1 Uncertainty and decision-making

If a man will begin with certainties, he shall end in doubts; but if he will be content to begin with doubts, he shall end in certainties.

Francis Bacon, *Advancement of learning*

Life is a school of probability.

Walter Bagehot

1.1 Introduction

Uncertainty is an essential and inescapable part of life. During the course of our lives, we inevitably make a long series of decisions under uncertainty – and suffer the consequences. Whether it be a question of deciding to wear a coat in the morning, or of deciding which soil strength to use in the design of a foundation, a common factor in these decisions is uncertainty. One may view humankind as being perched rather precariously on the surface of our planet, at the mercy of an uncertain environment, storms, movements of the earth's crust, the action of the oceans and seas. Risks arise from our activities. In loss of life, the use of tobacco and automobiles pose serious problems of choice and regulation. Engineers and economists must deal with uncertainties: the future wind loading on a structure; the proportion of commuters who will use a future transportation system; noise in a transmission line; the rate of inflation next year; the number of passengers in the future on an airline; the elastic modulus of a concrete member; the fatigue life of a piece of aluminium; the cost of materials in ten years; and so on.

In order to make decisions, we weigh our feelings of uncertainty, but our decision-making involves quite naturally another concept – that of utility, a measure of the desirability, or otherwise, of the various consequences of our actions. The two fundamental concepts of probability and utility are related dually, as we shall see. They are both subjective, probability since it is a function of our information at any given time, and utility – perhaps more obviously so – since it is an expression of our preferences. The entire theory is behavioural, because it is motivated by the need to make decisions and it is therefore linked to the exigencies of practical life, engineering and decision-making.

The process of design usually requires the engineer to make some big decisions and a large number of smaller ones. This is also true of the economic decision-maker. The consequences of all decisions can have important implications with regard to cost or safety. Formal decision analysis would generally not be required for each decision, although one's thinking should be guided by its logic. Yet there are many engineering and economic problems which justify considerable effort and investigation. Often the decisions taken will involve the safety of entire communities; large amounts of money will have been invested and the decisions will, when implemented, have considerable social and environmental impact. In the planning stages, or when studying the feasibility of such large projects, decision analysis offers a coherent framework within which to analyse the problem.

Another area of activity that is assisted by an approach based on decision theory is that of the writing of design standards, rules or codes of practice. Each recommendation in the code is in itself a decision made by the code-writer (or group of writers). Many of these decisions could be put 'under the microscope' and examined to determine whether they are optimal. Decision theory can answer questions such as: does the additional accuracy obtained by method 'X' justify the expense and effort involved? Codes of practice have to be reasonably simple: the simplification itself is a decision.

The questions of risk and safety are at the centre of an engineer's activities. Offshore structures, for example, are placed in a hostile environment and are buffeted by wind, wave and even by large masses of floating ice in the arctic and sub-arctic. It is important to assess the magnitude of the highest load during the life of the structure – a problem in the probabilistic analysis of **extremes**. Engineering design is often concerned with tradeoffs, such as that between safety and cost, so that estimating extreme demands is an important activity. Servicing the incoming traffic in a computer or other network is a similar situation with a tradeoff between cost and level of service.

Extremes can also be concerned with the minimum value of a set of quantities; consider the set of strengths of the individual links in an anchor chain. The strength of the chain corresponds to the minimum of this set. The strength of a material is never known exactly. Failure of a structural member may result from accumulated damage resulting from previous cycles of loading, as in fatigue failure. Flaws existing in the material, such as cracks, can propagate and cause brittle fracture. There are many uncertainties in the analysis of fracture and fatigue – for example, the stress history which is itself the result of random load processes. Because of the dependence of fracture on temperature, random effects resulting from its variation may need to be taken into account, for example in arctic structures or vessels constructed of steel.

