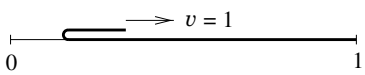
Problems

P1 Three small snails are each at a vertex of an equilateral triangle of side 60 cm. The first sets out towards the second, the second towards the third and the third towards the first, with a uniform speed of 5 cm min^{-1}. During their motion each of them always heads towards its respective target snail. How much time has elapsed, and what distance do the snails cover, before they meet? What is the equation of their paths? If the snails are considered as point-masses, how many times does each circle their ultimate meeting point?

P2 A small object is at rest on the edge of a horizontal table. It is pushed in such a way that it falls off the other side of the table, which is 1 m wide, after 2 s. Does the object have wheels?

P3 A boat can travel at a speed of 3 m s^{-1} on still water. A boatman wants to cross a river whilst covering the shortest possible distance. In what direction should he row with respect to the bank if the speed of the water is (i) 2 m s^{-1}, (ii) 4 m s^{-1}? Assume that the speed of the water is the same everywhere.

P4 A long, thin, pliable carpet is laid on the floor. One end of the carpet is bent back and then pulled backwards with constant unit velocity, just above the part of the carpet which is still at rest on the floor.

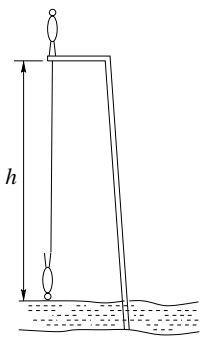


Find the speed of the centre of mass of the moving part. What is the minimum force needed to pull the moving part, if the carpet has unit length and unit mass?

P5 Four snails travel in uniform, rectilinear motion on a very large plane surface. The directions of their paths are random, (but not parallel, i.e. any two snails could meet), but no more than two snail paths can cross at any one point. Five of the $(4 \times 3)/2 = 6$ possible encounters have already occurred. Can we state with certainty that the sixth encounter will also occur?

P6 Two 20-g flatworms climb over a very thin wall, 10 cm high. One of the worms is 20 cm long, the other is wider and only 10 cm long. Which of them has done more work against gravity when half of it is over the top of the wall? What is the ratio of the amounts of work done by the two worms?

P7 A man of height $h_0 = 2$ m is bungee jumping from a platform situated a height $h = 25$ m above a lake. One end of an elastic rope is attached to his foot and the other end is fixed to the platform. He starts falling from rest in a vertical position.



The length and elastic properties of the rope are chosen so that his speed will have been reduced to zero at the instant when his head reaches the surface of the water. Ultimately the jumper is hanging from the rope, with his head 8 m above the water.

(i) Find the unstretched length of the rope.
(ii) Find the maximum speed and acceleration achieved during the jump.

P8 An iceberg is in the form of an upright regular pyramid of which 10 m shows above the water surface. Ignoring any induced motion of the water, find the period of small vertical oscillations of the berg. The density of ice is 900 kg m^{-3}.

P9 The suspension springs of all four wheels of a car are identical. By how much does the body of the car (considered rigid) rise above each of the wheels when its right front wheel is parked on an 8-cm-high pavement? Does the result change when the car is parked with both right wheels on

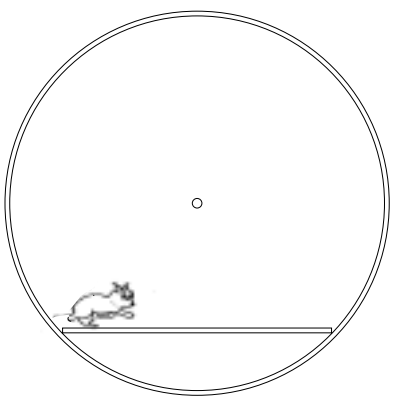
the pavement? Does the result depend on the number and positions of the people sitting in the car?

P10* In Victor Hugo's novel *les Misérables*, the main character Jean Valjean, an escaped prisoner, was noted for his ability to climb up the corner formed by the intersection of two vertical perpendicular walls. Find the minimum force with which he had to push on the walls whilst climbing. What is the minimum coefficient of static friction required for him to be able to perform such a feat?

P11 A sphere, made of two non-identical homogeneous hemispheres stuck together, is placed on a plane inclined at an angle of 30° to the horizontal. Can the sphere remain in equilibrium on the inclined plane?

P12 A small, elastic ball is dropped vertically onto a long plane inclined at an angle α to the horizontal. Is it true that the distances between consecutive bouncing points grow as in an arithmetic progression? Assume that collisions are perfectly elastic and that air resistance can be neglected.

