1

Introduction

In this opening chapter, we give an informal and qualitative overview – a pep talk – to help you appreciate why sustained nonequilibrium systems are so interesting and worthy of study.

We begin in Section 1.1 by discussing the big picture of how the Universe is filled with nonequilibrium systems of many different kinds, a consequence of the fact that the Universe had a beginning and has not yet stopped evolving. A profound and important question is then to understand how the observed richness of structure in the Universe arises from the property of not being in thermodynamic equilibrium. In Section 1.2, a particularly well studied nonequilibrium system, Rayleigh–Bénard convection, is introduced to establish some vocabulary and insight regarding what is a nonequilibrium system. Next, in Section 1.3, we extend our discussion to representative examples of nonequilibrium patterns in nature and in the laboratory, to illustrate the great diversity of such patterns and to provide some concrete examples to think about. These examples serve to motivate some of the central questions that are discussed throughout the book, e.g. spatially dependent instabilities, wave number selection, pattern formation, and spatiotemporal chaos. The humble desktop-sized experiments discussed in this section, together with theory and simulations relating to them, can also be regarded as the real current battleground for understanding nonequilibrium systems since there is a chance to compare theory with experiment quantitatively.

Next, Section 1.4 discusses some of the ways that pattern-forming nonequilibrium systems differ from the low-dimensional dynamical systems that you may have seen in an introductory nonlinear dynamics course. Some guidelines are also given to determine qualitatively when low-dimensional nonlinear dynamics may not suffice to analyze a particular nonequilibrium system. In Section 1.5, a strategy is given and explained for exploring nonequilibrium systems. We explain why fluid dynamics experiments have some advantages over other possible experimental systems and why certain fluid experiments such as Rayleigh–Bénard convection are especially attractive. Finally, Section 1.6 mentions some of the topics that we will *not* address in this book for lack of time or expertise.

1.1 The big picture: why is the Universe not boring?

When people look at the world around them or peer through telescopes at outer space, a question that sometime arises is: why is there something rather than nothing? Why does our Universe consist of matter and light rather than being an empty void? While this question remains unanswered scientifically and is intensely pursued by researchers in particle physics and cosmology, in this book we discuss a second related question that is also interesting and fundamental: why does the existing matter and light have an interesting structure? Or more bluntly: why is the Universe not boring?

For it turns out that it is not clear how the existence of matter and light, together with the equations that determine their behavior, produce the extraordinary complexity of the observed Universe. Instead of all matter in the Universe being clumped together in a single black hole, or spread out in a featureless cloud, we see with our telescopes a stunning variety of galaxies of different shapes and sizes. The galaxies are not randomly distributed throughout space like molecules in a gas but are organized in clusters, the clusters are organized in super-clusters, and these super-clusters themselves are organized in voids and walls. Our Sun, a fairly typical star in a fairly typical galaxy, is not a boring spherical static ball of gas but a complex evolving tangled medium of plasma and magnetic fields that produces structure in the form of convection cells, sunspots, and solar flares. Our Earth is not a boring homogeneous static ball of matter but consists of an atmosphere, ocean, and rocky mantle that each evolve in time in an endless never-repeating dynamics of weather, water currents, and tectonic motion. Further, some of the atoms on the surface of our Earth have organized themselves into a biosphere of life forms, which we as humans particularly appreciate as a source of rich and interesting structure that evolves dynamically. Even at the level of a biological organism such as a mammal, there is further complex structure and dynamics, e.g. in the electrical patterns of the brain and in the beating of the heart.

So again we can ask: why does the matter and light that exist have such interesting structure? As scientists, we can ask further: is it possible to explain the origin of this rich structure and how it evolves in time? In fact, how should we define or quantify such informal and qualitative concepts such as "structure" or "patterns" or "complexity" or "interesting?" On what details does this complexity depend and how does this complexity change as various parameters that characterize a system are varied?

