STELLAR ROTATION

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Observational basis

1.1 Historical development

The study of stellar rotation began at the turn of the seventeenth century, when sunspots were observed for the first time through a refracting telescope. Measurements of the westward motion of these spots across the solar disk were originally made by Johannes Fabricius, Galileo Galilei, Thomas Harriot, and Christopher Scheiner. The first public announcement of an observation came from Fabricius (1587–c. 1617), a 24-year old native of East Friesland, Germany. His pamphlet, *De maculis in Sole observatis et apparente earum cum Sole conversione*, bore the date of dedication June 13, 1611 and appeared in the *Narratio* in the fall of that year. Fabricius perceived that the changes in the motions of the spots across the solar disk might be the result of foreshortening, with the spots being situated on the surface of the rotating Sun. Unfortunately, from fear of adverse criticism, Fabricius expressed himself very timidly. His views opposed those of Scheiner, who suggested that the sunspots might be small planets revolving around an immaculate, nonrotating Sun. Galileo made public his own observations in *Istoria e Dimostrazioni intorno alle Macchie Solari e loro Accidenti*. In these three letters, written in 1612 and published in the following year, he presented a powerful case that sunspots must be dark markings on the surface of a rotating Sun. Foreshortening, he argued, caused these spots to appear to broaden and accelerate as they moved from the eastern side toward the disk center. The same effect made the sunspots seem to get thinner and slower as they moved toward the western side of the disk. Galileo also noticed that all spots moved across the solar disk at the same rate, making a crossing in about fourteen days, and that they all followed parallel paths. Obviously, these features would be highly improbable given the planetary hypothesis, which is also incompatible with the observed changes in the size and shape of sunspots.

The planetary hypothesis, championed by Scheiner among others, was thus convincingly refuted by Galileo. Eventually, Scheiner's own observations led him to realize that the Sun rotates with an apparent period of about 27 days. To him also belongs the credit of determining with considerably more accuracy than Galileo the position of the Sun's equatorial plane and the duration of its rotation. In particular, he showed that different sunspots gave different periods of rotation and, furthermore, that the spots farther from the Sun's equator moved with a slower velocity. Scheiner published his collected observations in 1630 in a volume entitled *Rosa Ursina sive Sol*, dedicated to the Duke of Orsini, who sponsored the work. (The title of the book derives from the badge of the Orsini family, which was a rose and a bear.) This was truly the first monograph on solar physics.

It is not until 1667 that any further significant discussion of stellar rotation was made. In that year the French astronomer Ismaël Boulliaud (1605–1694) suggested that the variability in light of some stars (such as Mira Ceti) might be a direct consequence of axial rotation, with the rotating star showing alternately its bright (unspotted) and dark (spotted) hemispheres to the observer. This idea was popularized in Fontenelle's *Entretiens sur la pluralité des mondes* – a highly successful introduction to astronomy that went through many revised editions during the period 1686–1742. To be specific, he noted "... that these fixed stars which have disappeared aren't extinguished, that these are really only half-suns. In other words they have one half dark and the other lighted, and since they turn on themselves, they sometimes show us the luminous half and then we see them sometimes half dark, and then we don't see them at all." * Although this explanation for the variable stars did not withstand the passage of time, it is nevertheless worth mentioning because it shows the interest that stellar rotation has aroused since its inception. As a matter of fact, nearly three centuries were to elapse before Boulliaud's original idea was fully recognized as a useful method of measuring the axial rotation of certain classes of stars, that is, stars that exhibit a detectable rotational modulation of their light output due to starspots or stellar plages.

For more than two centuries the problem of solar rotation was practically ignored, and it is not until the 1850s that any significant advance was made. Then, a long series of observations of the apparent motion of sunspots was undertaken by Richard Carrington and Gustav Spörer. They confirmed, independently, that the outer visible envelope of the Sun does not rotate like a solid body; rather, its period of rotation varies as a function of heliocentric latitude. From his own observations made during the period 1853–1861, Carrington derived the following expression for the Sun's rotation rate:

$$\Omega(\text{deg/day}) = 14^\circ 42 - 2^\circ 75 \sin^{7/4} \phi, \tag{1.1}$$

where ϕ is the heliocentric latitude. Somewhat later, Hervé Faye found that the formula

$$\Omega(\text{deg/day}) = 14^\circ 37 - 3^\circ 10 \sin^2 \phi \tag{1.2}$$

more satisfactorily represented the dependence of angular velocity on heliocentric latitude. Parenthetically, note that Carrington also found evidence for a mean meridional motion of sunspots. Convincing evidence was not found until 1942, however, when Jaakko Tuominen positively established the existence of an equatorward migration of sunspots at heliocentric latitudes lower that about 20° and a poleward migration at higher latitudes.

The spectroscope was the instrument that marked the beginning of the modern era of stellar studies. As early as 1871 Hermann Vogel showed that the Sun's rotation rate can be detected from the relative Doppler shift of the spectral lines at opposite edges of the solar disk, one of which is approaching and the other receding. Extensive measurements were made visually by Nils Dunér and Jakob Halm during the period 1887–1906. They showed a rotation rate and equatorial acceleration that were quite similar to those obtained from the apparent motion of sunspots. They concluded that Faye's empirical law

* Bernard le Bovier de Fontenelle, *Conversations on the Plurality of Worlds*, translation of the 1686 edition by H. A. Hargreaves, p. 70, Berkeley: University of California Press, 1990.

adequately represented the spectroscopic observations also, but their coverage of latitude was double that of the sunspot measurements. The first spectrographic determinations of solar rotation were undertaken at the turn of the twentieth century by Walter S. Adams at Mount Wilson Solar Observatory, California.

William de Wiveleslie Abney was the first scientist to express the idea that the axial rotation of single stars could be determined from measurements of the widths of spectral lines. In 1877, he suggested that the effect of a star's rotation on its spectrum would be to broaden all of the lines and that ". . . other conditions being known, the mean velocity of rotation might be calculated."* In 1893, while doubts were still being expressed with regard to measurable rotational motions in single stars, J. R. Holt suggested that axial rotation might be detected from small distortions in the radial velocity curve of an eclipsing binary. Thus, he argued,

> . . . in the case of variable stars, like Algol, where the diminution of light is supposed to be due to the interposition of a dark companion, it seems to me that there ought to be a spectroscopic difference between the light at the commencement of the minimum phase, and that of the end, inasmuch as different portions of the edge would be obscured. In fact, during the progress of the partial eclipse, there should be a shift in position of the lines; and although this shift is probably very small, it ought to be detected by a powerful instrument.†

Confirmation of this effect was obtained by Frank Schlesinger in 1909, who presented convincing evidence of axial rotation in the brightest star of the system δ Librae. However, twenty more years were to elapse before Abney's original idea resulted in actual measurements of projected equatorial velocities in single stars. This notable achievement was due to the efforts of Otto Struve and his collaborators during the period 1929–1934 at Yerkes Observatory, Wisconsin.

A graphical method was originally developed by Grigori Shajn and Otto Struve. The measurements were made by fitting the observed contour of a spectral line to a computed contour obtained by applying different amounts of Doppler broadening to an intrinsically narrow line-contour having the same equivalent width as the observed line. Comparison with an observed line profile gave the projected equatorial velocity $v \sin i$ along the line of sight. These early measurements indicated that the values of $v \sin i$ fell into the range 0–250 km s^{-1} and may occasionally be as large as 400 km s^{-1} or even more. As early as 1930 it was found that the most obvious correlation between $v \sin i$ and other physical parameters is with spectral type, with rapid rotation being peculiar to the earliest spectral classes. This was originally recognized by Struve and later confirmed by statistical studies of line widths in early-type stars by Christian T. Elvey and Christine Westgate. The O-, B-, A-, and early F-type stars frequently have large rotational velocities, while in late F-type and later types rapid rotation occurs only in close spectroscopic binaries. A study of rotational line broadening in early-type close binaries was also made by Egbert Adriaan Kreiken. From his work it is apparent that the components of these binaries have their rotational velocities significantly diminished with respect to single, main-sequence stars of the same spectral type. The following year, 1936, Pol Swings

* *Mon. Not. R. Astron. Soc.*, **37** (1877), p. 278.

† *Astronomy and Astro-Physics*, **12** (1893), p. 646.

properly established that in close binaries of short periods axial rotation tends to be either perfectly or approximately synchronized with the orbital motion.

