Modem Theory

At the heart of any modern communication system is the modem, connecting the data source to the communication channel. This first course in the mathematical theory of modem design introduces the theory of digital modulation and coding that underpins the design of digital telecommunications systems. A detailed treatment of core subjects is provided, including baseband and passband modulation and demodulation, equalization, and sequence estimation. The modulation waveforms for communication channels and digital recording channels are treated in a common setting and with unified terminology. A variety of more advanced topics is also covered, such as trellis codes, turbo codes, the Viterbi algorithm, block codes, maximum likelihood and maximum posterior probability, iterative demodulation, and jamming. Numerous end-of-chapter exercises are also included to test the reader’s understanding throughout. This insightful book is ideal for senior undergraduate students studying digital communications and is also a useful reference for practicing engineers.

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Modem Theory

An Introduction to Telecommunications

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In loving memory of
Gayle Jones Blahut (1962–2008)
— who always had the right touch
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The field of telecommunication consists of the theory and the practice of communication at a distance, principally electronic communication. Many systems for telecommunication now take the form of large, complex, interconnected data networks with both wired and wireless segments, and the design of such systems is based on a rich theory. Communication theory studies methods for the design of signaling waveforms to transmit information from point to point, as within a telecommunication system. Communication theory is that part of information theory that is concerned with the explicit design of suitable waveforms to convey messages and with the performance of those waveforms when received in the presence of noise and other channel impairments. Digital telecommunication theory, or *modem theory*, is that part of communication theory in which digital modulation and demodulation techniques play a prominent role in the communication process, either because the information to be transmitted is digital or because the information is temporarily represented in digital form for the purpose of transmission.

Digital communication systems are in widespread use and are now in the process of sweeping away even the time-honored analog communication systems, such as those used in radio, television, and telephony. The main task of communication theory is the design of efficient waveforms for the transmission of information over band-limited or power-limited channels. The most sweeping conclusion of information theory is that all communication is essentially digital. The nature of the data that is transmitted is unimportant to the design of a digital communication system. This is in marked contrast to analog communication systems, such as radio or television, in which the properties of the transmitted waveform are inextricably tied up with the properties of the application, and only weakly tied to considerations of the communication channel. To make the point more strongly, we can give a whimsical definition of a digital communication system as a communication system designed to best use a given channel, and an analog communication system as one designed to best fit a given source. The spectrum of a well-designed digital communication waveform is a good match to the passband characteristics of the channel; the only essential way in which the source affects the spectrum of the waveform is by the bit rate. In contrast, the spectrum of an analog communication waveform depends critically on the properties of the source.
The purpose of this book is to give a general introduction to the modulation waveforms and demodulation techniques that are central to the design of digital telecommunication systems. Moreover, because recording is essentially a communication process – from a past time to a future time – modulation techniques are also central to the design of magnetic and optical recording systems. Modulation waveforms for passband channels and for baseband channels, such as magnetic recording channels, are treated in a common setting and with unified terminology.

The topics of this book are confined to the modulation layer of communication theory. The topics at the modulation layer lie above other topics needed for the physical design of communication equipment and lie below topics in other layers of the theory that deal with networking, routing, and application sessions. These are topics for other books. The compaction and compression of source data, including analog or voice, are also topics that are not treated in this book. These, also, are topics for other books.

The material in this book consists, for the most part, of selected chapters from *Digital Transmission of Information* (published in 1990), which have been rewritten and expanded to fit the needs of a course in modem theory and digital telecommunications. Waveforms and modulators are studied in Chapters 2 and 5 for baseband and passband channels, respectively. Basic demodulators are developed in Chapters 3 and 6 for baseband and passband channels, respectively. More advanced methods of demodulation for channels with dispersion are studied in Chapter 4. In Chapter 3, the matched filter is introduced as a filter to maximize signal-to-noise ratio prior to a demodulation decision. A stronger statement of optimality of the matched filter is deferred to Chapter 7, where it is shown to be part of the maximum-likelihood demodulator for both coherent and noncoherent demodulation in gaussian noise. These first seven chapters contain the central ideas of modulation and demodulation, which are at the core of the theory of modems. The final five chapters then go deeper into the subject by developing some of the other topics that are needed to round out the foundations of the theory of modems. Chapter 8 treats methods of synchronizing the transmitter and the receiver so that they have the same time reference. Chapters 9 and 10 discuss methods of coding for communication channels to control errors. Rather than modulate one data bit, or a few data bits, at a time into a communication waveform, a coded representation modulates an entire message into a communication waveform so that cross-checks can eliminate errors. Chapter 9 discusses codes designed for an additive noise channel, Chapter 10 discusses codes designed for a discrete channel, usually binary. Finally, Chapters 11 and 12 advance the theory beyond the simple linear channel studied in most of the book. Chapter 11 studies the robustness of modems in the presence of simple nonlinearities and fading. Chapter 12 discusses techniques for the prevention of intentional disruption of communications by a malicious adversary known as a jammer.

Modern digital telephony, now in widespread use, is an almost miraculous system – partly wireless and partly wired – in which our everywhere environment is filled with a dense but faint electromagnetic fabric that millions of telephone users can tap into
to draw out conversations, text, and data almost without end, and certainly without awareness of the immense theory and the extensive industry that make this fabric possible. This is the real miracle of the recent decades, and this technological miracle has changed the world far more than has any political theory or practice. This book studies digital communication theory, which consists of the collection of waveform methods that underlie the telecommunication system. A deeper investigation of the merits of these methods is provided within the subject of information theory, which is only touched on in this book. The study of modems is a first step in understanding this wondrous wireless fabric, as well as the enormous and sophisticated wired backbone that underlies this global system. The study of information theory provides a fuller understanding of the optimality of these methods.

This book evolved within the rich environment of students and faculty at the University of Illinois. I could not have found a better set of colleagues anywhere with which to interact, and no environment more intellectually stimulating. The quality of the book has much to do with the typing skills of Mrs Frances Bridges and the editing skills of Mrs Helen Metzinger. And, as always, Barbara made it possible.
A man may expresse and signifie the intentions of his minde, at any distance of place by objects . . . capable of a twofold difference onely.

Sir Francis Bacon (1561–1626)