Chapter 1
The Transfer Laws of the Air-Sea Interface

1.1 Introduction

Hurricane Edouard had just passed by Cape Cod when I wrote these lines, after giving us a good scare, and keeping meteorologists of local TV stations out of bed all night. Approaching on a track along the East Coast, Edouard remained a category 3 hurricane, with 180 km/h winds, from the tropics to latitude 38° N. This is where it left the warm waters of the Gulf Stream behind, quickly to lose its punch over the much cooler Mid Atlantic Bight, and to be degraded to category 1, with 130 km/h winds, still enough to uproot a few trees on the Cape.

Edouard’s fury came from water vapor, as it ascends the “eye-walls” (Figure 1.1) that surround a hurricane’s core, condensing and releasing its latent heat of evaporation. The heat makes the moist air buoyant, turning the eyewalls into a giant chimney with an incredibly strong draft. The draft sucks in sea-level air, causing it to spiral toward the core in destructive winds and to drive waters against nearby coasts in storm surges. The fast air flow over warm water also ensures intense heat and vapor transfer to the air, sustaining the hurricane’s strength. Over colder water, where not enough water evaporates, the hurricane dies: The lifeblood of a hurricane is intense sea to air transfer of heat and water vapor. On the other hand, as hurricane winds whip the waters along, they transfer some of their momentum downward. The loss of momentum acts as a brake on the hurricane circulation, keeping the winds from completely getting out of hand.

A hurricane also mimics on a small-scale the global atmospheric circulation, which is similarly “fueled” by latent heat released from condensing water vapor. This happens in “hot towers,” concentrated updrafts of the InterTropical Convergence Zone (ITCZ), and also in somewhat less vigorous updrafts within extratropical storms. Many of the
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Figure 1.1 Mean structure of a mature hurricane (“Helene,” 26 Sept. 1958) in cross section, supposing axial symmetry. The left-hand half shows the boundaries of the eye-wall (solid lines, bending outward with height) and illustrates the cloud structure. The broken lines are contours of constant “equivalent potential temperature,” the absolute temperature in degrees Kelvin that the air would have with all of the latent heat in its vapor content released, and the pressure brought down to sea level pressure. In the right-hand half section, thin full lines are contours of constant wind speed in m s\(^{-1}\) (the thick lines repeat the eye wall boundaries), the broken lines are angular momentum contours, the dotted lines contours of temperature in °C. The maximum wind speed is in excess of 180 km/h. Note the stratiform cloud (dashed lines in the left half) extending to 13 km height, to the top of the troposphere, where the temperature is \(-55°\)C, \(= 218\) K. Satellites see this “cloud-top” temperature. From Palmén and Newton (1969).

latter draw their vapor supply from the warm Gulf Stream and its Pacific counterpart, the Kuroshio, ocean currents transporting massive amounts of heat from warm to cold regions. Hot towers make their presence known to travelers crossing the equator, and wake them from their slumber when updrafts toss around their jetliner, as high as 10 or 12 km above sea level. Heat release in the updrafts, and compensating cooling and subsidence, are part of a thermodynamic cycle that energizes various atmospheric circulation systems, including the easterly winds of the tropics and subtropics, and the westerlies of mid-latitudes. The winds in turn sustain sea to air heat and vapor transfer, supplying the fuel, moist air, for the updrafts. The associated air to sea transfer of momentum from the winds is again the control on the strength of the atmospheric circulation.

Important to the operation of hurricanes and to large-scale atmospheric and ocean circulation systems is therefore in what amount, and by what mechanisms, momentum, heat and vapor pass from one medium to the other. The rates of transfer, per unit time
and unit surface area, depend on a variety of conditions and processes; relationships between the rates and the variables influencing them are the "transfer laws" of the air-sea interface that we seek in this chapter. As all laws of physics, these too are distilled from observation, and, as most such laws, they are more or less accurate approximations. Their establishment requires painstaking work, hampered by difficulties of observation at sea. After nearly a century of research by many scientists from a variety of nations, there are still many uncertainties affecting the transfer laws.

1.2 **Flux and Resistance**

Transfers of momentum, heat and mass, are all *irreversible* processes. A number of texts deal with irreversible molecular processes of transfer, viscosity, heat conduction or diffusion, but their common thermodynamic characteristics have only engaged the interests of scientists relatively recently. De Groot’s seminal synthesis (1963) bears the title “Thermodynamics of Irreversible Processes,” while a later development (De Groot and Mazur 1984) is called “Non-equilibrium thermodynamics.” These monographs develop the subject for molecular transfer processes, and show that their laws have the general form:

\[
\text{Flux} = \text{Force} / \text{Resistance}
\]

where the “Force” has the character of a potential gradient, the “Resistance” of inverse conductivity.