At this juncture the problem was quietly abandoned for almost fifteen years. Interest in the measurements of axial rotation in stars was revived in 1949 by Arne Slettebak. Extensive measurements of rotational velocities were made during the 1950s and 1960s by Helmut A. Abt, Robert P. Kraft, Slettebak, and others. However, because the only observational technique available was to determine line widths in stars from photographic spectra, these studies were limited almost entirely to stars more massive than the sun ($M \gtrsim 1.5 M_\odot$) and to main-sequence or post–main-sequence stars. Since appreciable rotation disappears in the middle F-type stars, higher-resolution spectra are therefore required to measure rotational broadening in the late-type stars. In 1967, Kraft pushed the photographic technique to its limit to measure $v \sin i$ as low as 6 km s^{-1} in solar-type stars. Now, as early as 1933, John A. Carroll had suggested the application of Fourier analysis to spectral line profiles for rotational velocity determinations. In 1973, the problem was reconsidered by David F. Gray, who showed that high-resolution data make it possible to distinguish between the Fourier transform profile arising from rotation versus those arising from other broadening mechanisms. Since the late 1970s systematic studies of very slow rotators have been made by Gray, Myron A. Smith, David R. Soderblom, and others. Current techniques limit the measurement accuracy of projected rotational velocities to 2 km s^{-1} in most stars.

Periodic variations in the light output due to dark or bright areas on some rotating stars have also been used to determine the rotation periods of these stars. Although the principle of rotational modulation was suggested as early as 1667 by Ismaël Boulliaud, convincing detection of this effect was not made until 1947, when Gerald E. Kron found evidence in the light curve of the eclipsing binary AR Lacertae for surface inhomogeneities in its G5 component. The principle was therefore well established when in 1949 Horace W. Babcock proposed the so-called oblique-rotator model for the magnetic and spectrum variations of the periodic Ap stars. Kron's result was forgotten till 1966, when interest in the principle of rotational modulation was independently revived by Pavel Chugainov. A large body of literature has developed since the late 1960s. This work generally divides according to the method used to estimate the rotation periods, with the two types being (i) photometric monitoring of light variations produced by large starspot groups or bright surface areas and (ii) measurements of the periodic variation in strength of some emission lines that are enhanced in localized active regions in the chromosphere. These techniques have the advantage that a rotation period can be determined to much higher precision than $v \sin i$ and are free of the $\sin i$ projection factor inherent to the spectrographic method. Moreover, very accurate rotation periods can be derived even for quite slowly rotating stars at rates that would be impossible to see as a Doppler broadening of their spectral lines.

A different line of inquiry was initiated by the discovery of the so-called five-minute oscillations in the solar photosphere. The first evidence for ubiquitous oscillatory motions was obtained in the early 1960s by Robert B. Leighton, Robert W. Noyes, and George W. Simon. However, it is not until 1968 that Edward N. Frazier suggested that ". . . the well known 5 min oscillations are primarily standing resonant acoustic waves."*

* *Zeit. Astrophys.*, **68** (1968), p. 345.

Two years later, Roger K. Ulrich presented a detailed theoretical description of the phenomenon, showing that standing acoustic waves may be trapped in a layer beneath the solar photosphere. This model was independently proposed in 1971 by John W. Leibacher and Robert F. Stein. In 1975, Franz-Ludwig Deubner obtained the first observational evidence for these trapped acoustic modes. Soon afterward, it was realized that a detailed analysis of the frequencies of these many oscillatory modes could provide a probe of the solar *internal* rotation. Indeed, because axial rotation breaks the Sun's spherical symmetry, it splits the degeneracy of the nonradial modes with respect to the azimuthal angular dependence. A technique for measuring the solar internal rotation from these frequency splittings was originally devised by Edward J. Rhodes, Jr., Deubner, and Ulrich in 1979. Since 1984, following the initial work of Thomas L. Duvall, John W. Harvey, and others,* diverse methods have been used to determine the Sun's internal angular velocity.

1.2 The Sun

In Section 1.1 we briefly discussed the early measurements of the axial rotation of the Sun. With the advent of more sensitive instruments, however, Doppler and tracer measurements have shown that the solar atmosphere exhibits motions on widely different scales. Besides the large-scale axisymmetric motions corresponding to differential rotation and meridional circulation, velocity fields associated with turbulent convection and also with oscillatory motions at about a five-minute period have been observed. Considerable attention has focused on analysis of these oscillations since, for the very first time, they make it possible to probe the Sun's *internal* rotation.

1.2.1 Large-scale motions in the atmosphere

The solar surface rotation rate may be obtained from measurements of the longitudinal motions of semipermanent features across the solar disk (such as sunspots, faculae, magnetic field patterns, dark filaments, or even coronal activity centers), or from spectrographic observations of Doppler displacements of selected spectral lines near the solar limb. Each of the two methods for deriving surface rotation rates has its own limitations, although few of these limitations are common to both. Actually, the determination of solar rotation from tracers requires that these semipermanent features be both randomly distributed throughout the fluid and undergo no appreciable proper motion with respect to the medium in which they are embedded. In practice, no tracers have been shown to possess both characteristics; moreover, most of them tend to occur in a limited range of heliocentric latitudes. By the spectrographic method, rotation rates can be found over a wider range of latitudes. But then, the accuracy is limited by the presence of inhomogeneities of the photospheric velocity field and by macroscopic motions within coronal and chromospheric features, so that the scatter between repeated measurements is large.

Figure 1.1 assembles sidereal rotation rates obtained from photospheric Doppler and tracer measurements. The observations refer to the sunspots and sunspot groups, magnetic field patterns, and Doppler shifts. In all cases the relationships shown in Figure 1.1 are

* *Nature*, **310** (1984), pp. 19 and 22.

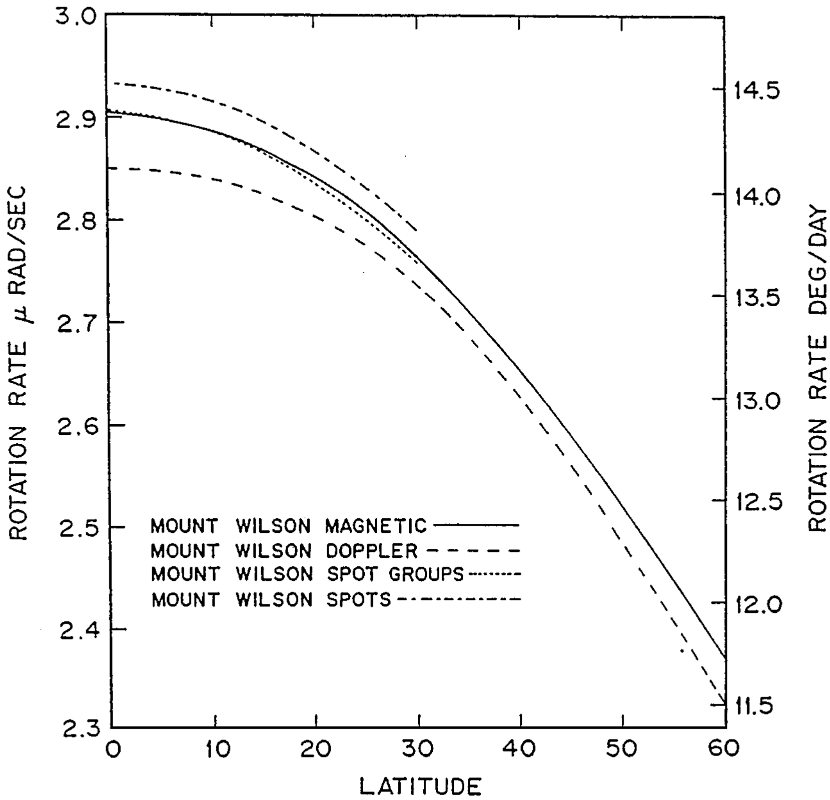

Fig. 1.1. Comparison of the solar differential rotation obtained by different methods. *Source:* Howard, R., *Annu. Rev. Astron. Astrophys.*, **22**, 131, 1984. (By permission. Copyright 1984 by Annual Reviews.)

smoothed curves obtained by fitting the data to expansions in the form

$$\Omega = A + B \sin^2 \phi + C \sin^4 \phi. \tag{1.3}$$

The decrease of angular velocity with increasing heliocentric latitude is clear. However, it is also apparent that different techniques for measuring the solar surface rotation rate yield significantly different results. In particular, the sunspot groups rotate more slowly in their latitudes than individual sunspots. Note also that the rotation rate for the magnetic tracers is intermediate between that for the individual spots and that for the photospheric plasma. It is not yet clear whether these different rotation rates represent real differences of rotation at various depths in the solar atmosphere or whether they reflect a characteristic behavior of the tracers themselves.

