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Excerpt
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1 Basic concepts and resistor circuits

1.1 Basics

We start our study of electronics with definitions and the basic laws that apply to *all* circuits. This is followed by an introduction to our first circuit element – the resistor.

In electronics, we are interested in keeping track of two basic quantities: the *currents* and *voltages* in a circuit. If you can make these quantities behave like you want, you have succeeded.

Current measures the flow of charge past a point in the circuit. The units of current are thus coulombs per second or *amperes*, abbreviated as A. In this text we will use the symbol I or i for current.

As charges move in circuits, they undergo collisions with atoms and lose some of their energy. It thus takes some work to move charges around a circuit. The work per unit charge required to move some charge between two points is called the *voltage* between those points. (In physics, this work per unit charge is equivalent to the difference in electrostatic potential between the two points, so the term *potential difference* is sometimes used for voltage.) The units of voltage are thus joules per coulomb or *volts*, abbreviated V. In this text we will use the symbol V or v for voltage.

In a circuit, there are sources and sinks of energy. Some sources of energy (or voltage) include batteries (which convert chemical energy to electrical energy), generators (mechanical to electrical energy), solar cells (radiant to electrical energy), and power supplies and signal generators (electrical to electrical energy). All other electrical components are sinks of energy.

Let's see how this works. The simplest circuit will involve one voltage source and one sink, with connecting wires as shown in Fig. 1.1. By convention, we denote the two sides of the voltage source as + and −. A positive charge moving from the − side to the + side of the source gains energy. Thus we say that the voltage across the source is positive. When the circuit is complete, current flows out of the + side of the source, as shown. The voltage across the component is negative when we

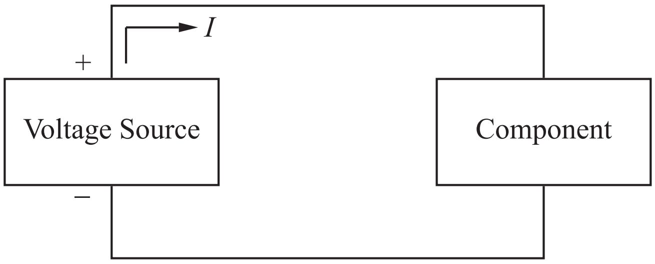

Figure 1.1 A simple generic circuit.

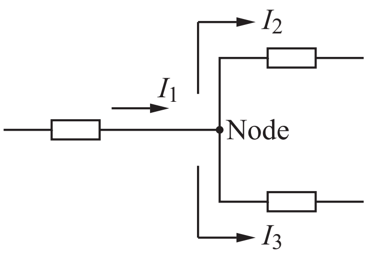

Figure 1.2 Example of Kirchoff's Current Law.

cross it in the direction of the current. We say there is a *voltage drop* across the component. Note that while we can speak of the current at any point in the circuit, the voltage is always between two points. It makes no sense to speak of the voltage at a point (remember, the voltage is a potential *difference*).

We can now write down some general rules about voltage and current.

1. The sum of the currents into a node (i.e. any point on the circuit) equals the sum of the currents flowing out of the node. This is Kirchoff's Current Law (KCL) and expresses conservation of charge. For example, in Fig. 1.2, $I_1 = I_2 + I_3$. If we use the sign convention that currents into a node are positive and currents out of a node are negative, then we can express this law in the compact form

$$\sum_k^{\text{node}} I_k = 0 \tag{1.1}$$

   where the sum is over all currents into or out of the node.
2. The sum of the voltages around any closed circuit is zero. This is Kirchoff's Voltage Law (KVL) and expresses conservation of energy. In equation form,

$$\sum_k^{\text{loop}} V_k = 0. \tag{1.2}$$

   Here we must use the convention that the voltage across a source is positive when we move across the source in the direction of the current and the voltage

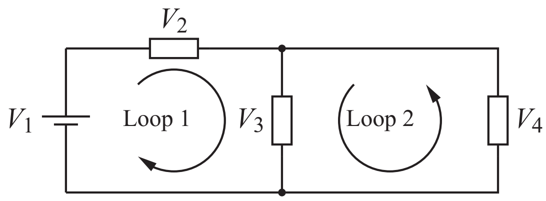

Figure 1.3 Example of Kirchoff's Voltage Law.

across a sink is negative when we move across the component in the direction of the current. If we traverse a source or sink in the direction opposite to the direction of the current, the signs are reversed. Figure 1.3 gives an example. Here we introduce the circuit symbol for an ideal battery, labeled with voltage V_1. The top of this symbol represents the positive side of the battery. The current (not shown) flows up out of the battery, through the component labeled V_2 and down through the components labeled V_3 and V_4. Looping around the left side of the circuit in the direction shown gives $V_1 - V_2 - V_3 = 0$ or $V_1 = V_2 + V_3$. Here we take V_2 and V_3 to be positive numbers and include the sign explicitly. Going around the right portion of the circuit as shown gives $-V_3 + V_4 = 0$ or $V_3 = V_4$. This last equality expresses the important result that components connected in parallel have the same voltage across them.

