waveheight with distance from shore (with "fetch" F), the characteristic wave's phase velocity increasing, wavenumber decreasing in the process. Under these idealized conditions (in "local equilibrium" with the wind), properties of wind waves depend on fetch F as well as on friction velocity u^*, and gravitational acceleration g. The phase velocity of the characteristic wave, C_p, or its nondimensional version C_p/u^* (known as "wave-age") serves as a surrogate variable for fetch, F. Nondimensional long-wave properties, such as $gH_{1/3}/u^{*2}$, only depend on one nondimensional parameter, conveniently wave-age.

The properties of the shortest surface structures, not always wave-like, also depend on surface tension σ. The kinematic version of this variable, $\gamma = \sigma/\rho$, is convenient in dimensional argument.

Wind waves facilitate momentum transfer, because horizontal pressure forces may act on their inclined faces, and contribute to τ_i, the net force of the air on the water surface per unit horizontal area. Pressure and shear forces on the interface are also what cause waves to grow with fetch. When waves decay, they hand over momentum to the water-side shear flow, adding to the momentum transferred from the air via viscous shear stress. Even while wind waves grow, they also continuously lose momentum to the water-side shear flow, to a small extent through viscous and turbulent drag on orbital motions, but mostly through "breaking," a complex turbulent overturning motion.

From our point of view in this chapter, wind waves may be thought to open another pathway of air-sea momentum transfer, on top of viscous shear. Somewhat surprisingly, while the long waves of the spectrum carry most of the horizontal momentum transport of the wind wave field, they neither gain nor lose momentum very fast, except on beaches, or perhaps in very strong winds. Short waves of the spectrum, on the other hand, are steep, efficient at extracting momentum from the air flow, prone to breaking, and thus short-lived. Circumstantial evidence suggests that they are responsible for a considerable fraction of the total air-sea momentum transfer. Viscous shear stress meanwhile remains active in momentum transfer: It is difficult to imagine circumstances under which fast air flow in contact with short or long waves would not exert viscous stress.

Returning to Equation 1.17, this summary of wave effects shows that the parameters r and C, quantifying wave influence on the velocity distribution in the surface layer, could depend on the force of gravity g, kinematic surface tension γ, friction velocity u^*, and the nondimensional parameter of wave age C_p/u^*. Viscosity and density of air and water still influence the shear-stress pathway of momentum transfer and should not be forgotten.

1.4 Flux and Force in Air-Sea Momentum Transfer

The flux of momentum from air to sea, alias effective interface shear stress $\tau_i = \rho u^{*2}$, is the Flux we wish to relate to a conjugate Force. In the bulk version of the viscous momentum transfer law, Equation 1.10, we found the velocity $U/\sqrt{2}$, realized in the upper portion of the growing air-side boundary layer, to be the conjugate Force. Something similar should prove a suitable choice again, the wind speed at a level well above the waves, say at $z = h$. The standard practical choice is $h = 10$ m. The Force $U(h)$ is then supposed to drive the Flux $\tau_i = \rho u^{*2}$. Putting $z = h$ in Equation 1.17 converts it into a complex implicit relationship between Flux and Force:

$$\frac{U(h)}{u^*} = \kappa^{-1} \ln\left(\frac{h}{r}\right) + C \tag{1.18}$$

where r and C also depend on u^*, as well as on other wave parameters, as just discussed. A nondimensional form of their functional relationship is:

$$C, \frac{gr}{u^{*2}} = func.\left(\frac{C_p}{u^*}, \frac{\gamma g}{u^{*4}}\right). \tag{1.19}$$

If viscosity has a significant effect on interface processes, also ν_a/ν_w and $u^{*3}/g\nu_w$ should be considered. Dividing the nondimensional Force $U(h)/u^*$ by the square root of gh/u^{*2} results in $U/\sqrt{gh}$, a more appropriate nondimensional Force, not containing the conjugate momentum Flux. The similar $u^*/\sqrt{gh}$ is a convenient nondimensional Flux variable. It should also be remembered here that the height h is a proxy for eddy size in the surface layer, an important physical factor in momentum transfer, not the incidental location of a recording instrument.

