

Decisions under Uncertainty

Probabilistic Analysis for Engineering Decisions

Decision-making and risk assessment are essential aspects of every engineer's life, whether this be concerned with the probability of failure of a new product within the warranty period or the potential cost, human and financial, of the failure of a major structure such as a bridge. This book helps the reader to understand the tradeoffs between attributes such as cost and safety, and includes a wide range of worked examples based on case studies. It introduces the basic theory from first principles using a Bayesian approach to probability and covers all of the most widely used mathematical techniques likely to be encountered in real engineering projects. These include utility, extremes and risk analysis, as well as special areas of importance and interest in engineering and for understanding, such as optimization, games and entropy. Equally valuable for senior undergraduate and graduate students, practising engineers, designers and project managers.

Ian Jordaan is University Research Professor in the Faculty of Engineering and Applied Science at the Memorial University of Newfoundland, Canada, and held the NSERC-Mobil Industrial Research Chair in Ocean Engineering at the university from 1986 to 1996. He is also president of Ian Jordaan and Associates Inc. and an advisor to C-CORE, an engineering R&D company based in St. John's. Prior to joining the university in 1986, he was Vice-President, Research and Development at Det Norske Veritas (Canada) Ltd., and before this he was a full professor in the Department of Civil Engineering at the University of Calgary, Canada. He is extensively involved in consulting activities with industry and government, and has pioneered methods of risk analysis for engineering in harsh environments. Recently he served on an expert panel of the Royal Society of Canada, studying science issues relative to the oil and gas developments offshore British Columbia. He is the recipient of a number of awards, including the Horst Leipholtz Medal and the P. L. Pratley Award, both of the Canadian Society for Civil Engineering.

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)

Decisions under Uncertainty

Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Faculty of Engineering and Applied Science
Memorial University of Newfoundland
St John's, Newfoundland, Canada



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by
Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521782777

© Jordaan 2005

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2005

A catalogue record for this publication is available from the British Library

Library of Congress cataloguing in publication data

Jordaan, Ian J., 1939–

Decisions under uncertainty: probabilistic analysis for engineering decisions / Ian J. Jordaan.

p. cm.

Includes bibliographical references and index.

ISBN 0 521 78277 5

1. Engineering—Statistical methods. 2. Decision making—Statistical methods.

3. Probabilities. I. Title.

TA340.J68 2004

620'.007'27—dc22 2004043587

ISBN 978-0-521-78277-7 Hardback

ISBN 978-0-521-36997-8 Paperback

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to in
this publication, and does not guarantee that any content on such websites is,
or will remain, accurate or appropriate. Information regarding prices, travel
timetables, and other factual information given in this work is correct at
the time of first printing but Cambridge University Press does not guarantee
the accuracy of such information thereafter.

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)

For Christina

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)

Contents

<i>List of illustrations</i>	<i>page</i> xii
<i>Preface</i>	xiii
<i>Acknowledgements</i>	xvi

1	Uncertainty and decision-making	1
1.1	Introduction	1
1.2	The nature of a decision	3
1.3	Domain of probability: subjectivity and objectivity	9
1.4	Approach and tools; rôle of judgement: indifference and preference	10
1.5	Probability and frequency	12
1.6	Terminology, notation and conventions	15
1.7	Brief historical notes	16
1.8	Exercise	19
2	The concept of probability	20
2.1	Introduction to probability	20
2.2	Coherence: avoiding certain loss; partitions; Boolean operations and constituents; probability of compound events; sets	26
2.3	Assessing probabilities; example: the sinking of the Titanic	37
2.4	Conditional probability and Bayes' theorem	43
2.5	Stochastic dependence and independence; urns and exchangeability	63
2.6	Combinatorial analysis; more on urns; occupancy problems; Bose–Einstein statistics; birthdays	67
2.7	Exercises	82

viii Contents

3	Probability distributions, expectation and prevision	90
3.1	Probability distributions	90
3.2	Discrete probability distributions; cumulative distribution functions	91
3.3	Continuous random quantities; probability density; distribution functions for the combined discrete and continuous case	103
3.4	Problems involving two or more dimensions	114
3.5	Means, expected values, prevision, centres of mass and moments	127
3.6	Random processes	144
3.7	Exercises	154
4	The concept of utility	162
4.1	Consequences and attributes; introductory ideas	162
4.2	Psychological issues; farmer's dilemma	164
4.3	Expected monetary value (<i>EMV</i>)	170
4.4	Is money everything? Utility of an attribute	172
4.5	Why do the π -values represent our utility? Expected utility	178
4.6	Attributes and measurement; transitivity	184
4.7	Utility of increasing and decreasing attributes; risk proneness	188
4.8	Geometric representation of expected utility	192
4.9	Some rules for utility	194
4.10	Multidimensional utility	197
4.11	Whether to experiment or not; Bayesian approach; normal form of analysis	203
4.12	Concluding remarks	210
4.13	Exercises	214
5	Games and optimization	220
5.1	Conflict	220
5.2	Definitions and ideas	222
5.3	Optimization	229
5.4	Linear optimization	237