P13 A small hamster is put into a circular wheel-cage, which has a frictionless central pivot. A horizontal platform is fixed to the wheel below the pivot. Initially, the hamster is at rest at one end of the platform.



When the platform is released the hamster starts running, but, because of the hamster's motion, the platform and wheel remain *stationary*. Determine how the hamster moves.

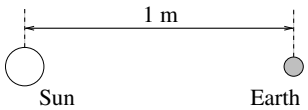
P14* A bicycle is supported so that it is prevented from falling sideways but can move forwards or backwards; its pedals are in their highest and lowest positions. A student crouches beside the bicycle and applies a horizontal force, directed towards the back wheel, to the lower pedal.

(i) Which way does the bicycle move?

(ii) Does the chain-wheel rotate in the same or opposite sense as the rear wheel?

(iii) Which way does the lower pedal move relative to the ground?

P15 If the solar system were proportionally reduced so that the average distance between the Sun and the Earth were 1 m, how long would a year be? Take the density of matter to be unchanged.



P16 If the mass of each of the members of a binary star were the same as that of the Sun, and their distance apart were equal to the Sun–Earth distance, what would be their period of revolution?

P17 (i) What is the minimum launch speed required to put a satellite into a circular orbit?

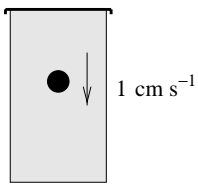
(ii) How many times higher is the energy required to launch a satellite into a polar orbit than that necessary to put it into an Equatorial one?

(iii) What initial speed must a space probe have if it is to leave the gravitational field of the Earth?

(iv) Which requires a higher initial energy for the space probe – leaving the solar system or hitting the Sun?

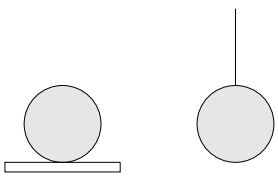
P18 A rocket is intended to leave the Earth's gravitational field. The fuel in its main engine is a little less than the amount that is necessary, and an auxiliary engine, only capable of operating for a short time, has to be used as well. When is it best to switch on the auxiliary engine: at take-off, or when the rocket has nearly stopped with respect to the Earth, or does it not matter?

P19 A steel ball with a volume of 1 cm^3 is sinking at a speed of 1 cm s^{-1} in a closed jar filled with honey. What is the momentum of the honey if its density is 2 g cm^{-3}?



P20 A gas of temperature T is enclosed in a container whose walls are (initially) at temperature T_1. Does the gas exert a higher pressure on the walls of the container when $T_1 < T$ or when $T_1 > T$?

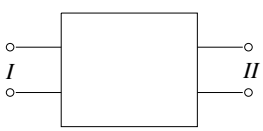
P21* Consider two identical iron spheres, one of which lies on a thermally insulating plate, whilst the other hangs from an insulating thread.



Equal amounts of heat are given to the two spheres. Which will have the higher temperature?

P22 Two (non-physics) students, A and B, living in neighbouring college rooms, decided to economise by connecting their ceiling lights in series. They agreed that each would install a 100-W bulb in their own rooms and that they would pay equal shares of the electricity bill. However, both decided to try to get better lighting at the other's expense; A installed a 200-W bulb and B installed a 50-W bulb. Which student subsequently failed the end-of-term examinations?

P23 If a battery of voltage V is connected across terminals I of the black box shown in the figure, a voltmeter connected to terminals II gives a reading of $V/2$; while if the battery is connected to terminals II, a voltmeter across terminals I reads V.

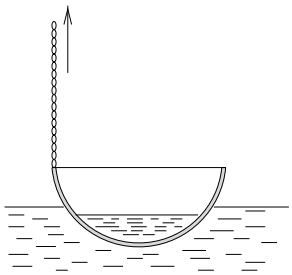


The black box contains only passive circuit elements. What are they?

P24 A bucket of water is suspended from a fixed point by a rope. The bucket is set in motion and the system swings as a pendulum. However, the bucket leaks and the water slowly flows out of the bottom of it. How does the period of the swinging motion change as the water is lost?

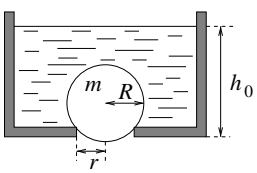
P25 An empty cylindrical beaker of mass 100 g, radius 30 mm and negligible wall thickness, has its centre of gravity 100 mm above its base. To what depth should it be filled with water so as to make it as stable as possible?