Equation 1.19, with possibly the viscous variables added, suggests a fairly complex momentum transfer law. The example of the laminar flow transfer law suggests, however, that some of the possible influences may not be noticeable. A drastic simplification would be if instead of Equation 1.18 we had just $U(h)/u^* = const$. A hypothesis to this effect in fact guided early years of research on momentum transfer, when the focus was on the wind-speed dependence of the momentum flux. Constant

$u^*/U(h)$ means constant drag coefficient $C_D = u^{*2}/U^2$. Within the limited wind speed range explored, and in light of considerable scatter in the observed value of the drag coefficient, a constant value seemed then a reasonable conclusion. Most data on momentum transfer are still presented today in the form: drag coefficient versus (dimensional) wind speed. The latter may be taken to be a proxy for nondimensional $U(h)/\sqrt{gh}$, with the denominator a constant scale velocity of about 10 m/s, for the usual reference height of $h = 10$ m.

1.4.1 Charnock's Law

Later work revealed that the drag coefficient increases with wind speed. Almost half a century ago, Charnock (1955) reported the distribution of wind speed with height over a reservoir, and expressed the results in the form:

$$\frac{U(z)}{u^*} = \kappa^{-1} \ln\left(\frac{gz}{u^{*2}}\right) + C \tag{1.20}$$

where C is a constant, not for just one velocity profile but at all observed wind speeds and directions, and according to Charnock approximately equal to 12.5.

Putting $z = h$ in Equation 1.20 brings it to the form of Equation 1.18, with $r = u^{*2}/g$, the waveheight scale, C a universal constant. We shall refer to it as Charnock's law. Another way to write it is:

$$\frac{U(h)}{\sqrt{gh}} = \frac{u^*}{\sqrt{gh}}\left[C - 2\kappa^{-1}\ln\left(\frac{u^*}{\sqrt{gh}}\right)\right]. \tag{1.21}$$

Because $u^*/\sqrt{gh}$ is always much less than 1.0, its logarithm is negative, so that the square-bracketed expression is positive, with a value typically around 30, and slowly decreasing with increasing u^*. Alternative statements of Charnock's law are:

$$C_D = (C - 2\kappa^{-1}\ln[u^*/\sqrt{gh}])^{-2}$$

$$R = (C - 2\kappa^{-1}\ln[u^*/\sqrt{gh}])\Big/u^*$$

with C_D the drag coefficient, R the Resistance to momentum transfer. The most convenient graphical representation of the law is friction velocity against wind speed, $u^* = \mathit{func}\,[U(h)]$, or the inverse of Equation 1.21, as Amorocho and DeVries (1980) pointed out some years ago. This minimizes the scatter of observed values.

1.4.2 Sea Surface Roughness

Over a "rough" solid surface, the experiments of Nikuradse (1933), using walls roughened by glued-on sand-grains of mean diameter r, showed the velocity distribution in the wall layer to be:

$$\frac{U(z)}{u^*} = \kappa^{-1}\ln\left(\frac{z}{r}\right) + 8.5. \tag{1.22}$$

A comparison with Charnock's law leads to the result that the sea surface behaves as a solid surface of "sand-grain roughness" r, where:

$$r = 3.064\frac{u^{*2}}{g}. \tag{1.23}$$

To take a typical situation, an 8 m s^{-1} wind calls forth a friction velocity of $u^* = 0.3$ m s^{-1}, and a sand-grain roughness of the sea surface of $r = 3$ cm or so. This contrasts with a characteristic waveheight of some 1.2 m in this wind at long fetch. Sand-grains 0.03 m in diameter closely packed on a smooth surface would mimic short waves in their effects on the velocity distribution over the sea surface in an 8 m s^{-1} wind. The comparison suggests that the sand-grain roughness length according to Equation 1.23 reflects the height of the surface disturbances mainly responsible for the drag of the air on the sea surface. We should add the caveat that the analogy with solid roughness is imperfect because wind waves are mobile, solid roughness elements are not, so that the mechanisms of momentum transfer may differ between them. To the extent that the analogy holds, it singles out short waves as the primary transferrers of momentum.