Chromospheric and coronal rotation measurements have also been reported in the literature. It seems clear from these results that the latitudinal gradient of angular velocity depends very much on the size and lifetime of the tracers located above the photosphere. To be specific, the long-lived structures exhibit smaller gradients than the short-lived ones, and the very long-lived coronal holes rotate almost uniformly. These noticeable differences remain poorly understood.

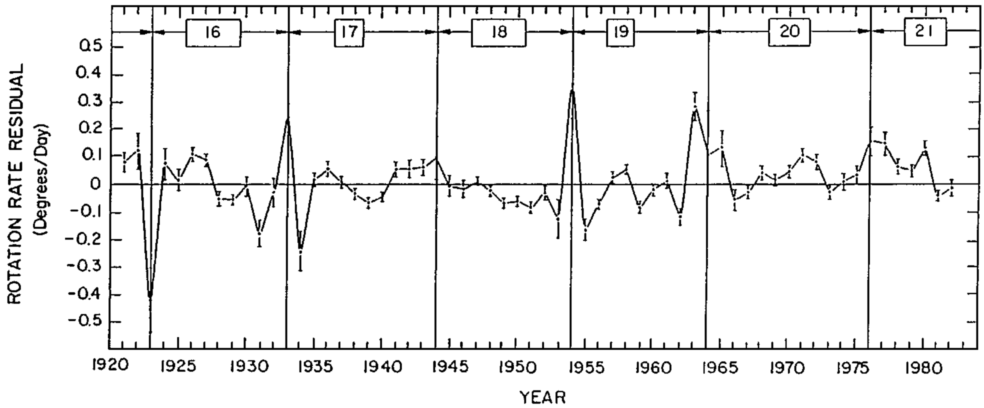

Fig. 1.2. Residuals of annual average sunspot rotation rates for the period 1921–1982. Solar cycle maxima timing and length are denoted by numbered boxes. Vertical lines denote year of sunspot minimum. *Source:* Gilman, P. A., and Howard, R., *Astrophys. J.*, **283**, 385, 1984.

Figure 1.1 merely illustrates the *mean* properties of the solar surface differential rotation. As was originally shown by Howard and LaBonte (1980), however, analysis of the residual motions in the daily Doppler measurements made at Mount Wilson suggests the presence of a *torsional oscillation* of very small amplitude in the photosphere. This oscillation is an apparently organized pattern of zonally averaged variations from a mean curve for the differential rotation, as defined in Eq. (1.3). The amplitude of the residuals constituting the torsional oscillation is of the order of 5 m s^{-1}. It is a traveling wave, with latitude zones of fast and slow rotation, that originates near the poles and moves equatorward over the course of a 22-year cycle. The latitude drift speed of the shear is of the order of 2 m s^{-1}. In the lower heliocentric latitudes, the torsional shear zone between the fast stream on the equator side and the slow stream on the pole side is the locus of solar activity. This coincidence strongly suggests that this torsional oscillation is somewhat related to the solar activity cycle.

Variations of the solar surface rotation rate over individual sunspot cycles have been reported by many investigators. Detailed analyses of the Mount Wilson sunspot data for the period 1921–1982 suggest that *on average* the Sun rotates more rapidly at sunspot minimum.* A similar frequency of rotation maxima is also seen in the Greenwich sunspot data for the years 1874–1976. The variability of the mean rotation rate is illustrated in Figure 1.2, which exhibits peaks of about 0.1 degree day^{-1} in the residuals near minima of solar activity. The Mount Wilson data also show variations from cycle to cycle, with the most rapid rotation found during cycles with fewer sunspots and less sunspot area.

* A similar result was obtained by Eddy, Gilman, and Trotter (1977) from their careful analysis of drawings of the Sun made by Christopher Scheiner (during 1625–1626) and Johannes Hevelius (during 1642–1644). During the earlier period, which occurred 20 years before the start of the Maunder sunspot minimum (1645–1715), solar rotation was very much like that of today. By contrast, in the later period, the equatorial velocity of the Sun was faster by 3 to 5% and the differential rotation was enhanced by a factor of 3. These results strongly suggest that the change in rotation of the solar surface between 1625 and 1645 was associated, as cause or effect, with the Maunder minimum anomaly.

Very recently, Yoshimura and Kambry (1993) have found evidence for a *long-term* periodic modulation of the solar differential rotation, with a time scale of the order 100 years. This modulation was observed in the sunspot data obtained by combining Greenwich data covering the period 1874–1976 and Mitaka data covering the period 1943–1992. Their analysis suggests that there exists a well-defined periodic variation in the overall rotation rate of the photospheric layers. To be specific, it is found that the surface rotation rate reaches a maximum at solar cycle 14, decreases to a minimum at cycle 17, and increases again to reach a maximum at cycle 21. Moreover, the time profile of the long-term modulation of the solar rotation is quite similar to the time profile of the solar-cycle amplitude modulation, but the two profiles are displaced by about 23 years in time. Further study is needed to ascertain whether this long-term modulation is strictly periodic or part of a long-term aperiodic undulation.

Several observational efforts have been made to detect a *mean* north–south motion on the Sun's surface. Unfortunately, whereas the latitudinal and temporal variations of the solar rotation are reasonably well established, the general features of the meridional flow are still poorly understood. Three different techniques have been used to measure these very slow motions: (i) the Doppler shift of selected spectral lines, (ii) the displacement of magnetic features on the solar disk, and (iii) the tracing of sunspots or plages. A majority of Doppler observations suggests a poleward motion of the order of 10 m s^{-1}, whereas others differ in magnitude and even in direction. Doppler data obtained with the Global Oscillation Network Group (GONG) instruments in Tucson from 1992 to 1995 indicate a poleward motion of the order of 20 m s^{-1}, but the results also suggest that the Sun may undergo episodes in which the meridional speeds increase dramatically. The analysis of magnetic features shows the existence of a meridional flow that is poleward in each hemisphere and is of the order of 10 m s^{-1}, which agrees with most of the Doppler measurements. On the contrary, sunspots or plages do not show a simple poleward meridional flow but a motion either toward or away from the mean latitude of solar activity, with a speed of a few meters per second. Analysis of sunspot positions generally shows equatorward motions at low heliocentric latitudes and poleward motions at high latitudes. Several authors have suggested that these discrepancies might be ascribed to the fact that different features are anchored at different depths in the solar convection zone. Accordingly, the meridional flow deep into this zone might be reflected by the sunspot motions, whereas the meridional flow in the upper part of this zone might be reflected by the other measurements. As we shall see in Section 5.2, these speculations have a direct bearing on the theoretical models of solar differential rotation.

1.2.2 Helioseismology: The internal rotation rate

The Sun is a very small amplitude variable star. Its oscillations are arising from a huge number of discrete modes with periods ranging from a few minutes to several hours. The so-called five-minute oscillations, which have frequencies between about 2 mHz and 4 mHz, have been extensively studied. They correspond to standing *acoustic* waves that are trapped beneath the solar surface, with each mode traveling within a well-defined shell in the solar interior. Since the properties of these modes are determined by the stratification of the Sun, accurate measurements of their frequencies thus provide a new window in the hitherto invisible solar interior.

To a first approximation, the Sun may be considered to be a spherically symmetric body. In that case, by making use of spherical polar coordinates (r, θ, φ), we can write the components of the Lagrangian displacement for each acoustic mode in the separable form

$$\boldsymbol{\xi} = \left(\xi_r P_l^m \,,\, \xi_h \frac{dP_l^m}{d\theta} \,,\, \xi_h \frac{P_l^m}{\cos\theta} \frac{\partial}{\partial\varphi} \right) \cos(m\varphi - \omega_{n,l} t), \tag{1.4}$$

where $P_l^m(\cos\theta)$ is the associated Legendre function of degree l and order m $(-l \le m \le +l)$. The eigenfunctions $\xi_r(r; n, l)$ and $\xi_h(r; n, l)$ define the radial and horizontal displacements of the mode. Both functions depend on the integer n, which is related to the number of zeros of the function ξ_r along the radius, and the integer l, which is the number of nodal lines on the solar surface. Because a spherical configuration has no preferred axis of symmetry, these eigenfunctions are independent of the azimuthal order m, so that to each value of the eigenfrequency $\omega_{n,l}$ correspond $2l + 1$ displacements. Rotation splits this degeneracy with respect to the azimuthal order m of the eigenfrequencies. Hence, we have

$$\omega_{n,l,m} = \omega_{n,l} + \Delta\omega_{n,l,m}. \tag{1.5}$$

Since the magnitude of the angular velocity Ω is much less than the acoustic frequencies $\omega_{n,l}$, perturbation theory can be applied to calculate these frequency splittings. One can show that

$$\Delta\omega_{n,l,m} = m \int_0^R \int_0^\pi K_{n,l,m}(r, \theta)\, \Omega(r, \theta)\, r\, dr\, d\theta, \tag{1.6}$$

where the rotational kernels $K_{n,l,m}(r, \theta)$ are functions that may be derived from a *non-rotating* solar model for which one has calculated the eigenfrequencies $\omega_{n,l}$ and their corresponding eigenfunctions. Given measurements of the rotational splittings $\Delta\omega_{n,l,m}$, it is therefore possible, in principle, to solve this integral equation for the angular velocity.

