Topics in Finite and Discrete Mathematics

Written for a broad audience of students in mathematics, computer science, operations research, statistics, and engineering, this textbook presents a short, lively survey of several fascinating noncalculus topics in modern applied mathematics. Coverage includes probability, mathematical finance, graphs, linear programming, statistics, computer science algorithms, and groups. A key feature is the abundance of interesting examples not normally found in standard finite mathematics courses, such as options pricing and arbitrage, tournaments, and counting formulas.

The author assumes a level of mathematical sophistication at the beginning calculus level; that is, students should have had at least a course in precalculus, and the added sophistication attained from studying calculus would be useful.

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Topics in Finite and Discrete Mathematics

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To Rebecca
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Preface

This text surveys many of the topics taught in discrete and finite mathematics courses. The topics chosen are widely applied in present-day industry and, at the same time, are mathematically elegant. Chapter 1 begins with such preliminaries as sets, mathematical induction, functions, and the division algorithm of algebra. Chapters 2 and 3 present combinatorics and probability. Chapter 4 introduces the modern approach to finance; it presents the concept of arbitrage and the arbitrage theorem and then uses them to analyze the no-arbitrage costs of options. Chapters 5 and 6 deal with graphs and their many applications. Chapter 7 introduces linear programming. Among other applications, we use the duality theorem to derive the arbitrage theorem as well as the minimax theorem of game theory. Chapter 8 presents sorting and searching techniques that are useful in computer science. Chapter 9 introduces the subject matter of statistics, presenting both its descriptive and inferential side. Chapter 10 deals with groups and permutations.

This book can be used for a course in discrete mathematics, or for one in finite mathematics, or for any course dealing with non–calculus-based applied mathematics. Calculus itself is not required, and a pre-calculus course should suffice as a prerequisite; the added mathematical sophistication attained from studying calculus would be useful. The text evolved from a seminar designed to introduce first-year undergraduates with a strong quantitative bent to the possibilities inherent in mathematics. Consequently, a key feature of the course, as well as of the text, is the emphasis on interesting examples.