#### Economics and the theory of games

This textbook offers a systematic, self-contained account of the main contributions of modern game theory and its applications to economics. Starting with a detailed description of how to model strategic situations, the discussion proceeds by studying basic solution concepts, their main refinements, games played under incomplete information, and repeated games. For each of these theoretical developments, there is a companion set of applications that cover the most representative instances of game-theoretic analysis in economics, e.g., oligopolistic competition, public goods, coordination failures, bargaining, insurance markets, implementation theory, signaling, and auctions. The theory and applications covered in the first part of the book fall under the so-called classical approach to game theory, which is founded on the paradigm of players' unlimited rationality. The second part shifts toward topics that no longer abide by that paradigm. This leads to the study of important topics such as the interplay between evolution and rationality, the behavioral dynamics induced by social learning, and how players might tackle the problem of multiple equilibria.

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> PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS The Edinburgh Building, Cambridge CB2 2RU, UK 40 West 20th Street, New York, NY 10011-4211, USA 477 Williamstown Road, Port Melbourne, VIC 3207, Australia Ruiz de Alarcón 13, 28014 Madrid, Spain Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

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First published 2003

Printed in the United States of America

Typeface Times 11/13 pt. System  $\bot T_F X 2_{\mathcal{E}}$  [TB]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data available

ISBN 0 521 77251 6 hardback ISBN 0 521 77590 6 paperback

To Olalla,

who happily entered the game as I was completing the last stages of this book

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## Preface

The twofold aim of this book is to provide both a wide coverage of modern game theory and a detailed account of many of its economic applications. The book is possibly too extensive to be fully covered in a single course. However, selected parts of it could be used in a variety of alternative courses, by adapting either the focus (e.g., altering the relative emphasis on theory and applications) or the technical difficulty of the discussion (e.g., approaching the more advanced topics less formally). I have written the book with the aim of rendering these different routes to using the book reasonably easy to pursue.

The material is organized in twelve chapters. The first nine of them embody the topics that generally would be included in a standard course of game theory and economic applications. In line with my objective of providing a smooth integration of theory and applications, these nine chapters display a repeated alternation of one and the other. Thus, on the one hand, there are five theory chapters that cover in turn the basic *Theoretical Framework* (Chapter 1), *Strategic-Form Analysis* (Chapter 2), *Refinements of Nash Equilibrium* (Chapter 4), *Incomplete Information* (Chapter 6), and *Repeated Interaction* (Chapter 8). In each of these five chapters, the first part is devoted to "core topics," while the more demanding discussion is gathered next under the heading of "supplementary material." In principle, most of the core topics could be taught at an undergraduate level, whereas the supplementary material typically would be covered only in more advanced (possibly graduate) courses.

Except for Chapter 1, each of the theory chapters has a subsequent companion one centered on applications (i.e., Chapters 3, 5, 7, and 9, respectively). These companion chapters include a thorough discussion of some of the most paradigmatic economic applications that rely on the corresponding theory. They are organized into three blocks or "general themes": *Oligopoly, Mechanism Design*, and *Markets*, with each of them including five different applications (labeled I through V). The study of these applications could be conducted in at least three ways. One possibility, of course, is to discuss them in association with the companion theory. A second option is to cover these applications in separate blocks, each block then being used for a monographic course on the respective topic. Still a third approach is to gather them in terms of comparative difficulty, selecting those applications in each block that are best suited to the target level. To facilitate this route, the harder applications are singled out by adding a star (\*) to their headings, a general convention that is also used throughout this book in other respects (e.g., to mark those exercises that are somewhat more challenging than the others).

#### xii Preface

The methodological standpoint adopted in the first nine chapters of the book is the classical one in the discipline – that is, players are assumed to know the game, behave rationally, and believe that others will do so as well. In recent times, however, there has been a strong move among game theorists to consider more realistic scenarios, in which players are assumed subject to limited (typically called "bounded") rationality. The last three chapters of the book are concerned with these developments. Thus, Chapter 10 focuses on the relationship between *evolution and rationality*, Chapter 11 discusses different models of *learning in games*, and Chapter 12 deals with issues of *equilibrium selection*. Some, or all, of these chapters could be used for a specific course on the subject, but they could also serve to complement selectively some of the subjects (either concerning theory or applications) that are studied earlier in the book.

I would like to conclude this brief Preface by thanking the large number of people who have helped me in a variety of different ways to complete this book. First, I must refer to the colleagues and students at the Universidad de Alicante and the Universitat Pompeu Fabra, where I have been teaching different courses on game theory in recent years. Even though they are too numerous to list in detail, it is all too clear to me how their constructive feedback all along has helped to improve the book very substantially. I want to single out, however, the role played by four doctoral students: Dunia López Pintado, Rafael López, Miguel Angel Meléndez Jiménez, and Fernando Luis Valli. They have invested much time and effort in reading the twelve chapters of the book very thoroughly and have provided numerous helpful suggestions. I also want to thank my colleague Giovanni Ponti, who helped me, generously and cheerfully, with the simulations and graphical illustrations of the various learning dynamics studied in Chapter 11. Finally, as always, my deepest sense of gratitude belongs to my family, whose support for my work has always been so generous, even at times when it was a powerful contender for time and attention. The fact that we have nevertheless managed quite well is the essential merit of Mireia, my partner in so many other endeavors.