Modeling Aggregate Behavior and Fluctuations in Economics

This book has two components: stochastic dynamics and stochastic random combinatorial analysis. The first discusses evolving patterns of interactions of a large but finite number of agents of several types. Changes of agent types or their choices or decisions over time are formulated as jump Markov processes with suitably specified transition rates: Optimizations by agents make these rates generally endogenous. Probabilistic equilibrium selection rules are also discussed, together with the distributions of relative sizes of the basin of attraction. As the number of agents approaches infinity, we recover deterministic macroeconomic relations of more conventional economic models. The second component analyzes how agents form clusters of various sizes. This has applications for discussing sizes or shares of markets by several types of agents, which involves some combinatorial analysis patterned after the population genetics literature. These are shown to be relevant to distributions of returns to assets, volatility of returns, and power laws.

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Stochastic Views of Interacting Agents

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To My Late Father

CONTENTS

PREFACE

This book is a sequel to Aoki (1996) in the loose sense that it is motivated by a similar set of considerations to its predecessor and shares some of the same objectives. It records my efforts, since the publication of that book, at evaluating and reformulating macroeconomic models that are employed by the mainstream economic profession. In this book, a stochastic point of view is taken to construct models for finite numbers of interacting agents. In other words, the book emphasizes models that focus on economic phenomena that involve stochastic laws, or stochastic regularities that govern economic phenomena.

To make this book more readily accessible to traditionally trained economists and graduate students in economics, it is more narrowly focused than my previous one, and it attempts to establish better links with some well-known models in the macroeconomic literature. This book is motivated by my strong desire to persuade some traditionally trained economists to phrase their questions in stochastic ways and apply some of the methods presented in it to their work.

Mainstream economists and graduate students of economics may wonder why to use stochastic models or what additional or new insights they yield or, if stochastic laws in economics are so useful, why they have not heard of them before. A short answer is that models with finite numbers of agents in appropriate stochastic contexts reveal interesting economic phenomena that are invisible in deterministic models with infinite numbers of (representative) agents. Traditional models wash out some important information about economies, but one would not know them. This finitary and stochastic approach provides more information about the economy than deterministic economic laws permit.

There are many areas of economics to which my approach applies. In speaking of inflation and unemployment, Tobin, in his presidential address at the American Economic Association Meeting in 1971, came close to describing stochastic laws and aggregate dynamics and fluctuations (in terms of Fokker–Planck equations, say), according to my way of modeling, when he said, "... stochastic macro-equilibrium, stochastic, because random intersectoral shocks keep individual labor markets in diverse states of disequilibrium;

macro-equilibrium, because the perpetual flux of particular markets produces fairly definite aggregate outcomes of unemployment and wages . . . ."

Another major class of examples is building business-cycle models. All kinds of theories are found in the literature, and new theories keep cropping up. The real business cycle (RBC) theory by Kydland and Prescott (1982) may arguably be the most influential current theory among mainstream economists. As typified by the RBC, a natural research strategy to study business cycles is to explain fluctuations as a direct outcome of the behavior of *individual agents*. The more strongly one wishes to interpret aggregate fluctuations as something "rational" or "optimal," the more one is led to this essentially microeconomic approach. The mission of this approach is to explain fluctuations as responses of individual agents to changes in their economic environments. The consumer's intertemporal substitution, for example, is a device to achieve this goal. This has been the standard approach in the mainstream economics in the last twenty years or so.

Surely, we would like to know the distribution of durations of "good" times and "bad" times. When models admit multiple equilibria, which equilibrium, if any, will the model settle in? How long can the system be expected to stay in one basin of attraction before it moves to another? And so on. This book presents a different approach to fluctuations.This alternative approach is based on the fact that economy consists of a large number of agents or sectors. (The population of a large industrialized economy, for example, contains of the order of 10^8 agents.) Even if agents intertemporally maximize their respective objective functions, their environments or constraints all differ and are always subject to idiosyncratic shocks. Our alternative approach emphasizes that an outcome of interactions of a large number of agents facing such incessant idiosyncratic shocks cannot be described by a response of the representative agent and calls for a model of stochastic processes. In a seminal work, Slutzky (1937) proposed a stochastic approach. We follow his lead in this book to build a stochastic model of fluctuations and growth.

Although studies of macroeconomy with many heterogeneous agents are not new, dynamic behavior of economies in disequilibrium is not satisfactorily analyzed. The traditional Walrasian economy is the egregious example. It focuses on price adjustment with the help of nonexistent auctioneer.

In a nutshell, this book formulates and analyzes a large but finite number of interacting economic agents as continous-time Markov chains with discrete state spaces. Dynamics are described in terms of the backward Chapman–Kolomogorov equations, also known as the master equations. We are interested in such questions as the existence of stationary probability distributions for some variables, of critical points of aggregate dynamics, and of fluctuations about locally stable equilibria, and in the distributions of relative sizes of the basins of attraction and associated probabilities, how they relate to the lengths

of business cycles, and so forth. The agents are assumed to be exchangeable rather than representative and have either a finite or countably infinite number of decisions to choose from, or they belong to a finite or countably infinite number of types or categories.

Unlike the jump Markov processes treated in standard textbooks on probability or stochastic processes, the transition rates of the processes in this book are endogenously determined via the value maximizations by the agents in the model. Using this framework, we take fresh looks at some well-known search models, such as the Diamond model and disequilibrium quantity adjustment models, as well as models for diffusions of innovations and endogenous growth. Formulations involving a few large clusters of agents in markets, and the implications for the volatility of returns on financial-asset markets, which may develop from interaction of many agents, are also examined using random combinatorial analysis. Such investigations lead to results not usually discussed in the traditional macroeconomic literature, such as the existence of power laws for some variables of interest, discoveries that some common laws apply to seemingly unrelated areas, and so on.

This book is aimed at advanced graduate students and practicing professionals in economics, as well as in some related areas, such as the recently formed area of econophysics. Some of the topics have been discussed in my graduate courses at UCLA and at Keio University, Tokyo, and at several conferences, workshops, and seminars. I wish to express appreciation to Professors R. Craine, K. Kawamata, A. Kirman, M. Marchesi, T. Lux, W. Semmler, H. Yoshikawa, and J.-B. Zimmermann for opportunities for presenting talks, and to Professors Y. Shirai, D. Costantini, U. Garibaldi, and D. Sornette for their useful comments on some parts of the topics in the book. I am particularly indebted to Professors Yoshikawa, Costantini, and Garibaldi for their help and guidance in overcoming my ignorance and misunderstandings. Simulations reported in this book were programmed by a former and a current graduate student at UCLA, J. Nagamine and R. Singh. I thank them for their help.

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