P26 Fish soup is prepared in a hemispherical copper bowl of diameter 40 cm. The bowl is placed into the water of a lake to cool down and floats with 10 cm of its depth immersed.



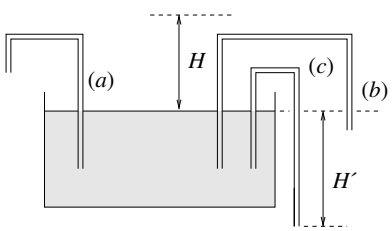
A point on the rim of the bowl is pulled upwards through 10 cm, by a chain fastened to it. Does water flow into the bowl?

P27 A circular hole of radius r at the bottom of an initially full water container is sealed by a ball of mass m and radius $R\,(>r)$. The depth of the water is now slowly reduced, and when it reaches a certain value, h_0, the ball rises out of the hole. Find h_0.



P28 Soap bubbles filled with helium float in air. Which has the greater mass – the wall of a bubble or the gas enclosed within it?

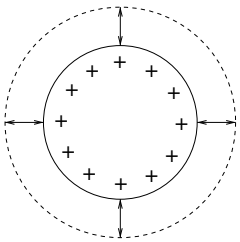
P29 Water which wets the walls of a vertical capillary tube rises to a height H within it. Three 'gallows', (a), (b) and (c), are made from the same tubing, and one end of each is placed into a large dish filled with water, as shown in the figure.



Does the water flow out at the other ends of the capillary tubes?

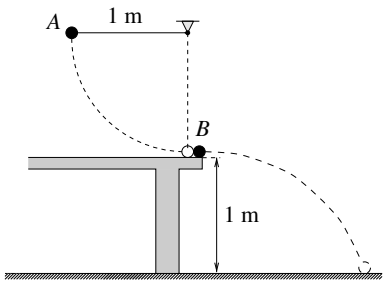
P30 A charged spherical capacitor slowly discharges as a result of the slight conductivity of the dielectric between its concentric plates. What are the magnitude and direction of the magnetic field caused by the resulting electric current?

P31 An electrically charged conducting sphere 'pulses' radially, i.e. its radius changes periodically with a fixed amplitude (*see figure*). The charges on its surface – acting as many dipole antennae – emit electromagnetic radiation. What is the net pattern of radiation from the sphere?



P32* How high would the male world-record holder jump (at an indoor competition!) on the Moon?

P33 A small steel ball B is at rest on the edge of a table of height 1 m. Another steel ball A, used as the bob of a metre-long simple pendulum, is released from rest with the pendulum suspension horizontal, and swings against B as shown in the figure. The masses of the balls are identical and the collision is elastic.



Considering the motion of B only up until the moment it first hits the ground:

(i) Which ball is in motion for the longer time?
(ii) Which ball covers the greater distance?

P34 A small bob is fixed to one end of a string of length 50 cm. As a

consequence of the appropriate forced motion of the other end of the string, the bob moves in a vertical circle of radius 50 cm with a uniform speed of 3.0 m s^{-1}. Plot, at 15° intervals on the circular path, the trajectories of both ends of the string, indicating on each the points belonging together.

P35 A point P is located above an inclined plane. It is possible to reach the plane by sliding under gravity down a straight frictionless wire, joining P to some point P' on the plane. How should P' be chosen so as to minimise the time taken?

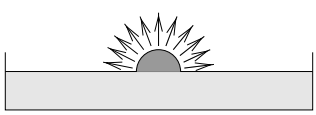
P36 The minute hand of a church clock is twice as long as the hour hand. At what time after midnight does the end of the minute hand move away from the end of the hour hand at the fastest rate?

P37 What is the maximum angle to the horizontal at which a stone can be thrown and always be moving away from the thrower?

P38* A tree-trunk of diameter 20 cm lies in a horizontal field. A lazy grasshopper wants to jump over the trunk. Find the minimum take-off speed of the grasshopper that will suffice. (Air resistance is negligible.)

P39* A straight uniform rigid hair lies on a smooth table; at each end of the hair sits a flea. Show that if the mass M of the hair is not too great relative to that m of each of the fleas, they can, by simultaneous jumps with the same speed and angle of take-off, exchange ends without colliding in mid-air.

P40 A fountain consists of a small hemispherical rose (sprayer) which lies on the surface of the water in a basin, as illustrated in the figure. The rose has many evenly distributed small holes in it, through which water spurts at the same speed in all directions.