While this book will explain some of what is known about these questions, especially at the laboratory level which allows controlled reproducible experiments, we can say at a hand-waving level why the Universe is interesting rather than boring: the Universe was born in a cosmological Big Bang and is still young when measured in units of the lifetime of a star. Thus the Universe has not yet lasted long enough to come to thermodynamic equilibrium: *the Universe as a whole is a nonequilibrium system*. Because stars are young and have not yet reached thermodynamic equilibrium, the nuclear fuel in their core has not yet been consumed. The flux of energy from this core through the surface of the star and out into space drives the complex dynamics of the star's plasma and magnetic field. Similarly, because the Earth is still geologically young, its interior has not yet cooled down and the flux of heat from its hot core out through its surface, together with heat received from the Sun, drives the dynamics of the atmosphere, ocean, and mantle. And it is this same flux of energy from the Earth and Sun that sustains Earth's intricate biosphere.

This hand-waving explanation of the origin of nonequilibrium structure is unsatisfactory since it does not lead to the quantitative testing of predictions by experiment. To make progress, scientists have found it useful to turn to desktop experimental systems that can be readily manipulated and studied, and that are also easier to analyze mathematically and to simulate with a computer. The experiments and theory described in this book summarize some of the systematic experimental and theoretical efforts of the last thirty years to understand how to predict and to analyze such desktop nonequilibrium phenomena. However, you should appreciate that much interesting research remains to be carried out if our desktop insights are to be related to the more complex systems found in the world around us. We hope that this book will encourage you to become an active participant in this challenging endeavor.

1.2 Convection: a first example of a nonequilibrium system

Before surveying some examples that illustrate the diversity of patterns and dynamics in natural and controlled nonequilibrium systems, we first discuss a particular yet representative nonequilibrium system, a fluid dynamics experiment known as Rayleigh–Bénard convection. Our discussion here is qualitative since we wish to impart quickly some basic vocabulary and a sense of the interesting issues before turning to the examples discussed in Section 1.3 below. We will return to convection many times throughout the book, since it is one of the most thoroughly studied of all sustained nonequilibrium systems, and has repeatedly yielded valuable experimental and theoretical insights.

A Rayleigh–Bénard convection experiment consists of a layer of fluid, e.g. air or water, between two horizontal plates such that the bottom plate is warm and

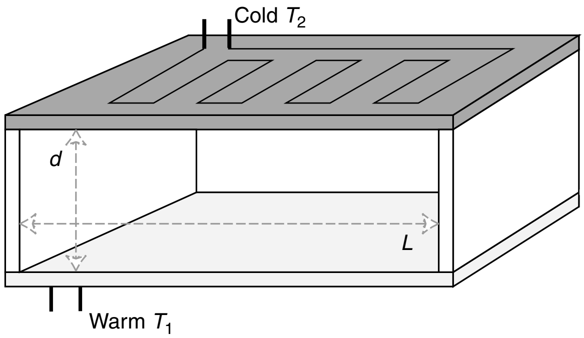

Fig. 1.1 Rayleigh–Bénard convection of a fluid layer between two horizontal plates is one of the simplest sustained nonequilibrium systems. The drawing shows a featureless square room of lateral width L and height d with copper-covered floor and ceiling, and supporting walls made of wood. By appropriate plumbing and control circuits, the floor and ceiling are maintained at constant temperatures of T_1 and T_2 respectively. When the temperature difference $\Delta T = T_1 - T_2$ is sufficiently large, the warm less-dense air near the floor and the cold more-dense air near the ceiling spontaneously start to move, i.e. convection sets in. The rising and falling regions of air eventually forms cellular structures known as convection rolls. The characteristic roll size is about the depth d of the air.

