LARGE-EDDY SIMULATIONS OF TURBULENCE

Large-Eddy Simulations of Turbulence is an ideal introduction for people new to large-eddy simulation (LES), direct numerical simulation, and Reynolds-averaged Navier–Stokes simulation and makes an excellent reference for researchers. Of particular interest in the text is the detailed discussion in Chapter 2 of vorticity, pressure, and the velocity gradient tensor, which are quantities useful for probing the results of a simulation - particularly when looking for coherent vortices and coherent structures. Chapters 4 and 5 feature an in-depth discussion of spectral subgrid-scale modeling. Although physical-space models are generally more readily applied, spectral models give insight into the requirements and limitations in subgrid-scale modeling and backscattering. A third special feature is the detailed discussion in Chapter 7 of LES of compressible flows - a topic previously accessible only in articles scattered throughout the literature. This will be of interest to those dealing with supersonic flows, combustion, astrophysics, and other related topics. Chapter 8 focuses on geophysical fluid dynamics with emphasis on rotating stratified shear flows. Interesting applications of LES to storm formation are given in particular.

Marcel Lesieur, Olivier Métais, and Pierre Comte form the nucleus of the Grenoble Equipe Modélisation et Simulation de la Turbulence (the Grenoble team for modeling and simulating turbulence) and play an important role in the development of subgrid-scale modeling of turbulent flows required for large-eddy simulation and in the implementation of large-eddy simulation methodology in research and applications. They were responsible for early research on spectral subgrid-scale closure and the use of the closure approach in developing the physical-space structure-function model. More recently they have made significant contributions to the development of modeling for compressible turbulent flows.

Large-Eddy Simulations of Turbulence

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Preface

In 1949, in an unpublished report to the U.S. Office of Naval Research, John von Neumann remarked of turbulence that

the great importance of turbulence requires no further emphasis. Turbulence undoubtedly represents a central principle for many parts of physics, and a thorough understanding of its properties must be expected to lead to advances in many fields.... [T]urbulence represents per se an important principle in physical theory and in pure mathematics.... These considerations justify the view that a considerable effort towards a detailed understanding of the mechanisms of turbulence is called for....¹

Few people today would disagree with these comments on the importance of understanding turbulence and, as implied, of its prediction. And, although the turbulence problem has still yet to be "solved," our understanding of turbulence has significantly advanced since that time; this progress has come through a combination of theoretical studies, often ingenious experiments, and judicious numerical simulations. In addition, from this understanding, our ability to predict, or at least to model, turbulence has greatly improved; methods to predict turbulent flows using large-eddy simulation (LES) are the main focus of the present book.

The impact of von Neumann is still felt today in the prediction of turbulent flows, both in his work on numerical methods and in the people and the research he has influenced. The genesis of the method of large-eddy simulation (or possibly more appropriately, "simulation des grandes échelles") was in the early 1960s with the research of Joe Smagorinsky. At the time, Smagorinsky was working in von Neumann's group at Princeton, developing modeling for dissipation and diffusion in numerical weather prediction. Doug Lilly, who later worked with Smagorinsky, realized the potential for simulating turbulent flows of Smagorinky's modeling work. When Lilly joined the National Center

¹ Quote provided by Russell J. Donnelly.

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for Atmospheric Research (NCAR), he encouraged NCAR's Jim Deardorff to pursue this line of research; Deardorff later completed the first series of LES, publishing his results in several important papers in the early 1970s. At that time at NCAR, Doug Lilly, Chuck Leith, Jim Deardorff, and later Jack Herring established a most stimulating environment for turbulence research. In addition to these first large-eddy simulations and other research, such as on the parameterization of boundary layer turbulence and studies of clear-air turbulence, the first direct numerical simulations were carried out at NCAR in that time period by Steve Orszag and Stu Patterson.

Research on large-eddy simulation is increasing rapidly as this methodology takes its place as a valuable numerical simulation tool along with direct numerical simulation and Reynolds-averaged Navier–Stokes simulation. As an example of this, a very informal survey using the Science Citation Index indicates that the number of archival papers with "large-eddy simulation" in their titles has increased almost geometrically in the past decade or so from 11 in 1990, to 25 in 1995, to 51 in 2000, and to 95 in 2003. In addition, with continuing improvements in numerical methods and also in subgrid modeling of turbulence, large-eddy simulation is being utilized more and more in applications. This can be seen, for example, in its implementation in most of the commercially available fluid dynamics codes. Undoubtedly it will become a principal tool in applications in the future.

This tremendous increase in the interest in, and of use of, LES demands the availability of books that describe the theory and modeling aspects of LES and that also give detailed examples of how it has been and can be applied. Such was the task of the authors of this book.

The three authors of this book, Marcel Lesieur, Olivier Métais, and Pierre Comte, who have formed the nucleus of the Grenoble Equipe Modélisation et Simulation de la Turbulence (the Grenoble team for modeling and simulating turbulence), are eminently qualified to write such a book. They have been very active in many developments in subgrid modeling of turbulent flows required for large-eddy simulation and also in the implementation of LES methodology in research and applications. Among other things they have been responsible for some of the first research on spectral subgrid-scale closure; using some of the ideas from this closure approach they developed the physical-space structure-function model, which has received considerable attention and use; and more recently they have led in developing modeling for compressible turbulent flows. Of course the readers will find out much more about their contributions in this book.

This book contains the basic information required for both a person new to the subject of large-eddy simulation and for use as a reference for the more experienced researcher. In addition, it contains several additional items the reader may find of special importance. The first of these items is contained in

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Chapter 2, where a detailed discussion is given of vorticity, pressure, and the velocity gradient tensor, which are quantities useful for probing the results of a simulation – in particular looking for coherent vortices and coherent structures. Since the approach of large-eddy simulation focuses on the large-scale motions, which are often coherent, it is important to have the appropriate tools available to examine the simulations for such features.

Another item of special importance is the in-depth discussion of spectral subgrid-scale modeling in Chapters 4 and 5. Although physical-space models are generally more readily applied, the spectral models give more insight into the requirements and limitations in subgrid-scale modeling and related issues, such as backscattering. A third special item in the book is its detailed discussion of the large-eddy simulation of compressible flows in Chapter 7 - a subject to which the authors have made important recent contributions, and information about which has, up till now, only been available in articles scattered throughout the literature. This topic will be of interest not only to those dealing with supersonic flows but also to those interested in combustion, astrophysics, and other related topics. In the final chapter the authors go back to the origins of large-eddy simulations and discuss applications to problems in geophysical fluid mechanics. The reader will become acquainted with examples of how large-eddy simulation can enable issues that are at such high Reynolds numbers that they are available only to large-eddy simulation to be addressed. Among the topics discussed are the effects of system rotation on turbulence and the generation of storms through baroclinic instabilities.

To learn a new topic, it is often best to have available examples worked out in some detail. One of the great merits of this book is that it is filled with many examples often taken from the research of the authors. These examples are supplemented by numerous animations, which are referenced at the end of the appropriate chapters and are available on the accompanying CD-ROM.

The authors have succeeded exceptionally well in providing a book that will be valuable for both the novice and the experienced user. The book will be useful as a text or reference in graduate courses on large-eddy simulation, and it should find a place on the reference shelf of both scientists and engineers who have interest in large-eddy simulation. The book should become a significant element in this rapidly developing field of turbulence simulation.

> James J. Riley June 2004