ix	Contents	
	5.5 Back to games: mixed strategies	257
	5.6 Discussion; design as a game against nature	265
	5.7 Exercises	266
6	Entropy	272
	6.1 Introductory comments	272
	6.2 Entropy and disorder	273
	6.3 Entropy and information: Shannon's theorem	277
	6.4 The second facet: demon's roulette	282
	6.5 Maximizing entropy; Maxwell–Boltzmann distribution; Schrödinger's nutshell	286
	6.6 Maximum-entropy probability distributions	291
	6.7 Nature of the maximum: interpretations	297
	6.8 Blackbodies and Bose–Einstein distribution	299
	6.9 Other facets; demon's urns	307
	6.10 Applications outside thermodynamics	310
	6.11 Concluding remarks	313
	6.12 Exercises	314
7	Characteristic functions, transformed and limiting distributions	317
	7.1 Functions of random quantities	317
	7.2 Transformation of distributions	318
	7.3 Convolutions	336
	7.4 Transforms and characteristic functions	340
	7.5 Central Limit Theorem – the model of sums	356
	7.6 Chebyshev inequality	367
	7.7 Laws of large numbers	370
	7.8 Concluding remarks	371
	7.9 Exercises	372
8	Exchangeability and inference	378
	8.1 Introduction	378
	8.2 Series of events; exchangeability and urns	382

x	Contents	
8.3	Inference using Bayes' theorem; example: binomial distribution; Bayes' postulate	389
8.4	Exchangeable random quantities	399
8.5	Examples; conjugate families of distributions	402
8.6	Basis for classical methods; reference prior distributions expressing 'indifference'	418
8.7	Classical estimation	426
8.8	Partial exchangeability	443
8.9	Concluding remarks	444
8.10	Exercises	446
9	Extremes	452
9.1	Objectives	453
9.2	Independent and identically distributed (<i>iid</i>) random quantities; de Méré's problem	455
9.3	Ordered random quantities; order 'statistics'	457
9.4	Introductory illustrations	459
9.5	Values of interest in extremal analysis	469
9.6	Asymptotic distributions	479
9.7	Weakest link theories	494
9.8	Poisson processes: rare extreme events	501
9.9	Inference, exchangeability, mixtures and interannual variability	506
9.10	Concluding remarks	510
9.11	Exercises	511
10	Risk, safety and reliability	514
10.1	What is risk? Its analysis as decision theory	514
10.2	Risks to the individual	518
10.3	Estimation of probabilities of system or component failure	526
10.4	Risk analysis and management	541
10.5	Structural systems	550
10.6	Broad issues	557
10.7	Exercises	558

xi Contents

11	Data and simulation	561
11.1	Introduction	561
11.2	Data reduction and curve fitting	562
11.3	Linear regression and least squares	580
11.4	Plotting positions	599
11.5	Monte Carlo simulation	603
11.6	Exercises	612
12	Conclusion	618
	Appendix 1 Common probability distributions	620
A1.1	Discrete distributions	620
A1.2	Continuous distributions	626
	Appendix 2 Mathematical aspects	637
A2.1	Stieltjes integrals	637
A2.2	Integration	640
A2.3	Solution to Lewis Carroll's problem and discussion	641
	Appendix 3 Answers and comments on exercises	643
	<i>References</i>	650
	<i>Index</i>	657

Illustrations

The artistic illustrations that grace the cover and appear periodically in the text have been prepared by Grant Boland, to whom I am most grateful.

Scales	<i>page</i> 10
Tweedledum and Tweedledee	14
Bruno de Finetti	18
Rule for Life	25
The Titanic	40
The Bald Soprano	45
Figure for minimum variance	134
Dice	170
The Entropy Demon	274
An urn	308
Another urn	384
Thomas Bayes	396
A fish	452
Dice	456
A third urn	458
The Twin Towers	522
Galton's apparatus	574
Playing cards	605

Preface

Probabilistic reasoning is a vital part of engineering design and analysis. Inevitably it is related to decision-making – that important task of the engineer. There is a body of knowledge profound and beautiful in structure that relates probability to decision-making. This connection is emphasized throughout the book as it is the main reason for engineers to study probability. The decisions to be considered are varied in nature and are not amenable to standard formulae and recipes. We must take responsibility for our decisions and not take refuge in formulae. Engineers should eschew standard methods such as hypothesis testing and think more deeply on the nature of the problem at hand. The book is aimed at conveying this line of thinking. The search for a probabilistic ‘security blanket’ appears as futile. The only real standard is the subjective definition of probability as a ‘fair bet’ tied to the person doing the analysis and to the woman or man in the street. This is our ‘rule for life’, our beacon. The relative weights in the fair bet are our odds on and against the event under consideration.

It is natural to change one’s mind in the face of new information. In probabilistic inference this is done using Bayes’ theorem. The use of Bayesian methods is presented in a rigorous manner. There are approximations to this line of thinking including the ‘classical’ methods of inference. It has been considered important to view these and others through a Bayesian lens. This allows one to gauge the correctness and accuracy of the approximation. A consistent Bayesian approach is then achieved. The link to decisions follows from this reasoning, in which there are two fundamental concepts. In our decision-making we use probability, a concept in which the Bayesian approach plays a special rôle, and a second concept, utility. They are related dually, as are other concepts such as force and displacement. Utility is also subjective: probabilities and utilities attach to the person, not to the event under consideration.