In materials science, we analyse potential material behaviour and this requires us to make inferences at a microscopic or atomic level. Here, the 'art of guessing' has developed in a series of stages and entropy – via its connection with information – is basic to this aspect of the art. The logic of the method is the following. Given a macroscopic measurement – such as temperature – how can one make a probabilistic judgement regarding the energy level of a particular particle of the material? Based on the assumption of equilibrium we state there is no change in average internal energy; we can then deduce probability values for all possible energy levels by maximizing entropy subject to the condition that the weighted sum of energies is constant. The commonly used distributions of energy such as Boltzmann's may be derived in this way. We shall also discuss recent applications of the entropy concept in new areas such as transportation. This approach should not be regarded as a panacea, but rather as a procedure with a logic which may or may not be appropriate or helpful in a particular situation.

1.2 The nature of a decision

We introduce the factors involved in decision-making in a relaxed and informal manner. Let us consider the problem of deciding whether or not to wear a coat in the morning: suppose that we are spending a winter in Calgary, a city near the Canadian Rockies. Warm westerly winds – the Chinook – transform bitterly cold weather (−20 °C, say) to relatively warm weather (10 °C, say!). It occasionally happens that the warm front drifts backwards and forwards so that the temperature changes quite abruptly between the two levels, cold and warm. Under such conditions – given, say, a warm morning – one is in some doubt as to whether to wear a coat or not.

One's dilemma is illustrated in Figure 1.1. The possible actions comprise an elementary action space: α_1 = *wear a coat*, α_2 = *don't wear a coat*. For each action, there are various 'states of nature' – here simplified to two states of nature for each action, defined by two symbols, θ_1 and θ_2. The symbol θ_1 = '*it turns cold during the day*', and θ_2 = '*it stays warm*

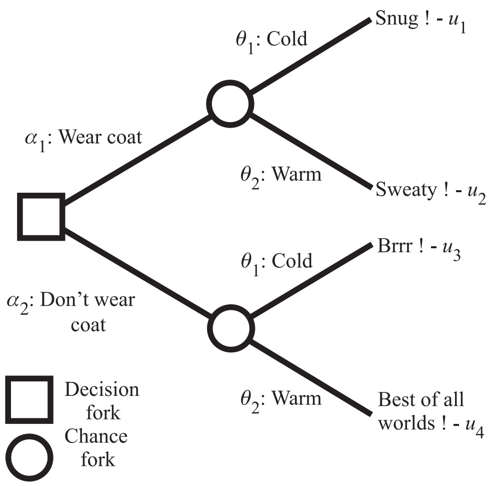


Figure 1.1 Uncertainty about the weather and its relation to wearing a coat in the morning

during the day'. Thus, there are four combinations of action and state of nature, and corresponding to each of these, there is a 'consequence', u_1, u_2, u_3 and u_4 as shown in Figure 1.1. A consequence will be termed a utility, U, a measure of value to the individual. Higher values of utility will represent consequences of greater value to the individual. The estimation of utilities will be dealt with later in Chapter 4; the concept is in no way arbitrary and requires careful analysis.

It is not difficult to reach the conclusion that the choice of decision (α_1 or α_2) depends on **two** sets of quantities only. These are the set of **probabilities** associated with the states of nature, $\{\mathbf{Pr}(\theta_1), \mathbf{Pr}(\theta_2)\}$, and the set of **utilities**, $\{u_1, u_2, u_3$ and $u_4\}$. The formal analysis of decisions will be dealt with subsequently, but it is sufficient here to note the dependence of the choice of decision on the two sets of quantities – probabilities and utilities. This pattern repeats itself in all decision problems, although the decision tree will usually be more complex than that given in Figure 1.1.

This example can also be used to illustrate a saying of de Finetti's (1974):

Prevision is not prediction.

Prevision has the same sense as 'expected value'; think of the centre of gravity of one's probabilistic weights attached to various possibilities. I recall an occasion when the temperature in Calgary was oscillating regularly and abruptly between 10 °C and −20 °C; the weather forecast was −5 °C. Yet nobody really considered the value of −5 °C as at all likely. If one attached probabilistic 'weights' of 50% at 10° and −20°, the value −5° would represent one's prevision – but certainly not one's 'prediction'.

