

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

TURBULENCE STRUCTURE AND
VORTEX DYNAMICS

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

The Isaac Newton Institute of Mathematical Sciences of the University of Cambridge exists to stimulate research in all branches of the mathematical sciences, including pure mathematics, statistics, applied mathematics, theoretical physics, theoretical computer science, mathematical biology and economics. The research programmes it runs each year bring together leading mathematical scientists from all over the world to exchange ideas through seminars, teaching and informal interaction.

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

TURBULENCE STRUCTURE AND
VORTEX DYNAMICS

edited by

J.C.R. Hunt

University College London

and

J.C. Vassilicos

University of Cambridge



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics
Edited by J. C. R. Hunt and J. C. Vassilicos
Frontmatter
[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge CB2 1RP, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, United Kingdom
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

Printed in the United Kingdom at the University Press, Cambridge

Typeset in 12pt Computer Modern

A catalogue record for this book is available from the British Library

ISBN 0 521 78131 0 hardback

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

CONTENTS

Contributors	vii
Introduction	ix
1. Motion and Expansion of a Viscous Vortex Ring: Elliptical Slowing Down and Diffusive Expansion <i>Yasuhide Fukumoto & H.K. Moffatt</i>	1
2. Stretching and Compression of Vorticity in the 3D Euler Equations <i>J.D. Gibbon, B. Galanti & R.M. Kerr</i>	23
3. Structure of a New Family of Stretched Non-Axisymmetric Vortices <i>Stéphane Le Dizès</i>	35
4. Core Dynamics of a Coherent Structure: a Prototypical Physical-Space Cascade Mechanism? <i>Dhoorjaty S. Pradeep & Fazle Hussain</i>	54
5. Fundamental Instabilities in Spatially-Developing Wing Wakes and Temporally-Developing Vortex Pairs <i>C.H.K. Williamson, T. Leweke & G.D. Miller</i>	83
6. Vortex Lines and Vortex Tangles in Superfluid Helium <i>Carlo F. Barenghi</i>	104
7. Evolution of Localized Packets of Vorticity and Scalar in Turbulence <i>A. Leonard</i>	127
8. Vortical Structure and Modeling of Turbulence <i>E.A. Novikov</i>	140
9. The Issue of Local Isotropy of Velocity and Scalar Turbulent Fields <i>Z. Warhaft</i>	144
10. Near-Singular Flow Structure: Dissipation and Eduction <i>J.C. Vassilicos</i>	152
11. Vortex Stretching versus Production of Strain/Dissipation <i>Arkadaj Tsinober</i>	164
12. Dynamics and Statistics of Vortical Eddies in Turbulence <i>J.C.R. Hunt</i>	192
13. Stability of Vortex Structures in a Rotating Frame <i>Claude Cambon</i>	244
14. LES and Vortex Topology in Shear and Rotating Flows <i>Marcel Lesieur, Pierre Comte & Olivier Métais</i>	269
15. Conditional Mode Elimination with Asymptotic Freedom for Isotropic Turbulence at large Reynolds Numbers <i>David McComb & Craig Johnston</i>	289

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

Contributors

C.F. Barenghi, Mathematics Department, University of Newcastle, Newcastle upon Tyne NE1 7RU, UK

C.F.Barenghi@newcastle.ac.uk

C. Cambon, Laboratoire de Mécanique de Fluides et d'Acoustique, UMR 5509, Ecole Centrale de Lyon, BP 163, 69131 Ecully Cedex France

cambon@mecaflu.ec-lyon.fr

Pierre Comte, Institut de Mécanique des Fluides, 2 rue Boussingault, 67000 Strasbourg Cedex, France

B. Galanti, Department of Chemical Physics, Weizmann Institute of Science, Rehovot, Israel 76100

galanti@chemphys.weizmann.ac.il

J.D. Gibbon, Department of Mathematics, Imperial College of Science, Technology & Medicine, 180 Queen's Gate, London SW7 2BZ, UK

j.gibbon@ic.ac.uk

Y. Fukumoto, Graduate School of Mathematics, Kyushu University 33, Fukuoka 812-8581, Japan

yasuhide@math.kyushu-u.ac.jp

J.C.R. Hunt, Department of Space and Climate Physics, University College, Gower St., London WC1, UK

jcrh@mssl.ucl.ac.uk

F. Hussain, Department of Mechanical Engineering, University of Houston, Houston TX 77204-4792, USA

fhussain@UH.edu

Craig Johnston, Department of Physics & Astronomy, The University of Edinburgh, The King's Buildings, Mayfield Road, Edinburgh EH9 3JZ, UK