Meteorologists have fallen into the habit of reporting data on air-sea momentum transfer in terms of a "roughness parameter" z_0 (a length) that combines r and C, defined by the following alternative statement of the logarithmic law:

$$\frac{U(z)}{u^*} = \kappa^{-1} \ln\left(\frac{z}{z_0}\right).$$

The parameter z_0 according to Charnock's law, with constant $C = 12.5$, is:

$$z_0 = 0.011\frac{u^{*2}}{g} \tag{1.24}$$

some 300 times smaller than the sand-grain roughness, with no relationship at all to any observable structures on a wind-blown sea surface.

One important point about the "roughness" of the sea surface, whichever way it is quantified, is that it is not an externally imposed parameter of the dimension of length. It arises from wind action on the water surface and could in principle depend on any or all of the wave parameters as well as viscosity. To the accuracy, and within the range of validity, of Charnock's law, it depends only on the two parameters u^* and g. To use a sea-surface roughness length in dimensional analysis as an external variable, side by side with u^* and g, is a serious conceptual error (unfortunately not uncommon, e.g., Maat et al., 1991).

1.4.3 Energy Dissipation

Does Charnock's law pass muster in nonequilibrium thermodynamics by conforming to Onsager's theorem? With $U(h)$ the Force, ρu^{*2} the Flux, their product equals by Charnock's law:

$$\rho u^{*2} U(h) = \rho u^{*3}\left(\kappa^{-1}\ln\left(\frac{gh}{u^{*2}}\right) + C\right). \tag{1.25}$$

The left-hand side of this equation is the work done by the wind stress on the air layer underneath the level h, energy transfer downward. As this is unquestionably dissipated in some manner by the underlying shear flow in air and water, and by the windsea, Onsager's theorem is satisfied by the bulk relationship that we call Charnock's law.

To examine the details of energy dissipation, we need the Turbulent Kinetic Energy (TKE) equation, derived from Reynolds' equations of motion for a turbulent fluid (see e.g., Businger 1982), in a simplified form, valid for the constant stress layer with unidirectional flow:

$$\frac{\partial \overline{E_t}}{\partial t} = -\overline{u'w'}\frac{dU}{dz} - \frac{\partial(\overline{w'p'}/\rho + \overline{w'E_t'})}{\partial z} - \varepsilon \tag{1.26}$$

where E_t is TKE defined in Equation 1.12, as energy per unit mass. Multiplied by density, the first term on the right contains the Reynolds stress $-\rho\overline{u'w'} = \tau_i = \rho u^{*2}$ multiplied by the mean velocity gradient, clearly the local Flux-Force product analogous to what we have seen in viscous momentum transfer. In this equation, the Flux-Force product plays the role of TKE production rate. The second term on the right is a divergence, of "pressure work," the velocity-pressure correlation, plus vertical flux of E_t. The divergence represents transfer of energy from one level to another; integrated from the interface up to some level, it yields pressure work transferring energy to wind waves. In the (nearly) constant stress layer above the waves, E_t is constant by the similarity principle, so that its flux, and its time-derivative on the left, both vanish. The same similarity principle also yields the velocity gradient (see Equation 1.14 on page 11), and the energy dissipation rate $\varepsilon = \rho u^{*3}/\kappa z$, that turns out to equal the TKE production rate (i.e., the local Flux-Force product). The flux-divergence term is then also insignificant above the waves. All these relationships are approximate and valid only from some level above the waves to levels where the Reynolds stress remains close to τ_i.

Integrating local energy dissipation from the lowest conceivable level where the constant stress layer formula holds, $z = u^{*2}/g$, to $z = h$, we find:

$$\int_{u^{*2}/g}^{h} \frac{\rho u^{*3}}{\kappa z}\,dz = \frac{\rho u^{*3}}{\kappa}\ln\left(\frac{gh}{u^{*2}}\right) = \rho u^{*2}(U(h) - Cu^{*}) \tag{1.27}$$

where the second equality comes from Charnock's law, showing the integrated dissipation to equal the downward energy transfer at level h, minus the downward energy transfer at level u^{*2}/g. With C about 12, $U(h)$ some $30u^{*}$, only 60% of the downward energy transfer is dissipated between the integration limits, the rest handed down to lower layers. Because u^{*2}/g is typically only 1 cm, most of the remaining dissipation must take place on the water side. It is indeed already stretching a point to suppose constant stress layer formulae valid so close to the sea surface, so that the energy transfer to the water side may be even greater than $C\rho u^{*3}$.