The decision problem just discussed – to wear a coat or not – is mirrored in a number of examples from industry and engineering. Let us introduce next the oil wildcatter who literally makes his or her living by making decisions under uncertainty. In oil or gas exploration, areas of land are leased, primarily by big companies, but also by smaller scale operators. Oil wells may be drilled by the company leasing the land or by other operators, on the basis of an agreement. Exploratory wells are sometimes drilled at some distance from areas in which there are producing wells. These wells are termed 'wildcats'. The risk element is greater, because there is less information about such wells; in the case of new fields, the term 'new field wildcats' is used to denote the wells. The probability of success even for a good prospect is often estimated at only 40%. Drilling operators have to decide whether to invest in a drilling venture or not, on the basis of various pieces of information. Those who drill 'wildcat' wells are termed 'wildcatters'.

We are only touching on the various uncertainties facing oil and gas operators; for instance, the flow from a well may be large or small; it may decline substantially with time; the drilling operation itself can sometimes be very difficult and expensive, the expense being impossible to 'predict'. There may be an option to join a consortium for the purpose of a drilling, lessening risk to an individual operator. The classic work is by Grayson (1960) who studied the implementation of decision theory to this field. He analysed the actual behaviour in the field of various operators.

Cambridge University Press
978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions
Ian Jordaan
Excerpt
More information

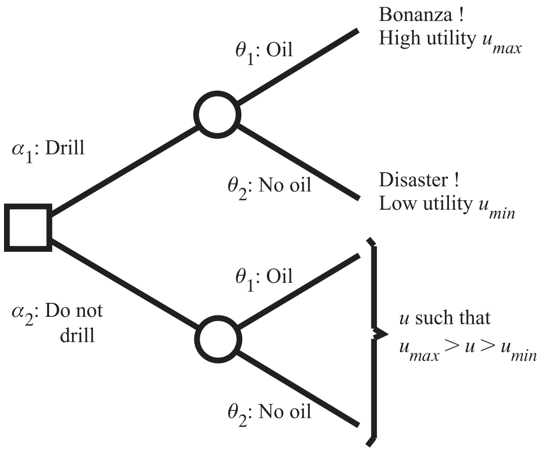


Figure 1.2 Oil wildcatter's dilemma

The wildcatter must decide whether or not to drill at a certain site before the lease expires. The situation is illustrated in the form of a decision tree in Figure 1.2. She realizes that if drilling is undertaken, substantial costs are involved in the drilling operation. On the other hand, failure to drill may result in a lost opportunity for substantial profits if there is oil present in the well. The decision must reflect an assessment of the probability of there being oil in the well. This assessment will almost invariably take into account a geologist's opinion. Thus, the action space is $A = \{\alpha_1 = \textit{drill for oil}, \alpha_2 = \textit{do not drill}\}$. The uncertain states of nature are $\Theta = \{\theta_1, \theta_2\}$ where $\Theta = \{\theta_1 = \textit{oil present}$ and $\theta_2 = \textit{no oil present}\}$. The choice of decision is again seen to depend on the values of U and the values $\{\mathbf{Pr}(\theta_1), \mathbf{Pr}(\theta_2)\}$.

In some decisions the states of nature differ, depending on the decision made. Consider the question of relocation of a railway that passes through a city. The trains may include cars containing chlorine, so that there is the possibility of release of poisonous gas near a human population. Figure 1.3 shows a decision tree for a problem of this kind. The consequences in terms of possible deaths and injuries are quite different for the two branches and might be quite negligible for the upper branch. Figure 1.3 illustrates a tradeoff between cost of relocation and risk to humans. This tradeoff between cost and safety is fundamental to many engineering decisions. Other tradeoffs include for example that between the service rate provided for incoming data and the queuing delay in data transmission. This is shown in Figure 1.4. Traffic in computer networks is characterized by 'bursts' in the arrival of data. The incoming traffic is aggregated and the bursts can lead to delays in transmission.

Figure 1.5 illustrates an everyday problem on a construction site. Is a batch of concrete acceptable? One might, for instance, measure air void content to ascertain whether the freeze–thaw durability is potentially good or bad. Bayes' theorem tells us how to incorporate this new information (Section 2.4). Non-destructive testing of steel structures is also rich in problems of this kind.