The book was written over many years during a busy life as a practising engineer. It is perhaps a good moment to present the perspective of an engineer using probabilistic concepts and with direct experience of engineering decision-making. Whilst a consistent approach to probability has been taken, engineering practice is often ‘messy’ by comparison to the mathematical solutions presented in the book. This can best be dealt with by the engineer – numerical methods, approximations and judgement must come into play. The book focusses on the principles underlying these activities. The most important is the subjective nature of the two concepts involved in decision-making. The challenge very often is to obtain consensus between engineers on design parameters, requiring a judicious addition of conservatism – but not too much! – where there are uncertainties which have not been formally analysed.

An important aspect for engineers is the link to mechanics. One can think of probability distributions as masses distributed over possible outcomes with the total mass being unity; mean values then become centres of mass; variances become moments of inertia; and other analogies appear. The Stieltjes integral is a natural way to obtain the mean values and moments; it is potentially of great usefulness to the engineer. It unifies common situations involving, for example, concentrated or distributed masses in mechanics or probability. But this has been included as an option and the usual combination of summation for discrete random quantities and Riemann integration for continuous ones has been used in most instances. Inference is treated from the Bayesian standpoint, and classical inference viewed as a special case of the Bayesian treatment is played down in favour of decision theory. This leads engineers to think about their problem in depth with improved understanding. Half hearted or apologetic use of Bayesian methods has been avoided. The derivation of the Bayesian results can be more demanding than the corresponding classical ones, but so much more is achieved in the results.

Use of this book

The book is intended as an introduction to probability for engineers. It is my sincere hope that it will be of use to practising engineers and also for teaching in engineering curricula. It is also my hope that engineering schools will in the future allow more time in their programs for modelling of uncertainty together with associated decision-making. Often probability is introduced in one course and then followed by courses such as those in 'Civil Engineering Systems'. This allows sufficient time to develop the full approach and not just pieces of 'statistics'. The present work will be of use in such situations and where it is considered beneficial for students to understand the full significance of the theory. The book might be of interest also in graduate courses.

I have often used extended examples within the text to introduce ideas, for example the collision probability of the Titanic in Chapter 2. More specific examples and illustrations are identified as 'examples' in the text. The use of urns, with associated random walks through mazes, assists in fundamental questions of assigning probabilities, including exchangeability and extremes. Raiffa's urn problem brings decision theory into the picture in a simple and effective manner. Simplified problems of decision have been composed to capture the essence of decision-making. Methods such as hypothesis testing have a dubious future in engineering as a result of their basic irrationality if applied blindly. Far better to analyse the decision at hand. But we do need to make judgements, for example, with regard to quality control or the goodness-of-fit of a particular distribution in using a data set. If wisely used, confidence intervals and hypothesis testing can assist considerably. The conjugate prior analysis in inference leads naturally to the classical result as a special case. Tables of values for standard distributions are so readily available using computer software that these have not been included.

Cambridge University Press

978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions

Ian Jordaan

Frontmatter

[More information](#)**xv** **Preface**

Optimization has been introduced as a tool in linear programming for game theory and in the maximization of entropy. But it is important in engineering generally, and this introduction might become a springboard into other areas as well. Chapter 11, dealing with data, linear regression and Monte Carlo simulation is placed at the end and collects together the techniques needed for the treatment of data. This may require a knowledge of extremes or the theory of inference, so that these subjects are treated first. But the material is presented in such a way that most of it can be used at any stage of the use of the book.

Acknowledgements

Special mention should be made of two persons no longer alive. John Munro formerly Professor at Imperial College, London, guided me towards studying fundamental aspects of probability theory. Bruce Irons, of fame in finite element work, insisted that I put strong effort into writing. I have followed both pieces of advice but leave others to judge the degree of success. My former students, Maher Nessim, Marc Maes, Mark Fuglem and Bill Maddock have taught me possibly more than I have taught them. My colleagues at C-CORE and Memorial University have been most helpful in reviewing material. Leonard Lye, Glyn George and Richard McKenna have assisted considerably in reviewing sections of the book. Recent students, Paul Stuckey, John Pond, Chuanke Li and Denise Sudom have been of tremendous assistance in reviewing material and in the preparation of figures.

I am grateful to Han Ping Hong of the University of Western Ontario who reviewed the chapter on extremes; to Ken Roberts of Chevron-Texaco and Lorraine Goobie of Shell who reviewed the chapter on risks; and to Sarah Jordaan who reviewed the chapter on entropy. Paul Jowitt of Heriot-Watt University provided information on maximum-entropy distributions in hydrology. Marc Maes reviewed several chapters and provided excellent advice.

Neale Beckwith provided unstinting support and advice.

Many of the exercises have been passed on to me by others and many have been composed specially. Other writers may feel free to use any of the examples and exercises without special acknowledgement.