Irreversible processes change the entropy of the system in which they occur. Entropy changes because it flows in and out of the system, and also because internal irreversible processes generate it. The rate of entropy generation, the internal entropy “source” term in the entropy balance, is always positive, according to the Second Law of thermodynamics. “To relate the entropy source explicitly to the various irreversible processes that occur in the system” is the main preoccupation of nonequilibrium thermodynamics (De Groot and Mazur, 1984). When only one Force is acting, the entropy production rate is proportional to the product of Flux and Force. Absorbing the proportionality factor in the Force, entropy production can be made equal to the Flux-Force product. With several Fluxes and their conjugate Forces present, a similar standardization of the Forces yields the entropy production rate as the sum of the Flux-Force products, a result known as “Onsager’s theorem.”

The transfer laws of the air-sea interface are also relationships between Fluxes and Forces in the sense of nonequilibrium thermodynamics. They are, however, the result of an interaction between turbulent flows in air and water, and wind waves on the sea surface, and are more complex than linear relationships between a Flux and a Force with a constant Resistance. They are empirical laws of physics depending on material properties, properties of the turbulent flows in air and water, and of wind waves. Their
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usual form is an implicit Flux-Force relationship:

\[ \text{func} \left( \text{Flux, Force, } \sum_{i=1}^{n} X_i \right) = 0 \] (1.2)

where the \( X_i \) are \( n \) variables having measurable influence on the transfer law.

An important requirement of a physical law is that it must be independent of units of measurement. This dictates the use of a consistent system of units, and leads to Buckingham’s theorem, according to which all physical laws are expressible as relationships between nondimensional combinations of variables, in appropriate products and quotients. Therefore, yet another way to state the transfer law of Equation 1.2 is:

\[ \text{func} \left( \sum_{i=1}^{m} N_i \right) = 0 \] (1.3)

where \( N_i \) are nondimensional combinations of the variables, including the Flux and the Force. Their number, \( m \), is less than the \( n + 2 \) of Equation 1.2, usually by the number of measurement units in the dimensional relationship 1.2. Such formulations of the transfer laws are most useful if either the Flux or the Force appears in only one of the \( N_i \); that variable can then be treated as the dependent one, the others deemed independent.

De Groot and Mazur (1984) discuss Flux-Force relationships valid locally, between heat flux and temperature gradient, and analogous quantities in other irreversible processes, while the most useful formulation of the air-sea transfer laws is between a property difference across a layer of air above the interface, and the flux across the interface, in what we might call a “bulk” relationship. To illustrate the difference between local and bulk relationships, and also to give a taste of the classical results of nonequilibrium thermodynamics, next we discuss viscous momentum transfer in a simple situation.

### 1.2.1 Momentum Transfer in Laminar Flow

Suppose that air and water are two semi-infinite viscous fluids in contact at the \( z = 0 \) plane, with the upper fluid impulsively accelerated to a velocity \( u = U = \text{const.} \) at time \( t = 0 \). In the absence of other forces, and as long as the flow remains laminar and unidirectional, shear stress between layers accelerates the lower fluid while retarding the upper one. The shear stress \( \tau \), force per unit area, equals viscosity times velocity gradient (see e.g., Schlichting 1960):

\[ \tau = \rho \nu \frac{\partial u}{\partial z} \] (1.4)

where \( \rho \) is density and \( \nu \) is kinematic viscosity. A layer of fluid between two levels \( \delta z \) apart experiences a net force equal to the difference in shear, which then accelerates
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the fluid:

$$\frac{\partial (\rho u)}{\partial t} = \frac{\partial \tau}{\partial z}$$  \hspace{1cm} (1.5)$$

where the left-hand side is mass times acceleration or rate of change of horizontal momentum $\rho u$. A legitimate interpretation of this relationship is that the shear stress is equivalent to vertical flux of horizontal momentum, the difference of which across the layer increases the local momentum.