Cambridge University Press
978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions
Ian Jordaan
Excerpt
More information

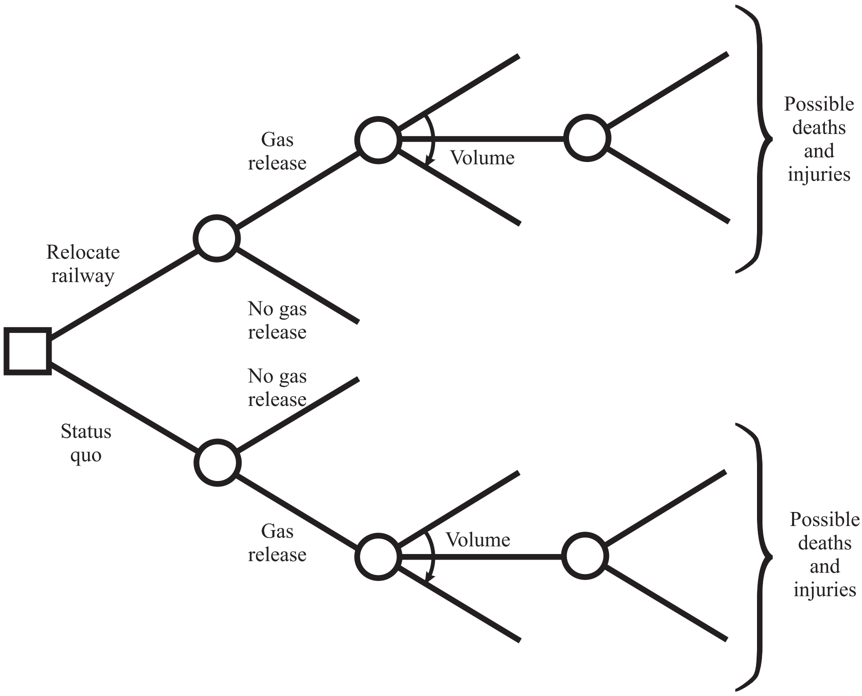


Figure 1.3 Rail relocation decision

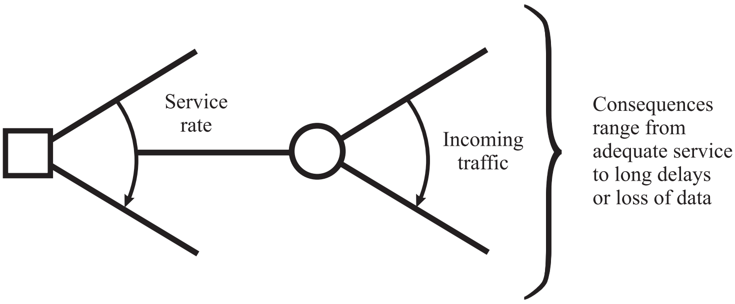


Figure 1.4 Decision on service rate for computer network

Let us return to the problems facing a code-writing committee. One of the many dilemmas facing such a committee is the question: should one recommend a simplified rule for use by engineers in practice? An example is: keep the span-to-depth ratio of a reinforced concrete slab less than a certain specified value, to ensure that the deflection under service loads is not excessive (rather than extensive analysis of the deflection, including the effects of creep, and so on). A further example is: should the 100-year wave height and period be the specified

Cambridge University Press
978-0-521-78277-7 - Decisions under Uncertainty: Probabilistic Analysis for Engineering Decisions
Ian Jordaan
Excerpt
More information

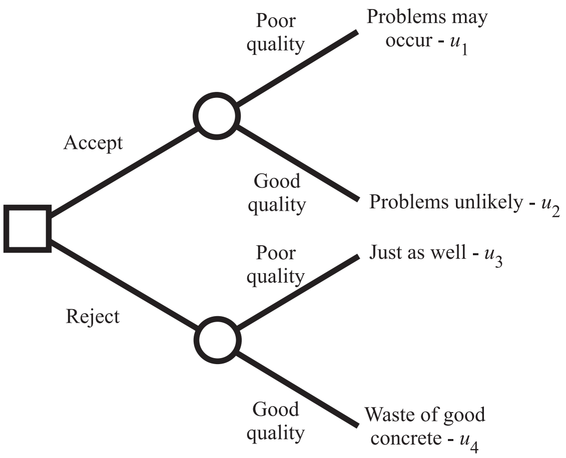


Figure 1.5 To accept or reject a batch of concrete posed as a problem of decision-making. Traditionally this is dealt with by hypothesis testing