Measurement of the rotational splitting $\Delta\omega_{n,l,m}$ provides a measure of rotation *in a certain region* of the Sun. In fact, the acoustic modes of progressively lower l penetrate deeper into the Sun, so that the information on the angular velocity in the deeper layers is confined to splittings of low-l modes. Similarly, because only when an acoustic mode is quasi-zonal can it reach the polar regions, the information on the angular velocity at high heliocentric latitudes is confined to splittings of low-m modes. Since the measured splittings for the low-l and low-m modes have comparatively larger relative errors, determination of the function $\Omega(r, \theta)$ thus becomes increasingly difficult with increasing depth and increasing latitude.

Several groups of workers have observed the splittings of acoustic frequencies that arise from the Sun's differential rotation. Figures 1.3 and 1.4 illustrate the inverted solution of Eq. (1.6) based on frequency splitting determinations from the latest GONG data (1996). Note that the equatorial rotation rate presents a steep increase with radius near $r = 0.7R_\odot$, thus suggesting the possibility of a discontinuity near the base of the convection zone. Note also that the equatorial rotation rate peaks near $r = 0.95R_\odot$, before decreasing with radius in the outermost surface layers. Figure 1.4 illustrates the latitudinal dependence of the inverted profile. In the outer convection zone, for latitude $\phi < 30^\circ$, the rotation rate is nearly constant on cylinders, owing to a rapidly rotating

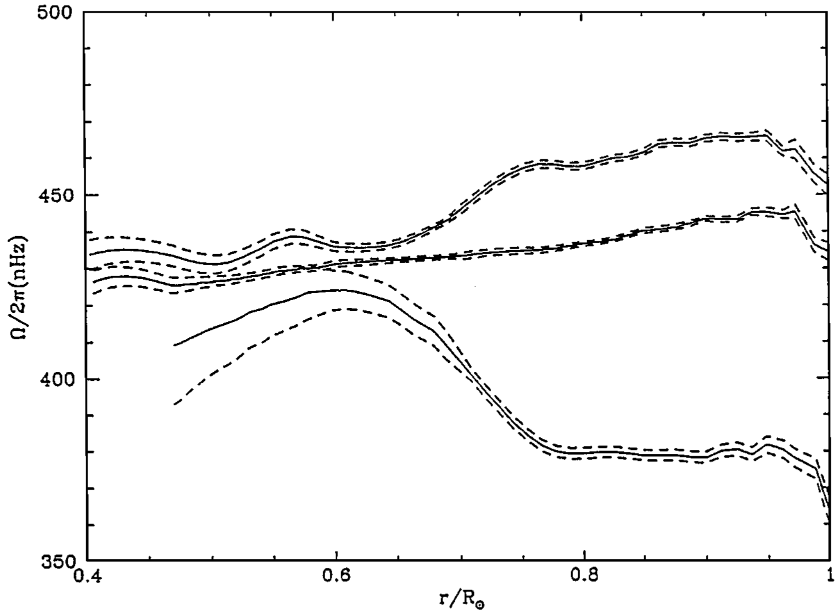

Fig. 1.3. Solar rotation rate inferred from the latest GONG data (1996). The curves are plotted as a function of radius at the latitudes of 0° (top), 30° (middle), and 60° (bottom). The dashed curves indicate error levels. *Source:* Sekii, T., in *Sounding Solar and Stellar Interiors* (Provost, J., and Schmider, F. X., eds.), I.A.U. Symposium No 181, p. 189, Dordrecht: Kluwer, 1997. (By permission. Copyright 1997 by Kluwer Academic Publishers.)

belt centered near $r = 0.95R_\odot$. At higher latitudes, however, the rotation rate becomes constant on cones. The differential character of the rotation disappears below a depth that corresponds to the base of the convection zone. This solution agrees qualitatively with the inverted profiles obtained by other groups. Perhaps the most interesting result of these inversions is that they show no sign of a tendency for rotation to occur at constant angular velocity on cylinders throughout the outer convection zone.

In summary, several inversion studies indicate that the rotation rate in the solar convection zone is similar to that at the surface, with the polar regions rotating more slowly than the equatorial belt. Near the base of the convection zone, one finds that there exists an abrupt unresolved transition to essentially uniform rotation at a rate corresponding to some average of the rate in the convection zone. This shear layer, which is known as the *solar tachocline,* is centered near $r = 0.7R_\odot$; recent studies indicate that it is quite thin, probably no more than $0.06R_\odot$. The actual rotation rate in the radiative core remains quite uncertain, however, because of a lack of accurately measured splittings for low-l acoustic modes. Several investigators have found that from the base of the convection zone down to $r \approx 0.1$–$0.2R_\odot$ their measurements are consistent with uniform rotation at a rate somewhat lower than the surface equatorial rate. Not unexpectedly, the rotation rate inside that radius is even more uncertain. Some studies suggest that the rotation rate of this inner core might be between 2 and 4 times larger than that at the surface. According to other investigators, however, it is more likely that this inner core rotates with approximately the same period as the outer parts of the radiative core. I shall not go into the disputes.

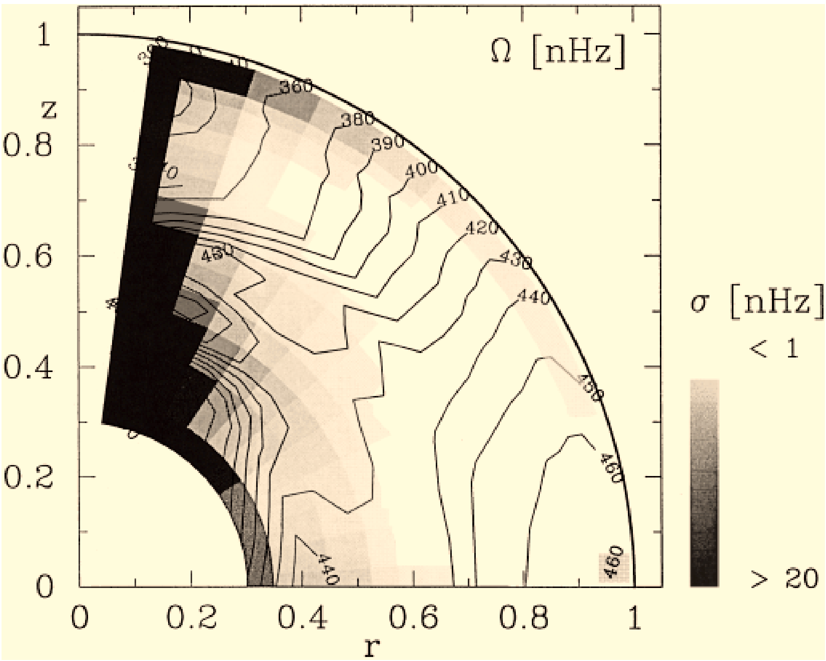

Fig. 1.4. Solar rotation rate as a function of normalized radius and latitude. Contours of isorotation are shown, superimposed on a gray-scale plot of the formal errors. A very dark background means a less reliable determination. *Source:* Korzennik, S. G., Thompson, M. J., Toomre, J., and the GONG Internal Rotation Team, in *Sounding Solar and Stellar Interiors* (Provost, J., and Schmider, F. X., eds.), I.A.U. Symposium No 181, p. 211, Dordrecht: Kluwer, 1997. (Courtesy of Dr. F. Pijpers. By permission; copyright 1997 by Kluwer Academic Publishers.)