What is the shape of the water 'bell' formed by the jets?

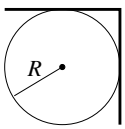
P41 A particle of mass m carries an electric charge Q and is subject to the combined action of gravity and a uniform horizontal electric field of strength E. It is projected with speed v in the vertical plane parallel to the field and at an angle θ to the horizontal. What is the maximum distance the particle can travel horizontally before it is next level with its starting point?

P42** A uniform rod of mass m and length ℓ is supported horizontally

at its ends by my two forefingers. Whilst I am slowly bringing my fingers together to meet under the centre of the rod, it slides on either one or other of them. How much work do I have to do during the process if the coefficient of static friction is μ_{stat}, and that of kinetic friction is μ_{kin} ($\mu_{\text{kin}} \leq \mu_{\text{stat}}$)?

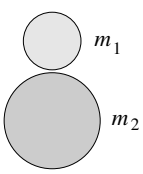
P43 Four identical bricks are placed on top of each other at the edge of a table. Is it possible to slide them horizontally across each other in such a way that the projection of the topmost one is completely outside the table? What is the theoretical limit to the displacement of the topmost brick if the number of bricks is arbitrarily increased?

P44 A plate, bent at right angles along its centre line, is placed onto a horizontal fixed cylinder of radius R as shown in the figure.



How large does the coefficient of static friction between the cylinder and the plate need to be if the plate is not to slip off the cylinder?

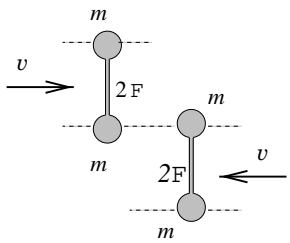
P45 Two elastic balls of masses m_1 and m_2 are placed on top of each other (with a small gap between them) and then dropped onto the ground. What is the ratio m_1/m_2, for which the upper ball ultimately receives the largest possible fraction of the total energy? What ratio of masses is necessary if the upper ball is to bounce as high as possible?



P46 An executive toy consists of three suspended steel balls of masses M, μ and m arranged in that order with their centres in a horizontal line. The ball of mass M is drawn aside in their common plane until its centre has been raised by h and is then released. If $M \neq m$ and all collisions are elastic, how must μ be chosen so that the ball of mass m rises to the greatest possible height? What is this height? (Neglect multiple collisions.)

P47 Two identical dumb-bells move towards each other on a horizontal air-cushioned table, as shown in the figure. Each can be considered as two point masses m joined by a weightless rod of length 2ℓ. Initially, they are not

rotating. Describe the motion of the dumb-bells after their elastic collision. Plot the speeds of the centres of mass of the dumb-bells as a function of time.



P48 Two small identical smooth blocks A and B are free to slide on a frozen lake. They are joined together by a light elastic rope of length $\sqrt{2}L$ which has the property that it stretches very little when the rope becomes taut. At time $t = 0$, A is at rest at $x = y = 0$ and B is at $x = L$, $y = 0$ moving in the positive y-direction with speed V. Determine the positions and velocities of A and B at times (i) $t = 2L/V$ and (ii) $t = 100L/V$.

P49* After a tap above an empty rectangular basin has been opened, the basin fills with water in a time T_1. After the tap has been closed, opening a plug-hole at the bottom of the basin empties it in a time T_2. What happens if both the tap and the plug-hole are open? What ratio of T_1/T_2 can cause the basin to overflow? As a specific case, let $T_1 = 3$ minutes and $T_2 = 2$ minutes.

P50 A cylindrical vessel of height h and radius a is two-thirds filled with liquid. It is rotated with constant angular velocity ω about its axis, which is vertical. Neglecting any surface tension effects, find an expression for the greatest angular velocity of rotation Ω for which the liquid does not spill over the edge of the vessel.

P51 Peter, who was standing by a racetrack, calculated that as one of the cars, in accelerating from rest to a speed of 100 km h^{-1}, used up x litres of fuel, it could increase its speed to 200 km h^{-1}, by using a further $3x$ litres of fuel. Peter, who has learned in physics that kinetic energy is proportional to the square of the speed, assumed that the energy content of the fuel was mainly converted into kinetic energy, i.e. he neglected air resistance and other types of friction.

A railway runs by the racetrack. Paul, who also knows some physics, saw the start of the race from the window of a train travelling at a speed of 100 km h^{-1} in the opposite direction to that of the car. He reasoned as