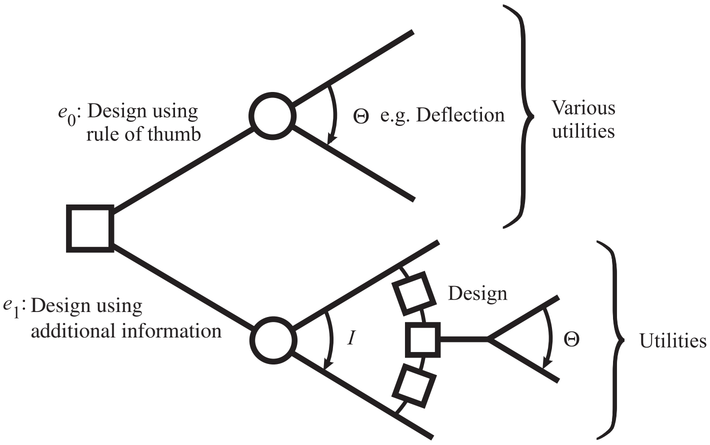


Figure 1.6 What recommendations to make for use in a design standard?

environmental indicators in the design of an offshore structure, in place of a more detailed description of the wave regime? These are numerous decisions of this kind facing the code-writer that can be analysed by decision theory. The decision tree would typically be of the kind illustrated in Figure 1.6. Rather than using the simplified rule, the committee might consider recommending the use of information 'I' which will be known at the time of design (e.g. the grade of steel or concrete, or a special test of strength if the structural material is available). This quantity could take several values and is therefore, from the point of view of the committee, a random quantity with associated probabilities.

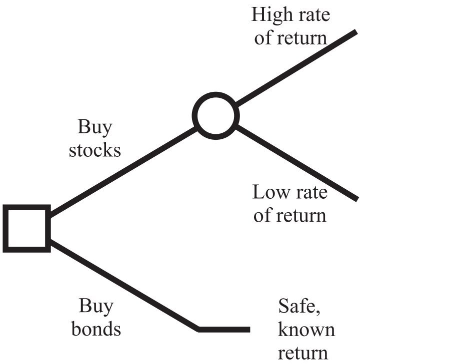


Figure 1.7 Risky and safe investment decisions

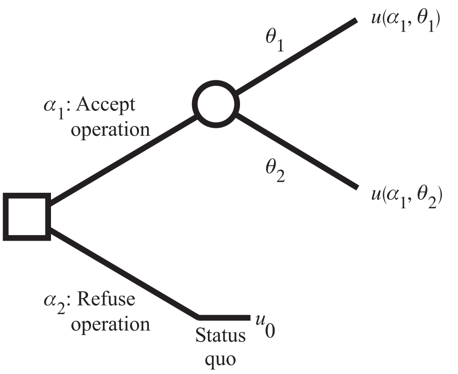


Figure 1.8 Dilemma in having an operation: θ_1 represents a successful operation and θ_2 an unsuccessful one

Economic decision-making naturally contains many decisions under uncertainty. Figure 1.7 illustrates the simple choice between stocks – a risky choice – and bonds with a known rate of return. We have stated that probability is a function of information, and consequently the better our information, the better our assessment of probabilities. The extreme of this would correspond to insider trading – betting on a sure thing, but not strictly legal or ethical!

Decisions in normal life can be considered using the present theory. Consider the following medical event.

'Patient x survives a given operation.'