1.3 Single stars

As was noted in Section 1.1, two basic methods have been used to measure rotational velocities of single stars. One of them consists of extracting rotational broadening from a spectral line profile, from which one infers the *projected* equatorial velocity $v \sin i$ along the line of sight. The other one consists of determining the modulation frequency of a star's light due to the rotation of surface inhomogeneities (such as spots or plages) across its surface. If observable, this modulation frequency is a direct estimate of the star's rotation period $P_{\rm rot}$, which is free of projection effects. Hence, given a radius R for the star, this period can be transformed into a *true* equatorial velocity v $(= \Omega R = 2\pi R/P_{\rm rot})$.

The spectrographic method has proven useful in determining the projected velocities for stars of spectral type O, B, A, and F. In fact, $v \sin i$ measurements can only be used in a statistical way because the inclination angle i is generally unknown. Evidence for random orientation of rotation axes is found in the lack of correlation between the measured values of $v \sin i$ and the galactic coordinates of the stars. For randomly oriented rotation axes, one can thus convert the average projected equatorial velocity $\langle v \sin i\rangle$ for a group of stars to an average equatorial velocity $\langle v\rangle$, taking into account that the average value $\langle \sin i\rangle$ is equal to $\pi/4$. Numerous statistical studies have been made over the period 1930–1970. The main results pertaining to stellar rotation have been assembled

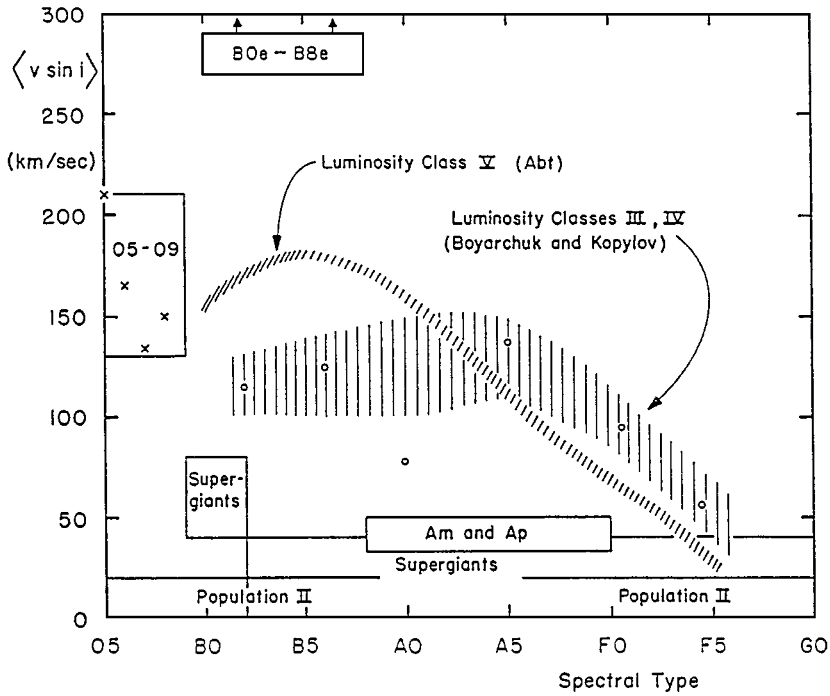

Fig. 1.5. Mean projected equatorial velocities for a number of different classes of stars as compared with normal main-sequence stars. *Source:* Slettebak, A., in *Stellar Rotation* (Slettebak, A., ed.), p. 5, New York: Gordon and Breach, 1970. (By permission. Copyright 1970 by Gordon and Breach Publishers.)

by Slettebak and are summarized in Figure 1.5. In this figure the mean observed rotational velocities for single, normal, main-sequence stars are compared with the mean observed $v \sin i$s for giant and supergiant stars, Be stars, peculiar A-type and metallic-line stars, and Population II objects.

The distribution of rotational velocities along the main sequence is quite remarkable: *Rotation increases from very low values in the F-type stars to some maximum in the B-type stars*. However, a different picture emerges when one considers the mean rotation periods rather than the mean equatorial velocities. This is illustrated in Table 1.1 which lists typical values of the masses, radii, equatorial velocities, angular velocities, and rotation periods. Note that the periods reach a minimum value of about 0.56 day near spectral type A5, and they increase rather steeply on both sides so that the G0- and O5-type stars have approximately the same rotation period. The large observed values $\langle v \rangle$ for the upper main-sequence stars are thus entirely due to the large radii of these stars.

The open circles in Figure 1.5 represent mean rotational velocities for stars belonging to the luminosity classes III and IV; they are connected by a broad cross-hatched band, thus suggesting uncertainties in the mean rotational velocities for the giant stars. According to Slettebak, the very low point at spectral type A0 can probably be interpreted in terms of selection effects. In any case, the broad band indicates that the early-type giants rotate more slowly than the main-sequence stars of corresponding spectral types, whereas for the late A- and F-types the giants rotate more rapidly than their main-sequence counterparts.

Table 1.1. *Average rotational velocities of main-sequence stars.*

Spectrum (class V)	M ($M_\odot$)	R ($R_\odot$)	v (km s^{-1})	Ω (10^{-5} s^{-1})	$P_{\rm rot}$ (days)
O5	39.5	17.2	190	1.5	4.85
B0	17.0	7.6	200	3.8	1.91
B5	7.0	4.0	210	7.6	0.96
A0	3.6	2.6	190	10.0	0.73
A5	2.2	1.7	160	13.0	0.56
F0	1.75	1.3	95	10.0	0.73
F5	1.4	1.2	25	3.0	2.42
G0	1.05	1.04	12	1.6	4.55

Source: McNally, D., *The Observatory,* **85**, 166, 1965.

This behavior can be interpreted as an evolutionary effect. As we know, the rapidly rotating B- and A-type main-sequence stars evolve to luminosity classes III and IV in later spectral types. But then, the drop in rotation as the star's radius increases is compensated by the steeper drop in rotation along the main sequence, so that the evolving star still has a larger equatorial velocity than its main-sequence counterpart. As we shall see in Section 6.5, the drop in rotation for the giants takes place between spectral types G0 III and G3 III; the drop for subgiants occurs a little earlier, at spectral types F6 IV to F8 IV.

Supergiants and Population II stars are shown schematically near the bottom of Figure 1.5. The supergiants of all spectral types do not show conspicuous rotations. They show no sudden decrease in rotation either, although rotational velocities up to 90 km s^{-1} are observed for spectral types earlier than F9. The apparent rotation velocities of Population II stars are also small, with $v \sin i$ values smaller than 30 km s^{-1}. Note also that the mean rotational velocities of the peculiar A-type stars and metallic-line stars are considerably smaller than the means for normal stars of corresponding spectral types. Finally, going to the other extreme, we note that the Be stars rotate most rapidly, and individual rotational velocities of 500 km s^{-1} have been observed by Slettebak. These stars are shown separately on Figure 1.5, with arrows indicating that their mean rotational velocities are in reality larger than shown. (As we shall see in Sections 6.3.2 and 6.3.4, however, there are no early-type stars with rotation rates anywhere near the critical rate at which centrifugal force balances gravity at the equator.) As a rule, the white dwarfs rotate rather slowly, with typical $v \sin i$ values of order 20 km s^{-1}, and none of them rotates faster than 60 km s^{-1}.

