Generalized Riemann Problems in Computational Fluid Dynamics

Numerical simulation of compressible, inviscid, time-dependent flow is a major branch of computational fluid dynamics. Its primary goal is to obtain accurate representation of the time evolution of complex flow patterns, involving interactions of shocks, interfaces, and rarefaction waves. The generalized Riemann problem (GRP) algorithm, developed by the authors for this purpose, provides a unifying "shell" that comprises some of the most commonly used numerical schemes for such flows. This monograph gives a systematic presentation of the GRP methodology, starting from the underlying mathematical principles, through basic scheme analysis and scheme extensions (such as reacting flow or two-dimensional flows involving moving or stationary boundaries). An array of instructive examples illustrates the range of applications, extending from (simple) scalar equations to computational fluid dynamics. Background material from mathematical analysis and fluid dynamics is provided, making the book accessible to both researchers and graduate students of applied mathematics, science, and engineering.

Matania Ben-Artzi is a professor of mathematics at the Institute of Mathematics, The Hebrew University of Jerusalem. His area of interest is mathematical physics, where pure mathematical analysis, theory of partial differential equations, and numerical analysis are applied in the study of various fundamental differential equations of physics.

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Preface

Computational fluid dynamics (CFD) is a relatively young branch of fluid dynamics, the other two being the experimental and the theoretical disciplines. Its rapid development was enabled by the spectacular progress in high power computers, as well as by a matching progress in numerical schemes.

The starting point for the formulation of CFD schemes is the governing equations. In fact, the term "fluid dynamical equations" is much too general and indeed ambivalent. In practice there exist numerous models of such equations. They reflect a variety of stipulations on the nature of the flow, such as compressibility, viscosity, or elasticity. They also involve various effects such as heat conduction or chemical reactions. A large portion of these models do not fall, mathematically speaking, under the category of "hyperbolic conservation laws," which is the subject matter of this monograph. We refer the reader to the book by Landau and Lifshitz [75] for a general survey of fluid dynamical models.

In this monograph we are concerned with time-dependent, inviscid, compressible flow, which is studied primarily in the "quasi-one-dimensional" geometric setting. This leads to a system of partial differential equations expressing the conservation of mass, momentum, and energy. There are various approaches to the numerical resolution of this system, such as the classical method of characteristics or the "artificial viscosity" scheme. Our focus here is on finite-difference approximations of the type referred to as "conservation law schemes." In the quasi-one-dimensional setting they are practically equivalent to "finite-volume schemes." At present, these schemes are the commonly preferred choice, since they are robust by virtue of their shock-capturing capability, and they may be readily extended to more than one space variable.

Rather than constructing a particular scheme, we try here to develop a methodology aimed at the derivation of high-resolution, second-order schemes, all of which extend the basic (first-order) Godunov scheme. It is based on the "heart of the matter," the analysis of the generalized Riemann problem (GRP). This problem is an extension of the classical Riemann problem (RP) of fluid dynamics, which is the initial value problem in which data consist of two *constant states* separated by a discontinuity at the origin. Loosely speaking, the corresponding GRP is defined by replacing those two states by *constant-gradient* states, also having a jump at the origin.

This monograph is devoted primarily to a mathematical introduction to conservation law schemes and to the development of the basic GRP methodology. However, we have also included (Part II) a number of applications to more representative cases of "scientific computing." Although these examples are not "algorithmically heavy," they serve to illustrate the kind of extensions (geometric or physical) that are invariably required for realistic CFD simulations.

Our collaboration on the GRP methodology began some twenty years ago. Over all these years we have benefited from numerous discussions with colleagues and students who helped us shape the method, its goals, and its presentation. Their ideas led oftentimes to joint work, as is witnessed by the bibliographic list. It is with real pleasure that we acknowledge their contribution to this monograph.

The preparation of this monograph was a demanding task to which we devoted considerable time and effort over the past few years. Throughout that time we have enjoyed the insights offered to us by A. Chorin, A. Birman, J.-P. Croisille, and O. Igra; their help is gratefully acknowledged. Special credit is due to J. Li whose participation was instrumental in shaping our treatment of (2-D) scalar conservation laws. Validation experiments are an important part of CFD research, and in that, as well as in other aspects, the cooperation of K. Takayama and other colleagues of the Shock Wave Research Center, Tohoku University, is gratefully acknowledged. Our thanks and appreciation are due to our colleague U. Feldman who set up the computer system used in the calculations and typesetting for this monograph.

Finally, it is with deep gratitude that we acknowledge the (silent) participation of our wives, Ofra and Linda, in this endeavor. Book writing invariably involves hard labor; their understanding and encouragement were instrumental in seeing it through to its successful conclusion.