How does this compare with viscous energy transfer across the interface? If all of the effective interface stress $\tau_i = \rho u^{*2}$ were viscous stress, the energy transfer would equal $\tau_i u_0$, with u_0 the velocity at the interface. Typically, the interface velocity is $u^{*}/3$, some 36 times smaller than Cu^{*}, so that downward energy transfer at the bottom of the constant stress layer dwarfs what viscous shear stress alone could conceivably accomplish. We return to this point in Chapter 3, where Figure 3.8 shows that, within one waveheight below the interface, energy dissipation is some 30 times greater than what turbulent shear flow on the water side is responsible for.

1.4.4 Buoyancy and Turbulence

The premises underlying Charnock's law are that in the constant stress layer over the sea, properties of turbulent flow depend only on the velocity and length scales, u^*, z, and that wave influences are expressible through the same velocity scale u^* plus the acceleration of gravity, g. In physical terms, the two chaotic processes arising from hydrodynamic instability, shear flow turbulence and windsea, between them govern air-sea momentum transfer, both behaving as, say, a perfect gas, their properties depending on just two variables.

The simple scaling of the turbulent flow in the constant stress layer no longer holds, however, when air density varies owing to heating, cooling, or evaporation. Upward sensible heat transfer or evaporation from the sea surface makes lower layers of air lighter than layers above, an unstable arrangement that results in chaotic gravitational convection, a species of turbulence different from the shear flow variety. Downward heat flux at the sea surface, on the other hand, generates air heavier than above, a stable arrangement. Whatever the source of density variations, the differential gravity force on a parcel lighter than its environment, known as buoyancy, tends to speed up upward eddy motions, while their greater than average density propels heavier parcels downward. Upward heat flux or evaporation thus intensifies vertical eddy motion, while downward heat flux does the opposite, retarding vertical motions arising from shear flow turbulence. Vertical eddy momentum transport, the Reynolds stress $-\rho\overline{u'w'}$, then also depends on heat and vapor fluxes at the air-sea interface.

The proximate cause of enhanced or depressed eddy motion in the presence of heat or vapor flux is the buoyancy or net upward gravitational force per unit mass, acting on a fluid parcel that is slightly lighter or heavier, as the case may be, than its average environment. If the density anomaly is ρ', the buoyancy is $b' = -g\rho'/\overline{\rho}$. The density *defect* $-\rho'$ is in turn proportional to the excess temperature θ' and excess vapor concentration χ' of an air parcel, $-\rho'/\overline{\rho} = \theta'/T + 0.61q'$, where T is absolute temperature and $q' = \chi'/\overline{\rho}$, specific humidity excess, q the standard variable in meteorology representing vapor concentration. The factor 0.61 comes from the different molecular weights of water vapor and air (see e.g., Garratt, 1992).

Eddy motions transport heat and humidity just as they transport momentum. The Reynolds fluxes of temperature and humidity are $\overline{w'\theta'}$ and $\overline{w'q'}$; the corresponding Reynolds flux of buoyancy is:

$$\overline{w'b'} = \frac{g}{T}\overline{w'\theta'} + 0.61g\overline{w'q'}. \tag{1.28}$$

Positive (upward) flux of heat or vapor implies positive buoyancy flux $\overline{w'b'}$.

From another point of view, the product of upward velocity and positive buoyancy force represents work done on the air parcel by the force of gravity, tending to increase its kinetic energy. By the same token, downward buoyancy flux implied by downward Reynolds flux of heat and vapor means work done against gravity, a sink for the kinetic energy of moving air parcels. The parcels must then somehow gain energy to sustain their motions. This interplay of turbulence and buoyancy is portrayed by the turbulent kinetic energy (TKE) equation, expanded from its form in Equation 1.26 to include buoyancy work:

$$\frac{\partial \overline{E_t}}{\partial t} = \overline{w'b'} - \overline{u'w'}\frac{dU}{dz} - \frac{\partial(\overline{w'p'}/\rho + \overline{w'E_t'})}{\partial z} - \varepsilon. \tag{1.29}$$

The first term on the right represents energy gain or loss on account of buoyancy, the other terms are as discussed above.