the upper plate is cool. As an example to visualize (but a bit impractical for actual experimentation as you will discover in Exercise 1.5), consider a square room whose lateral width L is larger than its height d, and in which all furniture, doors, windows, and fixtures have been removed so that there is only a smooth flat horizontal floor, a smooth flat horizontal ceiling, and smooth flat vertical walls (see Fig. 1.1). The floor and ceiling are then coated with a layer of copper, and just beneath the floor and just above the ceiling some water-carrying pipes and electronic circuits connected to water heaters are arranged so that the floor is maintained at a constant temperature T_1 and the ceiling is maintained at a constant temperature T_2.[1] Because copper conducts heat so well, any temperature variations within the floor or within the ceiling quickly become negligible so that the floor and ceiling can be considered as time-independent constant-temperature surfaces. The supporting sidewalls are made of some material that conducts heat poorly such as wood or Plexiglas.

A typical nonequilibrium experiment for the room in Fig. 1.1 would then be simply to fix the temperature difference $\Delta T = T_1 - T_2$ at some value and then to observe what happens to the air. "Observe what happens" can mean several

[1] Uniformly warming the floor and cooling the ceiling is not the usual way that a room is heated. Instead, a convector – a localized heat source with a large surface area – is placed somewhere in the room, and heat is lost through the windows instead of through the ceiling. (What we call a convector everyone else calls a radiator but this is poorly named since the air is heated mainly by convection, not by radiation.) But this nonuniform geometry is more complicated, and so less well suited, than our idealized room for experiment and analysis.

things depending on the questions of interest. By introducing some smoke into the room, the pattern of air currents could be visualized. A more quantitative observation might involve recording as a function of time t some local quantity such as the temperature $T(\mathbf{x}_0, t)$ or the x-component of the air's velocity $v_x(\mathbf{x}_0, t)$ at a particular fixed position $\mathbf{x}_0 = (x_0, y_0, z_0)$ inside the room. Alternatively, an experimentalist might choose to record some global quantity such as the total heat $H(t)$ transported from the floor to the ceiling, a quantity of possible interest to mechanical engineers and architects. These measurements of some quantity at successive moments of time constitute a time series that can be stored, plotted, and analyzed. A more ambitious and difficult observation might consist of measuring multivariate time series, e.g. measuring the temperature field $T(\mathbf{x}, t)$ and the components of the velocity field $\mathbf{v}(\mathbf{x}, t)$ simultaneously at many different spatial points, at successive instants of time. These data could then be made into movies or analyzed statistically. All of these observations are carried out for a particular fixed choice of the temperature difference ΔT and over some long time interval (long enough that any transient behavior will decay sufficiently). Other experiments might involve repeating the same measurements but for several successive values of ΔT, with each value again held constant during a given experiment. In this way, the spatiotemporal dynamical properties of the air in the room can be mapped out as a function of the parameter ΔT, and various dynamical states and transitions between them can be identified.

The temperature difference ΔT is a particularly important parameter in a convection experiment because it determines whether or not the fluid is in thermodynamic equilibrium. (It is precisely the fact that the nonequilibrium properties of the entire room can be described by a single parameter ΔT that constitutes the idealization of this experiment, and that motivated the extra experimental work of coating the floor and ceiling with copper.) If $\Delta T = 0$ so that the ceiling and floor have the same common temperature $T = T_1 = T_2$, then after some transient time, the air will be in thermodynamic equilibrium with zero velocity and the same uniform temperature T throughout. There is typically a transient time associated with approaching thermodynamic equilibrium because the air itself is rarely in such equilibrium without taking special precautions. For example, there might be a small breeze in the air when the door to the experimental room is closed or some part of the air may be a bit warmer than some other part because someone walked through the room. But as long as the room is sealed and the floor and ceiling have the same temperature, all macroscopic motion in the air will die out and the air will attain the same temperature everywhere.