To put the relation between stellar age and axial rotation on a firm quantitative basis, several authors have obtained projected equatorial velocities for stars belonging to open clusters and associations. Detailed statistical analyses have been made by Bernacca and Perinotto (1974) and Fukuda (1982). In Figure 1.6, which is derived from data presented by Fukuda, we compare the average rotational velocity loci for field and cluster stars. As was done in Figure 1.5, the data have been grouped to smooth out irregularities in the distributions of $\langle v \sin i \rangle$ along the main sequence (see also Section 6.3). Figure 1.6 shows that field and cluster stars of spectral type O, B, and A have mean projected rotational

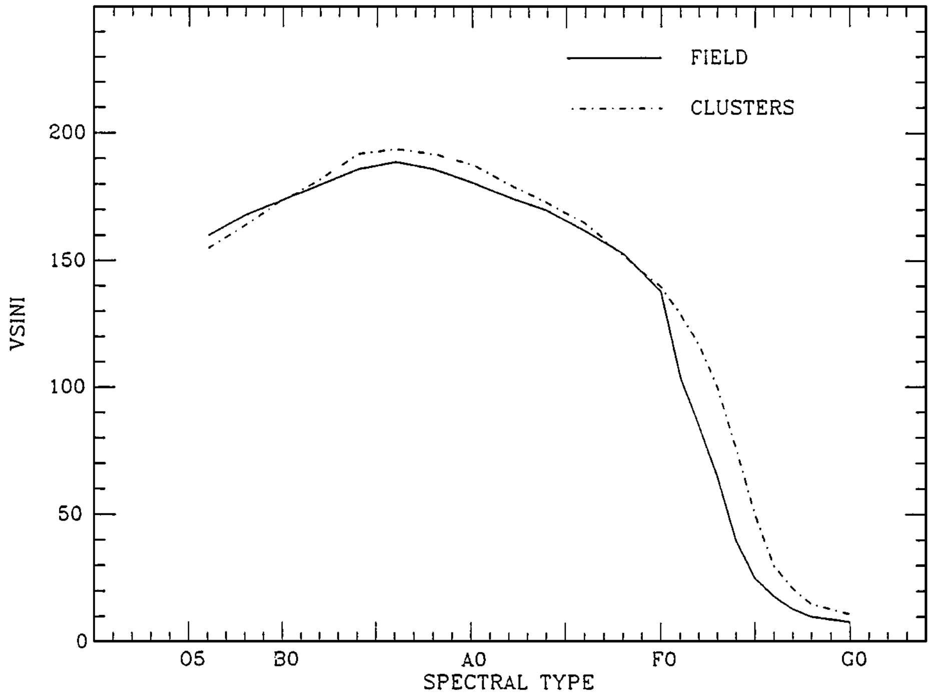

Fig. 1.6. Mean projected equatorial velocities for early-type field and cluster stars. Note that the open-cluster F dwarfs rotate more rapidly than their older, field counterparts. *Source:* Stauffer, J. R., and Hartmann, L. W., *Publ. Astron. Soc. Pacific*, **98**, 1233, 1986. (Courtesy of the Astronomical Society of the Pacific.)

velocities in the range 150–200 km s^{-1}. Within each spectral type, the mean rotational velocities of the field stars earlier than spectral type F0 are almost the same as those in clusters. Later than spectral type F0, however, the rotational velocities steeply decrease with increasing spectral type, dropping to below 20 km s^{-1} at spectral type G0. Note also that the F-type cluster stars, which are generally younger than the field stars, rotate more rapidly than their field counterparts. This result confirms Kraft's (1967) original finding that *the mean rotational velocities of late-F and early-G stars decline with advancing age*. This correlation between rotation and age was quantified shortly afterward by Skumanich (1972), who pointed out that the surface angular velocity of a solar-type star decays as the inverse square root of its age. To a good degree of approximation, we thus let

$$\Omega \propto t^{-1/2}, \tag{1.7}$$

which is known as *Skumanich's law*. (Other mathematical relations between rotation and age have been suggested, however.) As we shall see in Section 7.2, such a spin-down process is consistent with the idea that magnetically controlled stellar winds and/or episodic mass ejections from stars with outer convection layers continuously decelerate these stars as they slowly evolve on the main sequence.

An inspection of Figure 1.5 shows that appreciable rotational velocities are common among the *normal* O-, B-, and A-type stars along the main sequence, whereas they

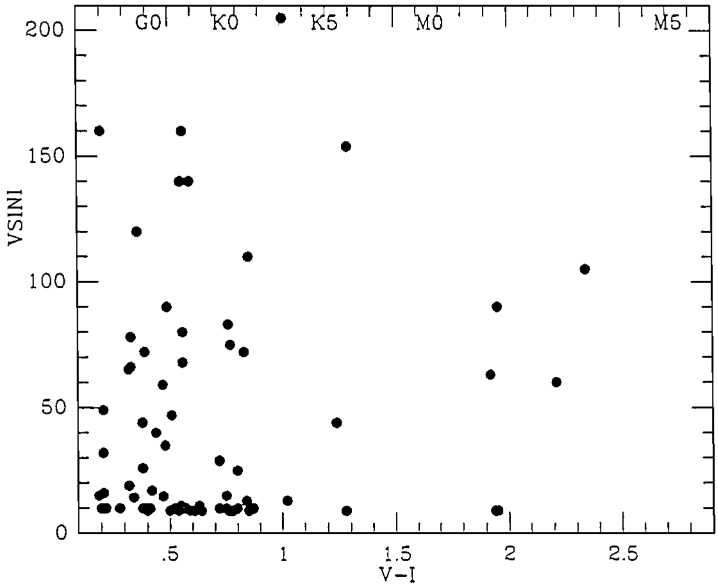

Fig. 1.7. Rotational velocity distribution for α Persei members. *Source:* Stauffer, J. R., Hartmann, L. W., and Jones, B. F., *Astrophys. J.*, **346**, 160, 1989.

virtually disappear near spectral type F5. Several photometric and spectroscopic studies made during the 1980s have confirmed that late-type, old field dwarfs with few exceptions are slow rotators, with true equatorial velocities less than 10 km s^{-1} in most stars. Fortunately, because continuous mass loss or discrete mass ejections cause spin-down of stars having convective envelopes, this sharp drop in rotational velocities along the main sequence is considerably reduced in younger stellar groups. Hence, clues to the rotational evolution of low-mass stars may be gained from the study of stars belonging to open clusters. This is illustrated in Figures 1.7 and 1.8, which depict, respectively, the rotational velocity distributions for lower main-sequence stars in the α Persei cluster (age $\sim$ 50 Myr) and in the Hyades (age $\sim$ 600 Myr). Figure 1.7 shows that the young α Persei cluster has a large number of very slowly rotating stars and a significant number of stars with projected equatorial velocities greater than 100 km s^{-1}. This is in contrast to the older Hyades, where G and K dwarfs are slow rotators, with the mean equatorial velocity appearing to decrease at least until spectral type K5. There is one prominent exception in Figure 1.8, however, a K8 dwarf that is the earliest known member of a population of relatively rapidly rotating late K- and M-type Hyades stars. These are genuine evolutionary effects that will be discussed in Section 7.4.2.

Other essential clues to the initial angular momentum distribution in solar-type stars can be obtained from the rotational velocity properties of low-mass, pre–main-sequence stars. These stars are commonly divided into two groups: the *classical* T Tauri stars, which have evidence of active accretion, and the *weak-line* T Tauri stars, which do not. Several photometric monitoring surveys have successfully determined rotation periods for a large number of these stars. It appears likely that most of the weak-line stars rotate faster than the classical T Tauri stars. Moreover, as was originally found by Attridge and Herbst

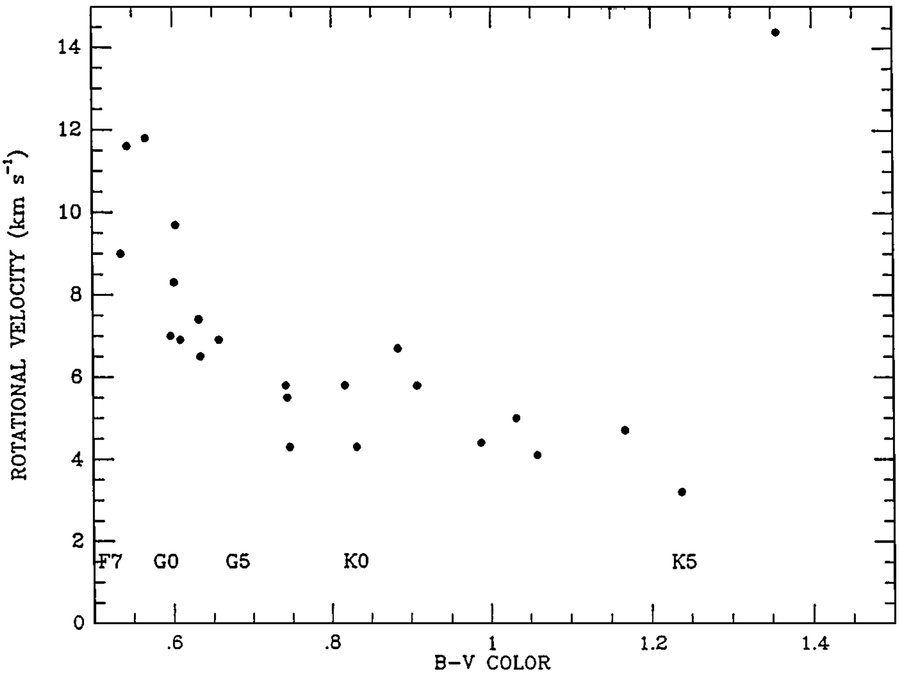

Fig. 1.8. Rotational velocity distribution for 23 Hyades stars. *Source:* Radick, R. R., Thompson, D. T., Lockwood, G. W., Duncan, D. K., and Baggett, W. E., *Astrophys. J.*, **321**, 459, 1987.