3. The power P provided or consumed by a circuit device is given by

$$P = VI \tag{1.3}$$

where V is the voltage across the device and I is the current through the device. This follows from the definitions:

$$VI = \left(\frac{\text{work}}{\text{charge}}\right)\left(\frac{\text{charge}}{\text{time}}\right) = \frac{\text{work}}{\text{time}} = \text{power}. \tag{1.4}$$

The units of power are thus joules per second or *watts*, abbreviated W. This law is of considerable practical importance since a key part of designing a circuit is to employ components with the proper power rating. A component with an insufficient power rating will quickly overheat and fail when the circuit is operated.

Finally, a word about prefixes and nomenclature. Some common prefixes and their meanings are shown in Table 1.1. As an example, recall that the unit *volts* is abbreviated as V, and *amperes* or *amps* is abbreviated as A. Thus 10^6 volts = 1 MV and 10^{-3} amps = 1 mA. Notice that case matters: 1 MA ≠ 1 mA.

Table 1.1 Some common prefixes used in electronics

Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

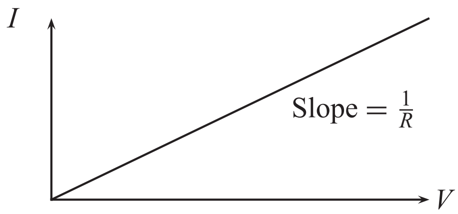

Figure 1.4 I–V curve for a resistor.

1.2 Resistors

A common way to represent the behavior of a circuit device is the I–V characteristic. This is a plot of the current I through the device as a function of applied voltage V across the device. Our first device, the resistor, has the simple linear I–V characteristic shown in Fig. 1.4. This linear relationship is expressed by Ohm's Law:

$$V = IR. \tag{1.5}$$

The constant of proportionality, R, is called the *resistance* of the device and is equal to one over the slope of the I–V characteristic. The units of resistance are *ohms*, abbreviated as Ω. Any device with a linear I–V characteristic is called a resistor.

The resistance of the device depends only on its physical properties – its size and composition. More specifically:

$$R = \rho \frac{L}{A} \tag{1.6}$$

Table 1.2 The resistivity of some common electronic materials

Material	ρ (10^{-8} Ωm)
Silver	1.6
Copper	1.7
Nichrome	100
Carbon	3500

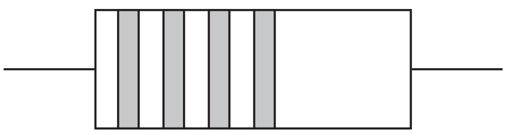

Figure 1.5 Value and tolerance bands on a resistor.

where ρ is the resistivity, L is the length, and A is the cross-sectional area of the material. The resistivity of some representative materials is given in Table 1.2.

The interconnecting wires or circuit board paths are typically made of copper or some other low resistivity material, so for most cases their resistance can be ignored. If we want resistance in a circuit we will use a discrete device made of some high resistivity material (e.g., carbon). Such resistors are widely used and can be obtained in a variety of values and power ratings. The low power rating resistors typically used in circuits are marked with color coded bands that give the resistance and the tolerance (i.e., the uncertainty in the resistance value) as shown schematically in Fig. 1.5.

As shown in the figure, the bands are usually grouped toward one end of the resistor. The band closest to the end is read as the first digit of the value. The next band is the second digit, the next band is the multiplier, and the last band is the tolerance value. The values associated with the various colors are shown in Table 1.3. For example, a resistor code having colors red, violet, orange, and gold corresponds to a value of $27 \times 10^3\ \Omega \pm 5\%$.

Resistors also come in variable forms. If the variable device has two leads, it is called a *rheostat*. The more common and versatile type with three leads is called a *potentiometer* or a "pot." Schematic symbols for resistors are shown in Fig. 1.6.

One must also select the proper power rating for a resistor. The power rating of common carbon resistors is indicated by the size of the device. Typical values are $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2 watts.

Table 1.3 Standard color scheme for resistors

Color	Digit	Multiplier	Tolerance (%)
none			20
silver		0.01	10
gold		0.1	5
black	0	1	
brown	1	10	
red	2	100	2
orange	3	10^3	
yellow	4	10^4	
green	5	10^5	
blue	6	10^6	
violet	7	10^7	
gray	8		
white	9		

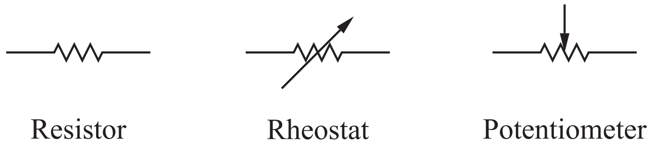

Figure 1.6 Schematic symbols for a fixed resistor and two types of variable resistors.