In this light, the previous relationship, Equation 1.4, is now seen as one between a Flux (of momentum) and a Force, the gradient of the velocity $\frac{\partial u}{\partial z}$, a local law, valid at any level $z$. The dynamics is contained in Equation 1.5. Multiplying that equation by $u$, we arrive at the energy balance:

$$\frac{\partial (\rho u^2/2)}{\partial t} = u \frac{\partial \tau}{\partial z}$$  \hspace{1cm} (1.6)$$

which, after rearrangement and substitution from Equation 1.4, transforms into:

$$\frac{\partial (\rho u^2/2)}{\partial t} = \frac{\partial}{\partial z} \left( \nu \frac{\partial (\rho u^2/2)}{\partial z} \right) - \rho \nu \left( \frac{\partial u}{\partial z} \right)^2.$$  \hspace{1cm} (1.7)$$

The first term on the right is the divergence of viscosity times the gradient of kinetic energy, legitimately interpreted as energy flux. The divergence of this quantity signifies vertical energy transfer from one location to another, leaving the total energy unchanged. The second term, however, is always negative, and signifies loss of mechanical energy, its transformation into heat through viscosity. The heat added to the air or water increases its entropy at the rate of heat generation divided by absolute temperature. This then is the entropy source term, locally, level by level, equal to the product of the Force $\frac{\partial u}{\partial z}$ and (by Equation 1.5) the Flux $\tau$, conforming to Onsager’s theorem.

The fluid properties, viscosity and density, are constant in either medium, but change at the interface: They will bear indices $a, w$, for air above, water below. Writing down Equations 1.4 and 1.5 separately for air and water, and eliminating $\tau$, we have two second order differential equations for $u$ to solve. The boundary conditions are as follows: Far above the interface the velocity is the undisturbed $U$, far below it is zero. At the interface, the velocity and the shear stress are continuous. The solution follows the standard approach to such problems, see e.g., Carslaw and Jaeger (1959). The results are:

$$u_a = u_0 \text{erfc} \left( \frac{z}{2 \sqrt{\nu a t}} \right) + U \text{erf} \left( \frac{z}{2 \sqrt{\nu a t}} \right)$$  \hspace{1cm} (1.8)$$

$$u_w = u_0 \text{erfc} \left( \frac{-z}{2 \sqrt{\nu w t}} \right)$$

where $u_0$ is the common interface velocity. The boundary condition of continuous interface stress yields a relationship for $u_0$:

$$\frac{u_0}{U} = \left( 1 + \frac{\rho_w \sqrt{\nu w}}{\rho_a \sqrt{\nu a}} \right)^{-1}.$$  \hspace{1cm} (1.9)$$
The solution represented by Equations 1.8 and 1.9 reveals the velocity distributions to be error functions and complementary error functions of the distance from the interface, portraying air-side and water-side boundary layers of thickness $2\sqrt{\nu t}$, which grow with the square root of time. The water-side velocities are much slower than the air-side ones: The typical value of $u_0/U$ is $1/200$. This can be anticipated from Equation 1.5, which shows accelerations to be inversely proportional to density. The density of water is about 800 times greater than the density of air, balanced somewhat in Equation 1.9 by the kinematic viscosity of the air being some 16 times greater than that of water.

From the solution we find the value of the interface stress, alias momentum flux from air to water:

$$\tau_i = \frac{U}{R}$$  \hspace{1cm} (1.10)

with

$$R = \frac{1}{\rho_a} \sqrt{\frac{\pi t}{\nu_a}} + \frac{1}{\rho_w} \sqrt{\frac{\pi t}{\nu_w}}$$

The result is clearly of the form of Equation 1.1, constituting a bulk relationship between the interface momentum flux and the undisturbed velocity difference between air and water, which plays the role of the conjugate Force. The Resistance $R$ consists of two additive components, identifiable as air-side and water-side resistance, respectively. Each component is proportional to the boundary layer thickness on that side, and inversely proportional to dynamic viscosity $\rho v$. With the values of material properties substituted, the air-side resistance turns out to be some 200 times greater than water-side resistance, so that the latter is for all practical purposes negligible.

The momentum transfer law must be reducible to a nondimensional form, containing fewer variables. One such form is:

$$C_D = \frac{2}{\sqrt{\pi}} \text{Re}^{-1}(1 + \left[ \frac{\rho_a}{\rho_w} \sqrt{\nu_a/\nu_w} \right])^{-1}$$  \hspace{1cm} (1.11)

with $C_D = \tau_i/\rho_a U^2$ a drag coefficient or nondimensional interface momentum flux, and $\text{Re} = 2U \sqrt{\nu_a t}/v$ a Reynolds number based on air-side boundary layer thickness. Counting the density ratio and the viscosity ratio as two separate parameters, the nondimensional version of the transfer law contains four variables, versus seven in the dimensional formulation. The reduction by three corresponds to the three units of measurement – mass, time and length – quantifying the dimensional variables.