C.Johnston@ed.ac.uk

R.M. Kerr, National Center for Atmospheric Research, P.O. Box 3000, Boulder, Colorado 80307, USA

kerrrobt@ncar.ucar.edu

S. le Dizès, Institut de Recherche sur les Phénomènes Hors Equilibre, Université Aix-Marseille, 12 avenue Général Leclerc, F 13003 Marseille, France

ledizes@marius.univ-mrs.fr

A. Leonard, Graduate Aeronautical Laboratories, California Institute of Technology, Pasadena, CA 91125, USA

tony@galcit.caltech.edu

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

viii

Contributors

M. Lesieur, Laboratoires de Ecoulements Géophysiques et Industriels, BP 53,
38041 Grenoble Cedex, France
Marcel.Lesieur@hmg.inpg.fr

T. Leweke, Institut de Recherche sur les Phénomènes Hors Equilibre, Université
Aix-Marseille, 12 avenue Général Leclerc, F 13003 Marseille, France
leweke@marius.univ-mrs.fr

W.D. McComb, Department of Physics & Astronomy, The University of Ed-
inburgh, The King's Buildings, Mayfield Road, Edinburgh EH9 3JZ., UK
wilm@holyrood.ed.ac.uk

Olivier Metais, LEGI, BP 53, 38041 Grenoble Cedex 9, France

G.D. Miller, Boeing Airplane Group, Seattle WA 98124, USA

H.K. Moffatt, Isaac Newton Institute for Mathematical Sciences, University
of Cambridge, 20 Clarkson Road, Cambridge CB3 0EH, UK
hkm2@newton.cam.ac.uk

E.A. Novikov, Institute for Nonlinear Science, University of California, San
Diego, La Jolla CA 92093-0402, USA
enovikov@ucsd.edu

D.S. Pradeep, Department of Mechanical Engineering, University of Houston,
Houston TX 77204-4792, USA

A. Tsinober, Department of Fluid Mechanics and Heat Transfer, Tel Aviv
University, Ramat Aviv 69978, Israel
tsinober@eng.tau.ac.il

J.C. Vassilicos, DAMTP, University of Cambridge, Cambridge, CB3 9EW,
UK
J.C.Vassilicos@damtp.cam.ac.uk

C.H.K. Williamson, Dept of Mechanical & Aerospace Engineering, Upson
Hall, Cornell University, Ithaca NY 14853, USA
cw26@cornell.edu

Z. Warhaft, Department of Mechanical & Aerospace Engineering, Upson Hall,
Cornell University, Ithaca NY 14853, USA
zw16@cornell.edu

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

Introduction

Leonardo da Vinci's drawings of eddies below waterfalls, John Constable's paintings of swirling and disintegrating cloud shapes and L.F. Richardson's Swiftian rhyme all show different aspects of the essential nature of turbulence. When expressed in prosaic scientific language the modern understanding of turbulence is that it is a collection of weakly correlated vortical motions, which, despite their intermittent and chaotic distribution over a wide range of space and time scales, actually consist of local characteristic 'eddy' patterns that persist as they move around under the influences of their own and other eddies' vorticity fields. Numerical simulations and experimental observations have now identified basic forms and even the 'life-cycles' of some of these structures. Some of them, for example, seem to appear as local shear layers, then evolve into vortex tubes and finally break up. In some cases quite extreme distortion and interaction between vortices lead to very large local velocities. These universal features occur in all highly turbulent flows. However, because the largest scale eddies extend across the whole flow and are strongly influenced by the boundary conditions they are not universal; nevertheless they tend to have the same characteristic forms in each type of turbulent flow.

In the Isaac Newton Institute (INI) programme on turbulence held between January and July 1999 there were several workshops and conferences on different aspects of the subject. All of them succeeded in bringing together physicists, engineers, mathematicians and experimentalists, as can be seen in this and other volumes and review articles describing the programme (Voke, Sandham & Kleiser 1999; Launder & Sandham 2000; Vassilicos 2000; Hunt, Sandham, Vassilicos, Launder, Monkewitz & Hewitt 2000).