(1992), the frequency distribution of rotation periods for the T Tauri stars in the Orion Nebula cluster is distinctly bimodal. Figure 1.9 illustrates the frequency distribution of known rotation periods for these stars, combining the data for the Trapezium cluster, the Orion Nebula cluster, and other T associations. *This combined distribution is clearly bimodal, with a sparsely populated tail of extremely slow rotators*. The implications of this bimodality will be further discussed in Section 7.4.1.

1.4 Close binaries

In Section 1.1 we pointed out that the early-type components of close binaries rotate more slowly than the average of single stars of the same spectral type. In contrast, whereas the rotational velocities of single main-sequence stars of spectral type F5 and later are quite small (i.e., less than 10 km s^{-1}), appreciable rotations are common among the late-type components of close binaries. It has long been recognized that the distribution of rotational velocities in the close binaries is caused mostly by tidal interaction between the components, although some other processes – such as stellar winds, gravitational radiation, and large-scale magnetic fields – may also play a definite role in some binaries. To be specific, all types of tidal interaction involve an exchange of kinetic energy and angular momentum between the orbital and rotational motions. If we neglect stellar winds, the total angular momentum will be conserved in the tidal process. However, due to tidal dissipation of energy in the outer layers of the components, the total kinetic energy will decrease monotonically. Accordingly, as a result of

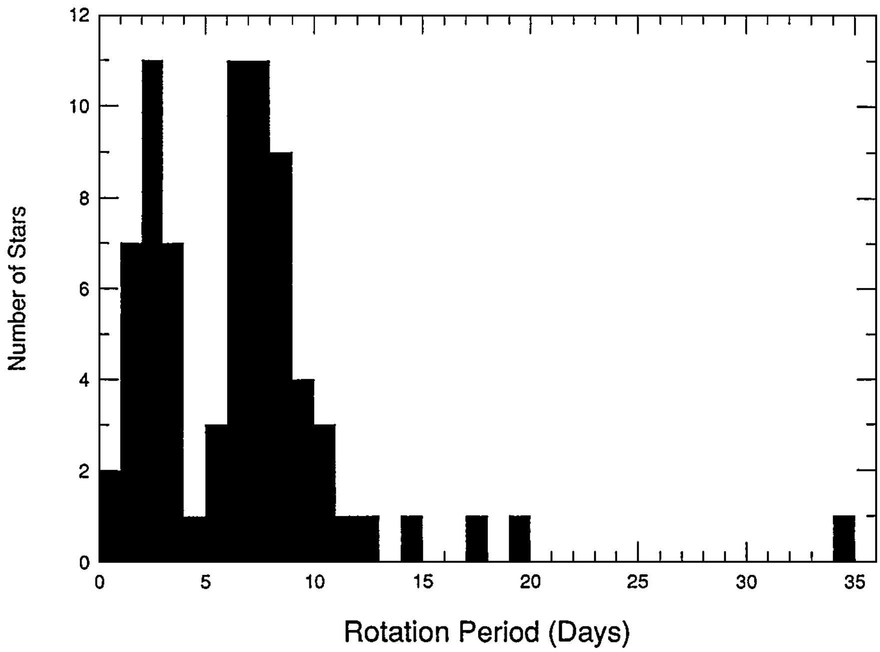

Fig. 1.9. Histogram showing the frequency distribution of rotation periods of T Tauri stars. This figure combines the data for the Trapezium cluster, the Orion Nebula cluster, and other T associations. *Source:* Eaton, N. L., Herbst, W., and Hillenbrand, L. A., *Astron. J.*, **110**, 1735, 1995.

various dissipative processes, a close binary starting from a wide range of initial spin and orbital parameters might eventually reach a state of minimum kinetic energy. This equilibrium state is characterized by a circular orbit, where the stellar spins are aligned and synchronized with the orbital spin.

As we shall see in Sections 8.2–8.4, however, in detached binaries the synchronization of the components proceeds at a much faster pace than the circularization of their orbits. Accordingly, the rotation of each component will quickly synchronize with the instantaneous orbital angular velocity *at periastron,*

$$\Omega_p = \frac{(1+e)^{1/2}}{(1-e)^{3/2}}\,\Omega_0, \tag{1.8}$$

where the tidal interaction is the most important during each orbital revolution. (As usual, e is the orbital eccentricity and Ω_0 is the mean orbital angular velocity.) Figure 1.10 illustrates this concept of *pseudo-synchronism* for a sample of selected eclipsing binaries with eccentric orbits for which we have accurate absolute dimensions. This figure compares the observed rotational velocities with the computed rotational velocities, assuming synchronization at periastron. We observe that most points scatter along the 45-degree line, indicating that pseudo-synchronization obtains in most close binaries of short orbital periods, either perfectly or approximately.

Observations show that an upper limit to the orbital period exists at which the observed rotational velocities begin to deviate very much from the synchronization (or

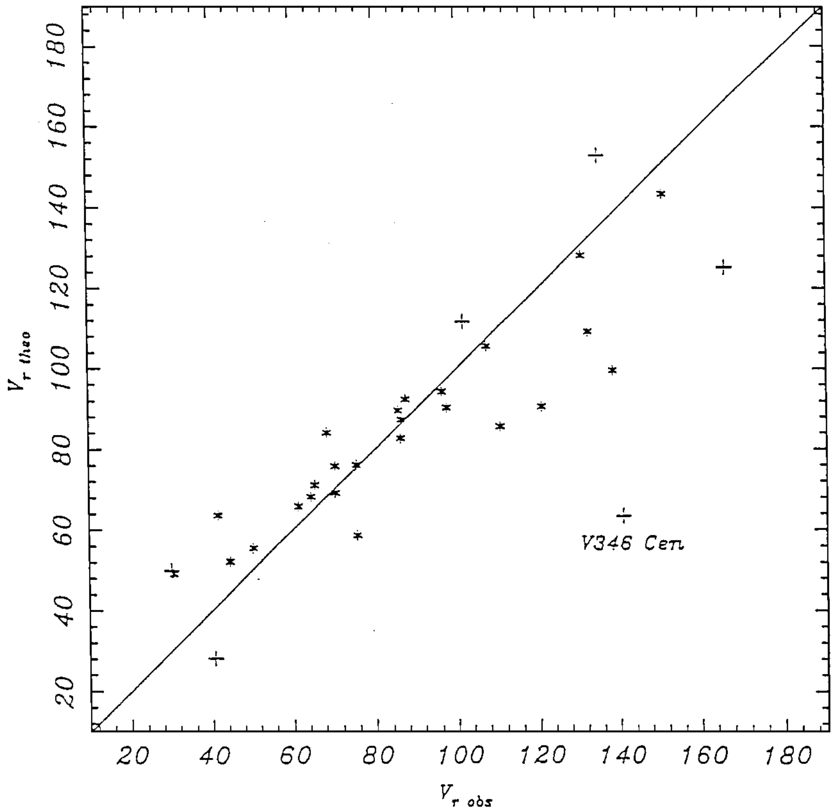

Fig. 1.10. Predicted versus observed rotational velocities assuming synchronization at periastron. The diagonal line is the locus of pseudo-synchronous rotation. *Source:* Claret, A., and Giménez, A., *Astron. Astrophys.*, **277**, 487, 1993.

pseudo-synchronization) period. As was originally noted by Levato (1976), the orbital period below which main-sequence binary components are still rotating in synchronism depends on spectral type. Specifically, he found that the largest orbital period for full synchronism is about 4–8 days in the early B spectral range, decreases to a minimum value of about 2 days at mid A-type, and increases up to 10–14 days at mid F-type. Subsequent investigations have confirmed that the tendency toward synchronization between the axial rotation and orbital revolution is indeed stronger in the F-type and later types than in the hotter ones. However, these studies have also demonstrated that *in the whole early spectral range synchronism (or pseudo-synchronism) extends up to binary separations substantially greater than previously held*. For example, the rotational properties of a large sample of early-type double-lined spectroscopic binaries have been investigated by Giuricin, Mardirossian, and Mezzetti (1985). Their statistical study indicates that a considerable tendency toward pseudo-synchronization extends up to a distance ratio $d/R \approx 20$ in the early-type (from O to F5) close binaries. (Here d is the mean distance between the components and R is the radius.) In fact, only for $d/R \gtrsim 20$ do pronounced deviations from synchronism at periastron become the rule in these binaries. In terms of orbital periods (for an easier comparison with Levato's underestimated upper limit