As soon as the temperature difference ΔT becomes nonzero (with either sign), the air can no longer be in thermodynamic equilibrium since the temperature is spatially nonuniform. One says that the air is driven out of equilibrium by the temperature difference since the nonequilibrium state is maintained as long as there is a temperature difference. For the case $\Delta T > 0$ of a warm floor and cool ceiling, as ΔT becomes larger and larger (but again held constant throughout any particular experiment), more and more energy flows through the air from the warm floor to the cooler ceiling, the system is driven further from equilibrium, and more and more complicated spatiotemporal dynamical states are observed. A temperature difference is not the only way to drive a system out of equilibrium as we will discuss in other parts of the book. Other possibilities include inducing relative motion (e.g. pushing water through a pipe which creates a shear flow), varying some parameter in a time-dependent fashion (e.g. shaking a cup of water up and down), applying an electrical current across an electrical circuit, maintaining one or more chemical gradients, or creating a deviation from a Maxwellian velocity distribution of particles in a fusion plasma.

For any particular mechanism such as a temperature difference that drives a system out of equilibrium, there are dissipative (friction-like) mechanisms that oppose this driving and act in such a way so as to restore the system to equilibrium. For the air convecting inside our room, there are two dissipative mechanisms that restore the air to a state of thermodynamic equilibrium if ΔT is set to zero. One is the viscosity of the fluid, which acts to decrease any spatial variation of the velocity field. Since it is known from fluid dynamics that the velocity of a fluid is zero at a material surface,[2] the only possible long-term behavior for a fluid approaching equilibrium in the presence of static walls is that the velocity field everywhere decays to zero. A second dissipative mechanism is heat conduction through the air. The warm regions of air lose heat to the cooler regions of air by molecular diffusion, and eventually the temperature becomes constant and uniform everywhere inside the room. These dissipative mechanisms of viscosity and heat conduction are always present, even when $\Delta T \neq 0$, and so one often talks about a sustained nonequilibrium system as a driven-dissipative system.

Rayleigh–Bénard convection is sometimes called buoyancy-induced convection for reasons that illustrate a bit further the driving and dissipative mechanisms competing in a nonequilibrium system. Let us consider an experiment in which the air in the room has reached thermal equilibrium with $\Delta T = 0$ and then the temperature difference ΔT is increased to some positive value. Small parcels of air near the floor will expand and so decrease in density as they absorb heat from the floor, while small parcels of air near the ceiling will contract in volume and increase in density as they lose heat to the ceiling. As illustrated in Fig. 1.2, buoyancy forces then appear that accelerate the lighter warmer fluid upwards and the heavier colder

[2] More precisely, the fluid velocity at a wall is zero in a frame of reference moving with the surface. Exercise 1.9 suggests a simple experiment using an electric fan to explore this point.

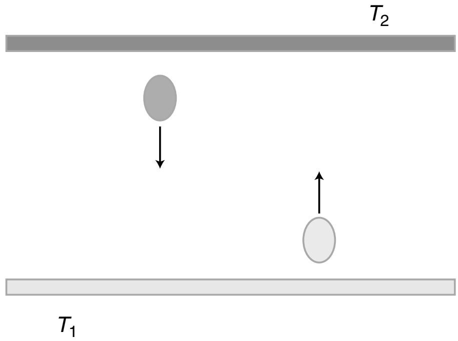

Fig. 1.2 Illustration of the driving and dissipative forces acting on small parcels of air near the floor and ceiling of the experimental room in Fig. 1.1 whose floor is warmer than its ceiling. The parcels are assumed to be small enough that their temperatures are approximately constant over their interiors. The acceleration of the parcels by buoyancy forces is opposed by a friction arising from the fluid viscosity and also by the diffusion of heat between warmer and cooler regions of the fluid. Only when the temperature difference $\Delta T = T_1 - T_2$ exceeds a finite critical value $\Delta T_c > 0$ can the buoyancy forces overcome the dissipation and convection currents form.

fluid downwards, in accord with the truism that "hot air rises" and "cold air falls." These buoyancy forces constitute the physical mechanism by which the temperature difference ΔT "drives" the air out of equilibrium. As a warm parcel moves upward, it has to push its way through the surrounding fluid and this motion is opposed by the dissipative friction force associated with fluid viscosity. Also, as the parcel rises, it loses heat by thermal conduction to the now cooler surrounding air, becomes more dense, and the buoyancy force is diminished. Similar dissipative effects act on a cool descending parcel.

