### NUMERICAL MODELING OF OCEAN CIRCULATION

Oceans are an essential part of the climate system. They dominate the hydrosphere and play a key role in natural climate variability. The modeling of ocean circulation is therefore very important, not only for its own sake, but also in terms of the prediction of weather patterns and the effects of climate change.

This book begins with an introduction to the basic computational techniques that are necessary for all models of the ocean and atmosphere, and the conditions they must satisfy. It contains descriptions of the workings of ocean models, the problems that must be solved in their construction, and how to evaluate computational results. Major emphasis is placed on those features that distinguish models of the ocean from other models in computational fluid dynamics, with the intention of examining ocean models critically, and determining what they do well and what they do poorly. Numerical analysis is introduced as needed, and exercises are included to illustrate major points. Additional resources are available at www.cambridge.org/9780521781824.

Developed from notes for a course taught in physical oceanography at the College of Oceanic and Atmospheric Sciences at Oregon State University, this book is ideal for graduate students of oceanography, geophysics, climatology and atmospheric science. It will also be of great interest to researchers in oceanography and atmospheric science.

ROBERT MILLER is a professor in the College of Oceanic and Atmospheric Sciences at Oregon State University, and is a member of the American Geophysical Union. Cambridge University Press 978-0-521-78182-4 - Numerical Modeling of Ocean Circulation Robert N. Miller Frontmatter <u>More information</u>

# NUMERICAL MODELING OF OCEAN CIRCULATION

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### Preface

This text grew out of notes for a course taught to graduate students in physical oceanography at the College of Oceanic and Atmospheric Sciences at Oregon State University. The students are typically in their second year of graduate school, having passed introductory courses in theoretical physical oceanography. This is the background assumed for the course. Most of the students at this point have seen some numerical analysis.

The course, and hence this text, is intended for all students of physical oceanography. Major emphasis is on those features that distinguish models of the ocean from other models in computational fluid mechanics. The intent is to examine ocean models critically, and determine what they do well and what they do poorly. We will ask when we can be confident that the model reflects nature, and when we can say that it is likely that we are looking at a feature of the model itself.

This is not a mathematics text as such, but it has a high mathematical content. Numerical analysis is introduced as needed. The reader may wish to consult supplementary references for basic numerical analysis of partial differential equations such as Sod (1985) (many typos, but reasonably current on fundamentals) or Richtmyer and Morton (1967), which is useful and commonly cited, though outdated. The reader might also find Isaacson and Keller (1966) or Allen *et al.* (1988) useful as general references. We will take examples from the two-volume work by Fletcher (1991) and the monograph by Leveque (1992). Durran (1999) contains useful and closely related material, from a slightly different viewpoint.

While implementation of detailed models has in the past been restricted to supercomputers at major computer laboratories, facilities at Oregon State University and many other institutions provide

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computing power of a magnitude available only at supercomputer centers a few years ago. Computational assignments and examples for this class were performed on computers at Oregon State University and at the National Center for Atmospheric Research (NCAR). Many, if not most, of the computing exercises presented here are now within the capabilities of common desktop computers and workstations. Examples and exercises are designed to illustrate questions of how models exhibit behavior typical of the real ocean and how and why they fail when they fail.

The scope of this text includes large-scale motions of the ocean in space and time. We consider the ocean as a shallow stratified fluid. By "shallow" we mean that we consider motions with horizontal scales much greater than the depth of the fluid. The motions which concern us evolve on timescales which are long compared to a day (or, more precisely, a "pendulum day," i.e., the period of a Foucault pendulum). Most of the motions considered here occur on large horizontal spatial scales. By "large" we will usually mean large compared to the distance the fastest internal wave travels in a day. While numerical ocean modeling would seem to be a diffuse subject that might not lend itself to a unified treatment in a single text, the majority of large-scale ocean models must deal with a number of common problems. Each way of solving these problems is a tradeoff, and a tactical choice that is suitable for one task may not be suitable for another.

Tides are not considered in this text, even though they are a prime example of shallow water motion. Tidal calculations for most purposes are performed without regard to stratification. More importantly, tidal calculations as performed in practice involve different computational strategies from those on which we focus here. Readers interested in the details of tidal calculations may refer to Foreman *et al.* (1993, 2000), Le Provost *et al.* (1994) or Ray (1993).

This work is intended as a text. It is not intended to review the state of the ocean modeling art. Rather its aim is to provide the student with the context in which discussion of numerical modeling is conducted. Attention is therefore confined to established facts and practices. For this reason, some very important topics, notably parameterization of turbulent processes, are omitted almost entirely, since even the basics of these topics are not well established in ocean modeling practice.

Chapters 2, 3 and 4 form the core of the ten-week course. There is usually time to treat one of the remaining chapters, and this is chosen

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according to class interests. Examples from the research literature are provided to illustrate the application of modeling techniques.

It is a pleasure to acknowledge the influence of many colleagues and mentors who have influenced my thinking on this topic. Conversations over the years with Andrew Bennett, Mark Cane, Michael Ghil, Ricardo Matano and Gary Sod, among others, have been particularly influential on my views of this subject. I would be especially pleased if mentors Alexandre Chorin and Allan Robinson were to see their fingerprints on this work. Linda Lamb's and Judy Scott's help with editing of the manuscript was invaluable, as was Dave Reinert's help with the figures. Acknowledgment is also made to the National Center for Atmospheric Research, which is sponsored by the National Science Foundation, for the computing time used in support of the classes and the compilation of results for this text.