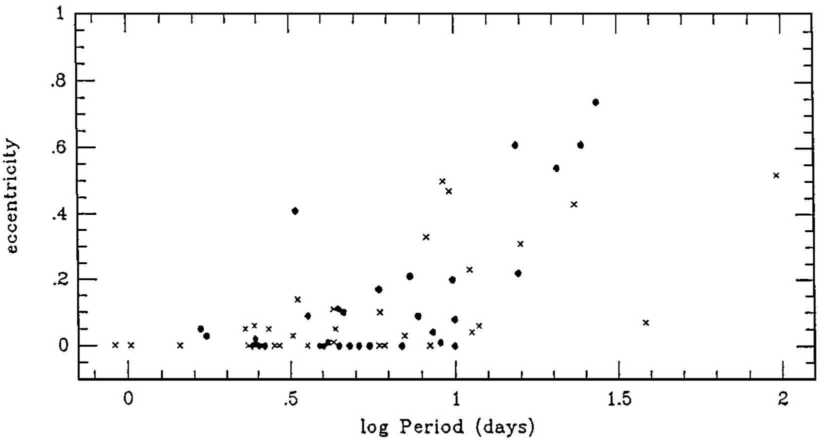

Fig. 1.11. Period–eccentricity distribution for a sample of spectroscopic binaries with A-type primaries. Single-lined binaries are shown as crosses; double-lined binaries are shown as filled circles. *Source:* Matthews, L. D., and Mathieu, R. D., in *Complementary Approaches to Double and Multiple Star Research* (McAlister, H., and Hartkopf, W. I., eds.), *A.S.P. Conference Series,* **32**, 244, 1992. (Courtesy of the Astronomical Society of the Pacific.)

periods), a limiting value of $d/R \approx 20$ corresponds to orbital periods of about 26, 18, and 13 days at spectral types B2, A0, and A5, respectively.

It is a well-known fact that circular (or nearly circular) orbits greatly predominate in short-period binaries. Since tidal interaction between the components of close binaries will tend to circularize their orbits, the precise determination of the cutoff period above which binaries display eccentric orbits appears to be a valuable test for the tidal theories. Giuricin, Mardirossian, and Mezzetti (1984) have studied the period–eccentricity distribution for a large sample of early-type detached binaries, excluding systems believed to have undergone (or to be undergoing) mass exchange between the components. They found that almost all binaries have circular or nearly circular orbits for orbital periods P smaller than 2 days. However, a mixed population of circular and eccentric orbits was found in the period range 2–10 days. Beyond $P = 10$ days all orbits are eccentric. A similar result was obtained by Matthews and Mathieu (1992), who investigated the period–eccentricity distribution of a sample of spectroscopic binaries with A-type primary stars. Figure 1.11 clearly shows that all binaries with orbital periods less than $P \approx 3$ days have circular or almost circular orbits (i.e., $e < 0.05$). Binaries with periods between 3 and 10 days are found with either circular or eccentric orbits, with the maximum eccentricity increasing with period. The longest-period circular orbit is at $P = 9.9$ days. This is exactly the kind of distribution one may expect to find for a sample of detached binaries with a *random* distribution of ages, where the population of circular and eccentric orbits becomes increasingly mixed as the Ps tend toward an upper limit period above which all orbits become eccentric.* For comparison, Figure 1.12 illustrates

* More recently, Mermilliod (1996, Fig. 2) has shown that this upper limit period was actually close to 25 days for a sample of 39 late-B and A-type binary stars belonging to open clusters. Note also that most of the O-type binaries with periods less than 30 days have circular orbits, whereas the long-period systems have eccentric orbits (Massey, 1982, p. 258).

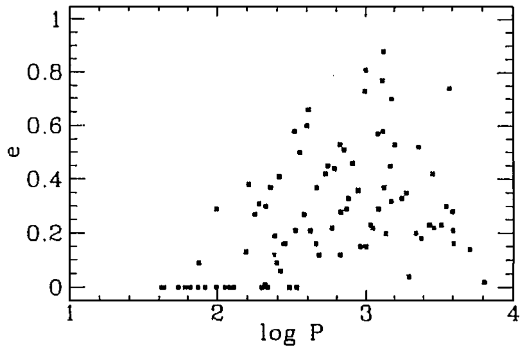

Fig. 1.12. Period–eccentricity distribution for a sample of spectroscopic binaries with red giant primaries. *Source:* Mermilliod, J. C., Mayor, M., Mazeh, T., and Mermilliod, J. C., in *Binaries as Tracers of Stellar Formation* (Duquennoy, A., and Mayor, M., eds.), p. 183, Cambridge: Cambridge University Press, 1992.

the period–eccentricity distribution of spectroscopic binaries with red giant primaries. Not unexpectedly, because red giants reach larger radii than main sequence stars, circular orbits are found for larger orbital periods. Note also the mixed population of circular and eccentric orbits in the period range 80–300 days. Again, this is caused by the mixing of all red giants, since the sample contains a range in age and mass.

It will be shown in Sections 8.2–8.4 that the degree of circularity of an orbit depends on how long the tidal forces have been acting on the components of a close binary. The study of binaries belonging to clusters is of particular interest, therefore, since these are the only stars for which one has some information about their ages. Mayor and Mermilliod (1984) were the first to study the orbital eccentricities for a *coeval* sample of late-type binaries in open clusters (33 red-dwarf binaries in the Hyades, Pleiades, Praesepe, and Coma Berenices open clusters). They found that all binaries with periods shorter than 5.7 days display circular orbits whereas all orbits with longer periods have significant eccentricities. More recently, it has been found that other coeval samples with different evolutionary ages exhibit transitions between circular and eccentric orbits at distinct cut-off periods. It is immediately apparent from Table 1.2 that *the transition period* $P_{\rm cut}$ *increases monotonically with the sample age* t_a. Accordingly, the observed t_a–$P_{\rm cut}$ relation strongly suggests that the circularization mechanism is operative *during* the main-sequence lifetime of the stars – pre–main-sequence tidal circularization is permitted but not required by present observations. This provides a very important test for the tidal mechanisms since the theoretical circularization time cannot exceed the sample age at cutoff period.

Tidal interaction in the RS CVn stars poses quite a challenging problem also. In fact, in these chromospherically active binaries there is still a tendency toward synchronization in the period range 30–70 days, up to $P = 100$ days. However, asynchronous rotators are present in all period groups, even among binaries with orbital periods of 30 days or less. In these systems one also finds that the rotation periods are either shorter or longer than the orbital periods, independent of the orbital eccentricities. As was shown by Tan, Wang, and Pan (1991), however, asynchronous RS CVn stars have orbital eccentricities that are larger, on the average, than the eccentricities of pseudo-synchronously rotating systems.

Table 1.2. *The observed t_a–P_{cut} relation.*

Binary Sample	Cutoff Period (day)	Age (Gyr)
Pre-main-sequence	4.3	0.003
Pleiades	7.05	0.1
Hyades/Praesepe	8.5	0.8
M67	12.4	4.0
Halo	18.7	17.6

Source: Mathieu, R. D., Duquennoy, A., Latham, D. W., Mayor, M., Mazeh, T., and Mermilliod, J. C., in *Binaries as Tracers of Stellar Formation* (Duquennoy, A., and Mayor, M., eds.), p. 278, Cambridge: Cambridge University Press, 1992.

These authors also found that the chromospheric activity in their sample of asynchronous binaries is lower, on the average, than in synchronous RS CVn stars. If so, then, other braking mechanisms (e.g., magnetically driven winds) must be interfering with tidal interaction in these giant binary stars. To make the problem even more complex, let us note that Stawikowski and Glebocki (1994) have found another basic difference between the synchronous and asynchronous long-period RS CVn stars, when their primary component is a late-type giant or subgiant: *Whereas for synchronously rotating stars the assumption about coplanarity of their equatorial and orbital planes is justified, in most asynchronous binaries the rotation axis of the primary is not perpendicular to the orbital plane*. A similar result was obtained by Glebocki and Stawikowski (1995, 1997) for late-type main-sequence binaries and short-period RS CVn stars with orbital periods shorter than about 10 days. Pseudo-synchronism and coplanarity will be further discussed in Section 8.2.1.

1.5 Bibliographical notes

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