As we have seen, in the absence of buoyancy flux, dependence of turbulence properties on only two scales implies that both the production and the dissipation terms are proportional to u^{*3}/z. The buoyancy flux in the constant stress layer depends, however, on heat and vapor fluxes imposed at the boundary, that is on other independent variables, and it cannot vary with just u^* and z. This then implies that some or all other terms in the TKE balance must vary with the buoyancy flux. The key additional external variable affecting the TKE balance is the interface buoyancy flux $B_0 = \overline{w'b'}(0)$. The properties of the mean flow as well as of the turbulence then depend on B_0 as well as on u^* and z. A modified similarity principle for the buoyancy-affected shear flow is that its properties depend on these three parameters only (Monin and Yaglom, 1971).

One way to take interface buoyancy flux into account is by means of a length scale, L, introduced into the literature by Obukhov (1946):

$$\frac{1}{L} = -\frac{\kappa B_0}{u^{*3}} \tag{1.30}$$

which serves as a proxy for B_0 in dimensional argument. Apart from the constant κ and the negative sign, both retained here to conform to historical custom, the Obukhov length contains only the two interface fluxes, of momentum (represented by u^*) and buoyancy. Negative B_0 or positive L signifies energy drain on the turbulence, positive B_0 or negative L extra energy supply. The meteorological literature refers to these as stable and unstable conditions, respectively.

Velocity gradients in the shear layer above the waves now depend on B_0, represented by L, as well as on u^* and z. Dimensional analysis leads to the following expanded version of Equation 1.14:

$$\frac{dU}{dz} = \frac{u^*}{\kappa z}\phi\left(\frac{z}{L}\right) \tag{1.31}$$

with $\phi(z/L)$ an unspecified function. Under "neutral" conditions, when the air is neither stable nor unstable, i.e., at vanishing B_0, hence $z/L \to 0$, $\phi(z/L)$ must tend to unity. Large positive B_0 generates vigorous convection and reduces surface stress-induced mechanical turbulence to insignificance. At moderately high positive B_0, or z/L of

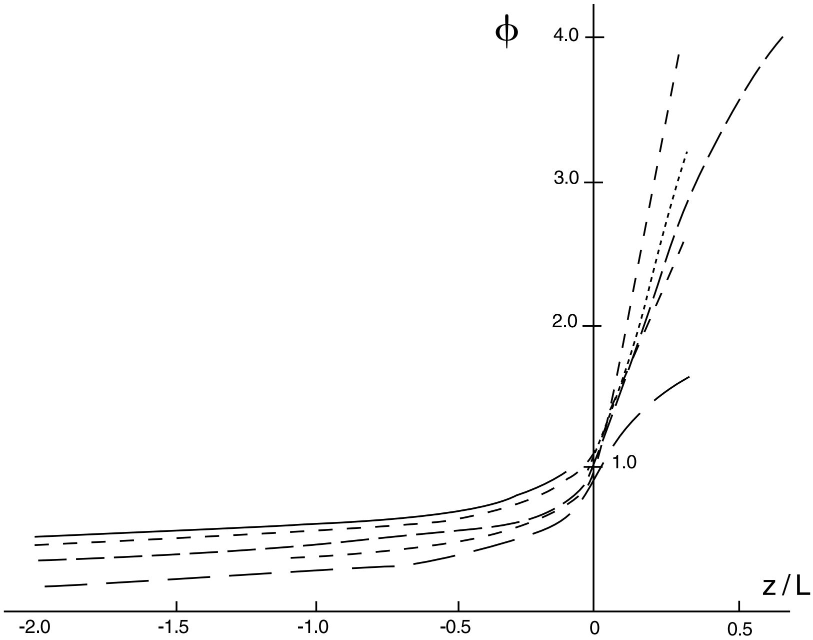

Figure 1.4 The empirical stability function $\phi(z/L)$ as recommended by different authors. Even at z/L close to zero, the uncertainty is seen to be high. From Yaglom (1977).

order -1, compound mechanical-convective turbulence prevails and Equation 1.31 is useful. At the other extreme, large negative buoyancy flux overwhelms mechanical turbulence to the point of completely eliminating it. At moderately high negative B_0 (i.e., positive and suitably small z/L), Equation 1.31 is again valid. The negative buoyancy flux in the TKE equation signifies work against gravity, that is, increase of potential energy as eddies bring lighter fluid down from higher levels. The production term must balance this loss of TKE, resulting in less vigorous shear flow turbulence, and sharper mean velocity gradients.