From this microscopic picture, we can understand the experimental fact that making the temperature difference ΔT positive is a necessary but not sufficient condition for the air to start moving since the buoyancy forces may not be strong enough to overcome the dissipative effects of viscosity and conduction. Indeed, experiment and theory show that only when the temperature difference exceeds a threshold, a critical value we denote as ΔT_c, will the buoyancy forces be sufficiently large that the air will spontaneously start to move and a persistent spatiotemporal structure will appear in the form of convection currents. If the room's width L is large compared to its depth d so that the influence of the walls on the bulk fluid can be ignored, a precise criterion for the onset of convection can be stated in the form

$$R > R_c. \tag{1.1}$$

Table 1.1. *The isobaric coefficient of thermal expansion* α, *the kinematic viscosity* ν, *and the thermal diffusivity* κ *for air, water, and mercury at room temperature* T = 293 K *and at atmospheric pressure. These parameters vary weakly with temperature.*

Fluid	α (K^{-1})	ν (m^2/s)	κ (m^2/s)
Air	3×10^{-3}	2×10^{-5}	2×10^{-5}
Mercury	2×10^{-4}	1×10^{-7}	3×10^{-6}
Water	2×10^{-4}	1×10^{-6}	2×10^{-7}

The parameter R is defined in terms of various physical parameters

$$R = \frac{\alpha g d^3 \Delta T}{\nu \kappa}, \tag{1.2}$$

and the critical value of R has the approximate value

$$R_c \approx 1708. \tag{1.3}$$

The parameters in Eq. (1.2) have the following meaning: g is the gravitational acceleration, about $9.8\,\mathrm{m/s^2}$ over much of the Earth's surface; $\alpha = -(1/\rho)(\partial\rho/\partial T)|_p$ is the fluid's coefficient of thermal expansion at constant pressure, and measures the relative change in density ρ as the temperature is varied; d is the uniform depth of the fluid; ΔT is the uniform temperature difference across the fluid layer; ν is the fluid's kinematic viscosity; and κ is the fluid's thermal diffusivity. Approximate values of the parameters α, ν, and κ for air, water, and mercury at room temperature ($T = 293$ K) and at atmospheric pressure are given in Table 1.1.

The combination of physical parameters in Eq. (1.2) is dimensionless and so has the same value no matter what physical units are used in any given experiment, e.g. System Internationale (SI), Centimeter-Gram-Seconds (CGS), or British. This combination is denoted by the symbol "R" and is called the Rayleigh number in honor of the physicist and applied mathematician Lord Rayleigh who, in 1916, was the first to identify its significance for determining the onset of convection. The pure number R_c is called the critical Rayleigh number R_c since it denotes the threshold that R must exceed for convection to commence. The value R_c can be calculated directly from the equations that govern the time evolution of a convecting fluid (the Boussinesq equations) as the criterion when the motionless conducting state of the fluid first becomes linearly unstable. The general method of this linear stability analysis is described in Chapter 2.

Despite its dependence on six parameters, you should think of the Rayleigh number R as simply being proportional to the temperature difference ΔT. The reason is that all the parameters in Eq. (1.2) except ΔT are approximately constant in a typical series of convection experiments. Thus the parameters α, ν, and κ in Eq. (1.1) depend weakly on temperature and are effectively fixed once a particular fluid is chosen. The acceleration g is fixed once a particular geographical location is selected for the experiment and the depth of the fluid d is typically fixed once the convection cell has been designed and is difficult to vary as an experimental parameter. Only the temperature difference ΔT is easily changed substantially and so this naturally becomes the experimental control parameter.

