The Dynamics of Fluidized Particles

Recent years have seen major progress in the development of equations to describe the motion of fluid–particle mixtures and their application to a limited range of problems. With rapid advances in numerical methods and computing power we are now presented with new opportunities to use direct integration of these equations in the solution of complex practical problems. However, results so obtained are only as good as the equations on which they are based, so it is essential to have a clear understanding of the underlying physics and the extent to which it is reflected properly in these equations.

In *The Dynamics of Fluidized Particles*, the author formulates these equations carefully and then to describe some important existing applications that serve to test their ability to predict salient phenomena. This account will be of value to both novices and established researchers in the field, and also to people interested in applying the equations to practical engineering problems.

Roy Jackson is a Professor of Engineering & Applied Science Emeritus at Princeton University. He has received many academic honours, including the School of Engineering Distinguished Teaching Award from Princeton University, and is a fellow of the Royal Society. The American Chemical Society has also recently published a “festschrift” in honour of his many research contributions.
The Dynamics of Fluidized Particles

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In memory of Susan
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Preface

This book addresses the motion of systems of solid particles immersed in a fluid that may be a liquid or a gas. The focus is on the range of particle concentrations of greatest interest in the operation of process plants, that is, solids volume fractions anywhere from a few percent to random close packing. As typical process applications we might mention hoppers and bunkers, dense fluidized beds, pneumatic transport lines, circulating fluidized beds, standpipes, cyclones, riser reactors, and slurry pipelines, but the same ideas can be used in nonprocess applications such as sediment transport, landslides, and avalanches. The book is intended as an introduction to this field for graduate students and others entering it for the first time but, by drawing together widely scattered material, it is hoped that it may also serve as a useful overview for more experienced workers. Most of the material is covered somewhere in the existing literature, to which the reader’s attention is directed, but some appears here for the first time, for example, parts of Chapters 3 and 4.

Many of the figures are taken from other publications and my thanks are due to the copyright holders for permission to reproduce this material. In certain cases these permissions are acknowledged in the captions of the figures in question but, in addition, I am indebted to the following organisations and individuals: Academic Press for Figures 5.1, 5.6, 5.7, and 6.32; Birkhäuser Verlag for Figure 5.26; The Institution of Chemical Engineers for Figures 5.4, 5.5, 5.9, 5.10, 5.14, 5.15, and 5.16; T. B. Anderson for Figures 4.8, 4.10, and 4.11; Y-M. Chen for Figures 7.4 and 7.5; B. Glasser for Figure 5.44; G. D. Cody for Figure 3.20; and T. J. Mountziaris for Figure 7.12.

I would not have undertaken the task of writing this book without urging by George Batchelor, whose influence has been a guiding light since my student days. A serious start on the work was made during a half year spent as a Visiting
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Fellow Commoner at Trinity College, Cambridge, in 1994 and I am grateful to the College for providing this opportunity for uninterrupted thought.

Such understanding of the subject as I have owes much to informal interactions over the years with my graduate students, and also with colleagues in academia and industry, among whom I should mention Sankaran Sundaresan, John Davidson, John Hinch, Jennifer Sinclair, Bud Homsy, Don Koch, John Gwyn, George Cody, and the late Yuri Buyevich. In particular, my collaboration with Sankaran Sundaresan over the past decade has been a source of special pleasure. At a more general level the intellectual atmosphere of the Chemical Engineering Department at Princeton and stimulating discussions with Dudley Saville, Ioannis Kevrekidis, Pablo Debenedetti, Bill Russel, and Sandra Troian have served to sharpen my fluid-mechanical wits in many ways. In addition Pablo Debenedetti, in his role as department chairman, has been most supportive of my literary efforts. Finally, I must acknowledge the invaluable help provided by Patti Weiss, who has taken care of many of those time-consuming details under whose weight the project might otherwise have foundered.