Boundary layer meteorologists have explored buoyancy effects on the atmospheric surface layer in detail and proposed several different empirical formulae for the function $\phi(z/L)$, separately for stable and unstable conditions. Figure 1.4 after Yaglom (1977) shows some of these. We may conclude from the differences between the formulae that the corrections are known only within a factor of two, and that only at small $|z/L|$. Most widely used are the formulae summarized by Deardorff (1968); they are, in the stable case, $L > 0$:

$$\phi\left(\frac{z}{L}\right) = 1 + \beta\frac{z}{L} \tag{1.32}$$

and in the unstable case, $L < 0$:

$$\phi\left(\frac{z}{L}\right) = \frac{1}{(1 - \alpha z/L)^{1/4}} \tag{1.33}$$

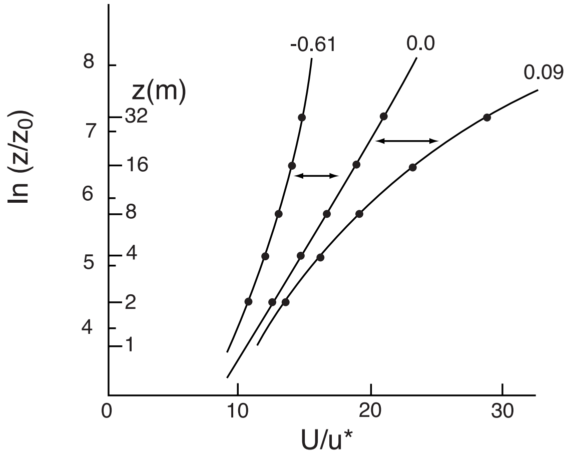

Figure 1.5 Velocity profiles over land in stable, neutral, and unstable conditions, marked by a parameter ("gradient Richardson number"), of the same sign as, but inversely proportional to, the Obukhov length L. Arrows mark typical departures from the logarithmic neutral profile, of some $\Delta U = 5u^*$. Over the ocean, typical departures are generally somewhat less. From Garratt (1992).

with α, β constants. Integration now recovers the logarithmic law plus correction terms depending on z/L:

$$\frac{U(z)}{u^*} = \kappa^{-1} \ln\left(\frac{z}{r}\right) + C + \kappa^{-1}\psi\left(\frac{z}{L}\right) \tag{1.34}$$

with:

$$\psi\left(\frac{z}{L}\right) = \beta\frac{z}{L}$$

in the stable case; while in the unstable case we have:

$$\psi\left(\frac{z}{L}\right) = -\left[\ln\left(\frac{1+x^2}{2}\right) + 2\ln\left(\frac{1+x}{2}\right) - 2\tan^{-1}(x) + \pi/2\right]$$

where $x = (1 - \alpha z/L)^{1/4}$. In the unstable case, at constant u^*, the more vigorous turbulence reduces the wind speed at a fixed level, compared to the undisturbed wall layer, while less vigorous stirring under stable conditions increases it, (Figure 1.5).

Putting $z = h$ and $r = u^{*2}/g$ in Equation 1.34 yields a corrected form of Charnock's law that connects the Force $U(h)$ to the three interface fluxes, of momentum, heat and vapor, the latter two through B_0. Taking the buoyancy-related term in Equation 1.34 to the left-hand side, we are back at Charnock's law, but for a "corrected," or "neutral," nondimensional velocity, $U(h)/u^* - \kappa^{-1}\psi(h/L)$:

$$\frac{U(h)}{u^*} - \kappa^{-1}\psi\left(\frac{h}{L}\right) = \kappa^{-1}\ln\left(\frac{gh}{u^{*2}}\right) + C \tag{1.35}$$

with the same constant C as before, and with correction terms as given following Equation 1.34. This is how observations on momentum flux are usually reported, corrected for buoyancy flux to a neutral value of the wind speed.