You should also note that the numerator $\alpha g d^3 \Delta T$ in Eq. (1.2) is related to quantities that determine the buoyancy force, while the denominator $\nu\kappa$ involves quantities related to the two dissipative mechanisms so Eq. (1.1) indeed states that instability will not occur until the driving is sufficiently strong compared to the dissipation. Most nonequilibrium systems have one or more such dimensionless parameters associated with them and these parameters are key quantities to identify and to measure when studying a nonequilibrium system.

What kind of dynamics can we expect for the air if the Rayleigh number R is held constant at some value larger than the critical value R_c? From Fig. 1.2, we expect the warm fluid near the floor to rise and the cool fluid near the ceiling to descend but the entire layer of ascending fluid near the floor cannot pass through the entire layer of descending fluid near the ceiling because the fluid is approximately incompressible. What is observed experimentally is pattern formation: the fluid spontaneously achieves a compromise such that some regions of fluid rise and neighboring regions descend, leading to the formation of a cellular convection "pattern" in the temperature, velocity, and pressure fields. The distance between adjacent rising and falling regions turns out to be about the depth of the air. Once the air begins to convect, the dynamics becomes too complicated to understand by casual arguments applied to small parcels of air and we need to turn to experiments to observe what happens and to a deeper mathematical analysis to understand the experimental results (see Figs. 1.14 and 1.15 below in Section 1.3.2). However, one last observation can be made. The motion of the fluid parcels inside the experimental system transport heat and thereby modify the temperature gradient that is felt in a particular location inside the system. Thus the motion of the medium changes the balance of driving and dissipation in different parts of the medium, and this is the reason why the dynamics is nonlinear and often difficult to understand.

The general points we learn from the above discussion about Rayleigh–Bénard convection are the following. There are mechanisms that can drive a system out of thermodynamic equilibrium, such as a flux of energy, momentum or matter through the system. This driving is opposed by one or more dissipative mechanisms such as viscous friction, heat conduction, or electrical resistance that restore the system to thermal equilibrium. The relative strength of the driving and dissipative mechanisms can often be summarized in the form of one or more dimensionless parameters, e.g. the Rayleigh number R in the case of convection. Nonequilibrium systems often become unstable and develop an interesting spatiotemporal pattern when the dimensionless parameter exceeds some threshold, which we call the critical value of that parameter. What happens to a system when driven above this threshold is a complex and fascinating question which we look at visually in the next section and then discuss in much greater detail throughout the rest of the book. However, the origin of the complexity can be understood qualitatively from the fact that transport of energy and matter by different parts of the pattern locally modifies the balance of driving and dissipation, which in turn may change the pattern and the associated transport.

1.3 Examples of nonequilibrium patterns and dynamics

1.3.1 Natural patterns

In this section we discuss examples of pattern-forming nonequilibrium systems as found in nature while in the next section we look at prepared laboratory systems, such that a nonequilibrium system can be carefully prepared and controlled. These examples help to demonstrate the great variety of dynamics observed in pattern-forming nonequilibrium systems and provide concrete examples to keep in mind as we try to identify the interesting questions to ask.

We begin with phenomena at some of the largest length and time scales of the Universe and then descend to human length and time scales. An example of an interesting pattern on the grandest scales of the Universe is the recently measured organization of galaxies into sheets and voids shown in Fig. 1.3. Observation has shown that our Universe is everywhere expanding, with all faraway galaxies moving away from each other and from the Earth, and with the galaxies that are furthest away moving the fastest. The light from a galaxy that is moving away from Earth is Doppler-shifted to a longer wavelength (becomes more red) compared to the light coming from an identical but stationary galaxy. By measuring the extent to which known spectral lines are red-shifted, astronomers can estimate the recessional speed v of a galaxy and convert this speed to a distance d by using the so-called Hubble law $v = H_0 d$, where the Hubble constant H_0 has the approximate value $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (and a megaparsec Mpc is about 3×10^{19} km or about 3×10^6 light years).

Figure 1.3 summarizes such distance measurements for about 100 000 galaxies out to the rather extraordinary distance of about four billion light years which is