Suppose that the patient (you, possibly) has an illness which causes some pain and loss of quality of life. Your doctors suggest an operation which, if successful, will relieve the pain; there is the possibility that you will not survive the operation and its aftermath. Your dilemma is illustrated in Figure 1.8. This example is perhaps a little dramatic, but the essential point is that again one's decision depends on the probabilities $\mathbf{Pr}(\theta_1)$ and $\mathbf{Pr}(\theta_2)$ (Figure 1.8) and one's utilities. One would be advised to carry out research in assessing the probabilities; the utilities must clearly be the patient's. The example points to the desirability of strong participation by patients in medical decisions affecting them.

1.3 Domain of probability: subjectivity and objectivity

The domain of probability is, in a nutshell, that which is possible but not certain. Probabilities can be thought of as 'weights' attached to what one considers to be possible. It therefore becomes essential to define clearly what one considers to be possible. The transition from 'what is possible' to 'certainty' will result from a well-defined measurement, or, more generally, a set of operations constituting the measurement. The specification or statement of the 'possible outcomes' must then be objective in the sense just described. We must be able, at least in principle, to determine its truth or falsity by means of measurement. To take an example, consider the statement:

E: 'We shall receive more than 1 cm of rain on Friday.'

We assume that it is Monday now. We need to contrive an experiment to decide whether or not there will be more than 1 cm of rain on Friday. We might then agree that statement E will be true or false according to measurements made using a rain gauge constructed at a meteorological laboratory.

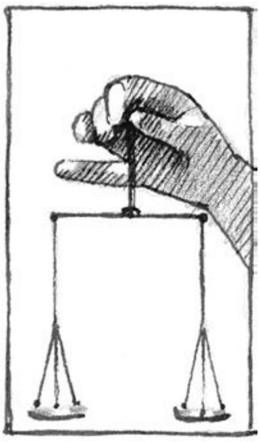
Having specified how we may decide on the truth or falsity of E we are in a position to consider probabilities. Since it is now Monday, we can only express an opinion regarding the truth or falsity of E. This 'opinion' is our probability assignment. It is subjective since we would base it on our knowledge and information at the time. There is no objective means of determining the truth or falsity of our probability assignment. This would be a meaningless exercise. If a spectacular high pressure zone develops on Thursday, we might change our assignment.

The same logic as above should be applied to any problem that is the subject of probabilistic analysis. The statement of the possible outcomes must be objective, and sufficient for the problem being analysed. It is instructive to ponder everyday statements in this light. For example, consider 'his eyes are blue', 'the concrete is understrength', 'the traffic will be too heavy', 'the fatigue life is adequate'... We should tighten up our definitions in these cases before we consider a probabilistic statement.

The approach that we have followed in defining what is possible echoes Bridgman's operational definition of a concept: the concept itself is identified with the set of operations which one performs in order to make a measurement of the concept; see Bridgman (1958). For example, length is identified with the physical operations which are used in measuring length, for instance laying a measuring rod along the object with one end coinciding with a particular mark, taking a reading at the other end, and so on. We shall suggest also that probability can be defined using the operational concept, but tied to the decision-maker's actions in making a fair bet, as described in the next chapter.

1.4 Approach and tools; rôle of judgement: indifference and preference

The theory that we are expounding deals with judgements regarding events. These judgements fall into two categories, corresponding to the two fundamental quantities introduced in Section 1.2 that are involved in decision-making: probabilities and utilities. The basic principle in dividing the problem of decision into two parts is that probability represents our judgement – regardless of our desires – as to the likelihood of the occurrence of the events under consideration, whereas utility represents our judgement of the relative desirability of the outcomes of the events. For example, we assess the likelihood (probability) of an accident in an impartial manner, taking into account all available information. The consequences of the accident may be highly undesirable, possibly involving loss of life, leading to our estimate of utility. But these factors do not affect our judgement of probability.



Probability will be defined formally in the next chapter; for the present introductory purposes, we introduce one of our tools, the urn containing balls of different colours. The urn could be a jar, a vase, a can or a bag – or one's hand, containing coloured stones. Our primitive ancestors would probably have used stones in their hand. It is traditional in probability theory to use urns with coloured balls, let us say red and pink, identical – apart from colour – in weight, size and texture. Then the balls are mixed and one is drawn without looking at the colour.