In the Symposium on Vortex Dynamics and Turbulence Structure there were lectures and discussions on a number of key questions that have engaged turbulence researchers for many years. What is the overall significance for turbulent flows of vortical structures? How should one study their persistence and characteristic structure; do they correspond to some kind of eigensolutions of the basic equations or of some reduced form of these equations; what are their geometrical statistics and their stability, given that they exist in a chaotic environment with many other structures surrounding them? How do they interact or not interact with each other and with surrounding turbulence, and what are their dissipative properties? Are the near-singularities of the turbulence or the conjectured finite-time singularities related to the vortical or other (e.g. straining) structures, and if so what kind? What are the Eulerian and Lagrangian properties of such structures, and how do their conditional statistics relate to the well-established unconditional Eulerian and

Lagrangian statistics (e.g. spectra, energy cascades up- and down-scale, relative motions of particles) and the scaling properties of the entire flow? To what extent can turbulence be represented in terms of space-filling functions such as Fourier or Chebychev basis functions or is it necessary to work in terms of localised functions such as wavelets.

The articles in this volume address all these questions. Most involve mathematical analysis, but some describe numerical simulations and experimental results that focus on these questions. Some of the papers focused on the deterministic kinds of vortical motion that characterise eddy motions, while others also relate these studies to the overall statistics of the turbulent flows which can be measured more readily than the details of individual eddies. Only one paper is exclusively concerned with the statistical dynamics of turbulence.

Deterministic analyses were applied to isolated vortices, to their response when subject to large scale rotational and irrotational straining, and to their interaction with each other. In some situations large scale straining is a reasonable ‘mean-field’ approximation for the average effects of all other vortices. But in other situations it is necessary to consider specific interactions between small numbers of vortices. **Fukumoto & Moffatt** analyse the effect of viscosity on the motion of a vortex ring, and how the diffusion of vorticity changes its motion. The straining of vortices are considered in three papers; **Gibbon, Galanti & Kerr** consider the general mathematical properties of the stretching and compression of vorticity, including the surprising fact that its tendency to become a singularity at any point in the flow is related to the overall properties of the flow.

There are many different ways that finite amplitude vortices can be stretched and distorted, and **Le Dizes** presents an analysis of a new family of stretched non-axisymmetric vortices. As elongated vortices are stretched and distorted by external straining fields, oscillations and waves can develop and lead to the formation of new structures and ultimately to the total breakdown into small scale chaotic motion. The basic mechanisms of these ‘core dynamics’ are reviewed by **Pradeep & Hussain**. In some cases the external motions are caused by adjacent vortices and then the instability and transformations are coupled in a global sense, as shown in the experimental paper of **Williamson, Leweke & Miller**. In ‘classical’ fluids such as air and water at ambient temperature, the vorticity in a vortex diffuses out of vortices or is exchanged when vortices interact as a result of molecular diffusion. In superfluids at very low temperatures these diffusion processes do not occur and therefore vortices move and interact with each other according to the theory of ideal inviscid flow. However certain quantum effects also lead to dissipative phenomena such as reconnection. This is the motivation of **Barenghi’s** paper on ideal fluid turbulence and its relation to normal fluid turbulence.

Other papers here show how a combination of deterministic and statistical

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)*Introduction*

xi

analyses of turbulent velocity fields is leading to a better understanding of the qualitative characteristics of the eddy motion in turbulence as well as to quantitative predictions. Much research is based on the assumption that this is the key to improving the approximate models of turbulence (such as Large Eddy Simulation and spectral models) and to assessing their accuracy and range of application. **Leonard** analyses, following the earlier ideas of Synge & Lin (1943), the dynamics and kinematics of small individual eddies or packets of vorticity, strained by eddy motions with larger length scales. He explores the limits when the lengths of the strained eddies become comparable with the larger ones, and tend to form elongated and randomly twisted 'ribbons'. The consequences for the spectra are worked out.

Novikov explains why this dynamical interaction implies that small scale turbulence may not be as statistically independent of the large scales as is assumed in Kolmogorov's theory; there may be fewer degrees of freedom and some aspects of their motion may be 'slaved' to the larger scales on some 'slow' manifold. He derives some statistical conditions based on this concept. However the eddy motions do need to be considered because they determine the intermittency of turbulence which he explains as being crucial to the interpretation of the overall turbulence statistics.

Warhaft's discussion of experimental measurements of small scale turbulence also takes up this theme. The higher the order of the statistical moments the more they are anisotropic. These are associated with small scale organised structures, in which there are strong local gradients in both the velocity and scalar fields. He demonstrates that the structures can be defined more precisely if measurements are made at three rather than two points simultaneously, which has been usual up to now.

Flow visualisation and experiments have indicated that these structures are quite geometrically complex, often approximating to sheets of vorticity and scalars wound up into spiral forms which correspond to a type of ideal mathematical singularity. **Vassilicos** analyses such velocity fields and their effects on the diffusion of scalars; he also shows how these types of eddy can be detected when they occur at random positions in numerical simulations of turbulent flows. He demonstrates how such structures are consistent with the 'anomalous' scaling laws found in statistical correlations in fully developed turbulence.