In Equation 1.11, the density-viscosity ratio term is small compared to unity, so that a sufficiently accurate form of the transfer law is the much simpler: $C_D = \frac{2}{\sqrt{\pi}} \text{Re}^{-1}$. A lesson to be learned here is that not all variables playing a role in momentum transfer necessarily have a significant impact on the interface transfer law: Nobody could argue that the density or viscosity of water is irrelevant to momentum flux, yet neither significantly affects it in this example.

Does the bulk relationship, Equation 1.10, conform to Onsager’s theorem? The total energy dissipation is the integral of the local value $\rho v(\partial U/\partial z)^2$. Using the approximate
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1.3.1 Turbulence, Eddies and Their Statistics

Turbulence consists of a continuous succession of chaotic movements by parcels of fluid, analogous perhaps to molecular agitation, but occurring on a much larger than molecular scale. Moving parcels of fluid displace other fluid that eventually has to fill in the space vacated. This is known as continuity. Irregular and ephemeral closed flow structures arise in this manner, loosely called eddies. The details of eddy motion are complex, yet “stochastic” average properties of the flow (averages over many “realizations” in statistical theory, time-mean properties in practice) obey ascertainable laws, not unlike laws that quantify the macroscopic effects of molecular agitation.

The chaotic motions of turbulence are three-dimensional, so that at a fixed point there are velocity fluctuations along all three coordinate axes, $u'$, $v'$, $w'$, even if the mean velocity has the same “alongwind” direction, $\overline{u} > 0$, $\overline{v} = 0$, $\overline{w} = 0$ (primes distinguish fluctuations from mean quantities carrying overbars). The mean square velocity fluctuations are then nonzero and their square roots provide measures of eddy velocity, a velocity “scale,” such as $u_m = \sqrt{\overline{u'^2}}$. They also define the important Turbulent Kinetic Energy, TKE per unit mass in J kg$^{-1}$:

$$E_t = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$  \hspace{1cm} (1.12)

Eddies also stir up the fluid, and if some fluid property is unevenly distributed, they try to equalize it. Thus, when mean flow momentum $\rho \overline{u}$ varies in the vertical, fluctuating vertical eddy motions of velocity $u'$ bring faster fluid from the momentum-rich region, which locally appears as excess velocity, positive $u'$. Averaged, the effects of these eddy motions add up to eddy transport of momentum, $\rho \overline{u'w'}$, also known as Reynolds flux of momentum or Reynolds stress, after Osborne Reynolds who first formulated equations of motion for a turbulent fluid with Reynolds stresses included.
Another important turbulence property is characteristic eddy size. This can again be assigned only from statistical properties of the flow, traditionally from a two-point correlation function, such as $u'(x)u'(x + r)$, the mean product of the vertical velocity component at two along-wind locations a distance $r$ apart. See Townsend (1956) for a fuller discussion. Figure 1.2 shows the typical shape of such a correlation function, with a negative correlation loop required by continuity, on the principle of “what goes up must come down.” The distance where the correlation function drops to zero is a measure of eddy size, or a “length-scale” of turbulence, say $\ell$.

Correlation functions contain more information. According to a well-known theorem of statistics, a two-point velocity correlation function is the Fourier transform of an energy spectrum that assigns portions of kinetic energy to wavenumbers $k$ (radians per unit length), and vice versa, the spectrum function $\phi(k)$ is the Fourier transform of the correlation function. The most useful correlation function in this context is $u'(x)u'(x + r)$, between alongwind velocity fluctuations at downwind distances. The corresponding energy spectrum of turbulence peaks at a wavenumber $k_p$, which is close to $\ell^{-1}$ derived from the $u'(x)u'(x + r)$ correlation. An alternative choice for eddy length scale is then $\ell = k_p^{-1}$. A physical interpretation of the spectrum is that reciprocal wavenumbers are characteristic dimensions of smaller and larger eddies, the values of the spectrum function a measure of their energy.

Apart from length and velocity scales, an important property of turbulence is the rate at which it dissipates energy, conventionally denoted by $\varepsilon$, in $W kg^{-1}$. Energy dissipation is the work of the sharpest instantaneous velocity gradients that occur in the eddying motion; viscous shear stress times the velocity gradient being the rate at which mechanical energy is converted into heat. Laboratory observations of many different types of turbulent shear flow revealed the general “similarity” principle that the dissipation rate is proportional to $u'_m/\ell$, varying from one part of the flow to another with the velocity and length scales as this product does. The proportionality constant
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changes, however, with the boundary conditions on the shear flow, as well as with the different possible choices for velocity and length scales of the turbulence.

