Flatland

An Edition with Notes and Commentary

Flatland, Edwin Abbott Abbott's story of a two-dimensional universe, as told by one of its inhabitants who is introduced to the mysteries of threedimensional space, has enjoyed an enduring popularity from the time of its publication in 1884. This fully annotated edition enables the modernday reader to understand and appreciate the many "dimensions" of this classic satire with commentary on language and literary style, including numerous definitions of obscure words and an appendix on Abbott's life and work. Historical commentary, writings by Plato and Aristotle, and citations from Abbott's other writings work together to show how this tale relates to Abbott's views of society in late-Victorian England and classical Greece. Approaching the book from a mathematical stance, additional notes and illustrations enhance the usefulness of *Flatland* as an elementary introduction to higher-dimensional geometry.

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Flatland

by Edwin A. Abbott

An Edition with Notes and Commentary by

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The Mathematical Association of America



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It is altogether fitting that there be a "Cambridge *Flatland*." Abbott graduated from the University of Cambridge, as did most of his closest friends and many of his students. His son was a fellow of Jesus College, Cambridge, and his daughter attended Girton College. His last great work, the fourteen-volume "Diatessarica" series, was printed in Cambridge at the University Press. We believe that he would be pleased that Cambridge University Press is publishing *Flatland*, and it is to his memory that this edition is dedicated.

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