Tsinober analyses the dynamical equations governing correlations between the straining and vorticity fields of small scale turbulence, in order to clarify the relative roles of vortex stretching and straining, or relative advection, in producing even smaller scales and thence dissipation. His results suggest that it may be necessary to consider a cycle of stretching and straining of eddy motions to understand the full dynamics; indeed the simple, rather static concept of vortex stretching is quite inconsistent with the production

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)

of smaller scales. Like Betchov in 1956 he has the temerity to propose an amendment to L.F. Richardson's rhyme about the roles of great whirls and lesser whirls in the cascade process!

Hunt's paper is similar to Leonard's in assuming that the analysis of the non-linear interactions in turbulence can be usefully idealised as a sequence of events when small scale vortices are strained by large scale motions. He discusses how the weakly non-linear effects cause the vortex sheets to roll up, or become unstable. Curiously there is a geometrical problem to be solved: how to define the changes in these shapes, which are associated with the cycle of growth, transformation and breakup of small scale eddies, that Tsinober analyses using statistical data in his paper. Hunt also reviews an outstanding kinematical question about turbulence as to when and to what extent spectra reflect on the one hand the forms of the eddies themselves, especially their singularities, and on the other, the distribution of their amplitudes with wavenumber (or frequency).

Cambon takes up the question, touched on by Hunt, that when vortices are formed in turbulence, for example as a result of straining by larger scales, various kinds of waves and instabilities tend to grow. He reviews and relates a number of current mathematical techniques used for analysing these perturbations. He points out how some are local and some global; some are based on eigen solutions, while others are based on general linear solutions more dependent on initial conditions. Many interesting special cases are described in detail, and reasons are given why cyclonic eddies are more stable than anticyclonic.

In numerical simulations the resolution is now fine enough for even small scale flow structure to be described for high Reynolds number turbulence. **Lesieur, Comte and Metais** use Large Eddy Simulation techniques to examine the structure of the vortices that form in shear flows and rotating flows. They explain how the vortices contribute to the statistical distribution of kinetic energy in the turbulence, as well as describing in some detail how the different scales and orientations of vortices are related in these chaotic flows, which have a high degree of local organisation.

On long enough timescales it is likely that the internal eddy structure is unimportant, in which case turbulence can be analysed rather like a viscoelastic fluid, based on the concepts and methods of statistical physics. **McComb & Johnston** use methods involving the Renormalisation Group. In conjunction with novel assumptions about the statistical independence of the small eddy scales, they derive quantitatively the energy spectrum of turbulence and new results about the internal 'eddy viscosity' that controls the energy transfer between eddy scales. These methods may well have wider applications to more complex flows in future.

We, and we believe all the speakers at the workshop, are extremely grateful

Cambridge University Press

978-0-521-78131-2 - Turbulence Structure and Vortex Dynamics

Edited by J. C. R. Hunt and J. C. Vassilicos

Frontmatter

[More information](#)*Introduction*

xiii

to Geoff Hewitt, Peter Monkewitz and Neil Sandham for their invaluable contribution to the organisation of the Isaac Newton Institute's Turbulence Research Programme: to Keith Moffatt and the wonderful staff of the Isaac Newton Institute, ERCOFTAC, the European Commission, the Royal Academy of Engineering and the Industrial Working Group under the chairmanship of Michael Reeks, for their support; and to the Isaac Newton Institute for sponsoring and hosting the workshops.

References

- Hunt, J.C.R., Sandham, N., Vassilicos, J.C., Launder, B.E., Monkewitz, P.A. & Hewitt, G.F., 2000, 'Developments in turbulence research: a review based on the 1999 Programme of the Isaac Newton Institute, Cambridge', submitted to *J. Fluid Mech.*
- Launder, B.E. & Sandham, N.D. (eds.), 2000, *Closure Strategies for Turbulent and Transitional Flows*, Cambridge University Press.
- Synge, J.L. & Lin, C.C., 1943, 'On a statistical model of isotropic turbulence', *Trans. R. Soc. Canada*, **37**, 45–79.
- Vassilicos, J.C., (ed.), 2000, *Intermittency in Turbulent Flows*, Cambridge University Press.
- Voke, P.R., Sandham, N., & Kleiser, L. (eds.), 1999, *Direct Large-Eddy Simulation III*, Proceedings of the Isaac Newton Institute Symposium/ERCOFTAC Workshop, Cambridge UK, 12–14 May 1999, ERCOFTAC Series, Vol. 7, Kluwer Academic Publishers.

Julian Hunt

Christos Vassilicos