The same similarity principle applies to other properties or effects of turbulence and constitutes the great simplifying factor in an otherwise almost untreatably complex phenomenon: once we have information on the variation of the velocity and length scales of turbulence in space or time, we are often able to quantify other properties of the flow. This was first recognized by Ludwig Prandtl, who introduced the concept of a “mixing length” for the eddy length scale, and used it to considerable advantage in constructing theories for different species of turbulent shear flow subject to simple boundary conditions, such as the flow in boundary layers, jets and wakes. The empirical finding, that two independent variables characterizing a turbulent shear flow are sufficient to describe other flow properties, is analogous to the thermodynamic principle that two state variables are all that is needed to determine different properties of a pure substance.

Energy dissipation occurs in the sharpest velocity gradients and therefore at the smallest scales (i.e., at the highest wavenumbers). Kolmogorov (1941) hypothesized that the spectrum function well beyond the peak of the spectrum toward the dissipative range (in the “inertial subrange”) depends only on the wavenumber \( k \) and the energy dissipation \( \varepsilon \) (instead of separately on \( \ell \) and \( u_\infty \)). This implies by dimensional reasoning:

\[
\phi(k) = a \varepsilon^{2/3} k^{-2/3}
\]  

(1.13)

with \( a \) a constant, equal to 0.47 according to Lumley and Panofsky (1964). Observations of the spectrum in the intermediate range thus yield the rate of energy dissipation. Recalling that \( \varepsilon \) is proportional to \( u_\infty^3/\ell \), known \( \ell \) allows the velocity scale to be determined: This is the basis of the so-called “dissipation method” of determining wind stress (see below).

According to the similarity principle of turbulence, the Reynolds stresses should be proportional to density times the square of the velocity scale, \(-\rho u'w' = \text{const.} u_\infty^2\), choosing \( u_\infty \) for the velocity scale, as suggested above. An alternative legitimate choice for the velocity scale is therefore the “friction velocity” \( u^* = \sqrt{-u'w'} \), particularly useful where the Reynolds stress is constant in a region of the flow. This is (nearly) true of the airflow at low levels over the sea, where the Reynolds stress differs little from \( \tau_i \), the effective shear force on the interface (that includes any pressure forces acting on wind waves), alias momentum flux from air to water.

1.3.2 The Air-side Surface Layer

Air flow above the sea is variable, but changes in atmospheric conditions take place slowly enough to regard the mean wind speed steady at a few tens of meters above the interface, in what we will call a surface layer. Nor does the mean wind direction vary noticeably with height in this layer, only the wind speed: \( \mathbf{\tau} = U(z), \mathbf{v} = 0, \mathbf{w} = 0 \). The mean velocity is thus a function only of the distance \( z \) above the mean position of the
interface (known as the Mean Sea Level, MSL). What makes the problem of observing any property in the surface layer very difficult, is that, under wind, the interface is a highly irregular surface that also rapidly changes its shape. The visible structures on the wind-blown interface are wind waves in common parlance, but to avoid even a suggestion of regular parallel-crested water waves we will call them collectively the “windsea.” A U.S. Navy Hydrographic publication (Bigelow and Edmondson, 1947) distinguishes between “sea” and “swell,” two different wave-like phenomena, “sea” under storms, “swell” what is left over from a storm, more or less regular parallel-crested waves propagating away from the region where the storm generated the “sea.” Windsea is a less confusing name than sea, and is certainly descriptive.

Figure 1.3 illustrates the surface layer above the windsea, indicating the air flow, eddies possibly tied to individual irregular waves, and a spar buoy with instruments to observe the mean wind at different levels. Smith (1978) gave details of such a “stable platform”; it was designed to withstand waves of 18 m height crest to trough, albeit protruding only 12.5 m above MSL. In moderate winds, waves are only 2 m height or less, and instruments on platforms similar to Smith’s (e.g., fixed towers or ship masts) are able to determine the mean wind at several levels.

Such observations have revealed that, some distance above the windsea, the turbulent air flow has the same character as over a solid boundary, in what is known as a “wall layer.” The mean velocity increases with distance above the sea surface, while the Reynolds flux of momentum, \( \rho u'w' \), that dwarfs viscous stress, is approximately constant with height from just above the waves to 10 m or more, and equal to the effective interface stress \( \tau_i \). The latter, the net horizontal force on the interface, includes pressure forces on the inclined surfaces of wind waves, as well as shear stress. The effective interface shear stress defines the friction velocity, \( u^* = \sqrt{\tau_i/\rho} \), which then serves as the velocity scale of the turbulent flow in the entire surface layer. Above the waves, where the flow has the character of a wall layer, also described as the constant stress layer, observations have shown the eddy length scale \( \ell \) to be proportional to the