As noted in Eq. (1.3), the power consumed by a device is given by $P = VI$, but for resistors we also have the relation $V = IR$. Combining these we obtain two power relations specific to resistors:

$$P = I^2 R \tag{1.7}$$

and

$$P = V^2/R. \tag{1.8}$$

1.2.1 Equivalent circuit laws for resistors

It is common practice in electronics to replace a portion of a circuit with its functional equivalent. This often simplifies the circuit analysis for the remaining portion of the circuit. The following are some equivalent circuit laws for resistors.

1.2.1.1 Resistors in series

Components connected in series are connected in a head-to-tail fashion, thus forming a line or series of components. When forming equivalent circuits, any

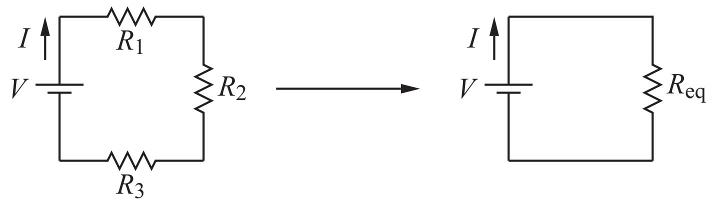

Figure 1.7 Equivalent circuit for resistors in series.

number of resistors in series may be replaced by a single equivalent resistor given by:

$$R_{eq} = \sum_i R_i \tag{1.9}$$

where the sum is over all the resistors in series. To see this, consider the circuit shown in Fig. 1.7. We would like to replace the circuit on the left by the equivalent circuit on the right. The circuit on the right will be equivalent if the current supplied by the battery is the same.

By KCL, the current in each resistor is the same. Applying KVL around the circuit loop and Ohm's Law for the drop across the resistors, we obtain

$$\begin{aligned} V &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \\ &= IR_{eq} \end{aligned} \tag{1.10}$$

where

$$R_{eq} = R_1 + R_2 + R_3. \tag{1.11}$$

This derivation can be extended to any number of resistors in series, hence Eq. (1.9).

1.2.1.2 Resistors in parallel

Components connected in parallel are connected in a head-to-head and tail-to-tail fashion. The components are often drawn in parallel lines, hence the name. When forming equivalent circuits, any number of resistors in parallel may be replaced by a single equivalent resistor given by:

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \tag{1.12}$$

where the sum is over all the resistors in parallel. To see this, consider the circuit shown in Fig. 1.8. Again, we would like to replace the circuit on the left by the equivalent circuit on the right.

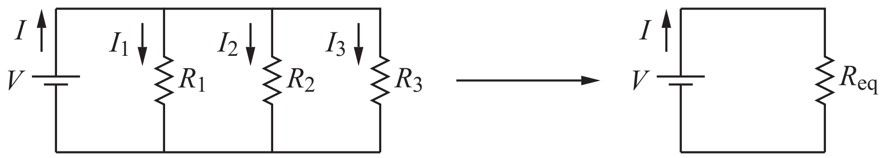

Figure 1.8 Equivalent circuit for resistors in parallel.

First, note that KCL requires

$$I = I_1 + I_2 + I_3. \tag{1.13}$$

Since the resistors are connected in parallel, the voltage across each one is the same, and, by KVL is equal to the battery voltage: $V = I_1R_1$, $V = I_2R_2$, $V = I_3R_3$. Solving these for the three currents and substituting in Eq. (1.13) gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_{eq}} \tag{1.14}$$

where

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{1.15}$$

Again, this derivation can be extended to any number of resistors in parallel, hence Eq. (1.12).

A frequent task is to analyze two resistors in parallel. Of course, for this special case of Eq. (1.12) we get $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$. It is often more illuminating to write this as an equation for R_{eq} rather than $\frac{1}{R_{eq}}$. After some algebra, we get

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}. \tag{1.16}$$

This special case is worth memorizing.

Example For the circuit shown in Fig. 1.9, how much current flows through the 20 kΩ resistor? What must its power rating be?

Solution As we will see, there is more than one way to solve this problem. Here we use a method that relies on basic electronics reasoning and our resistor equivalent circuit laws. We want the current through the 20 kΩ resistor. If we knew the voltage across this resistor (call this voltage V_{20k}), we could then get the current from Ohm's Law. In order to get the voltage across the 20 kΩ resistor, we need the voltage across the 10 kΩ resistor since, by KVL, $V_{20k} = 130 - V_{10k}$. In order to get the voltage across the 10 kΩ resistor, we need to know the current through

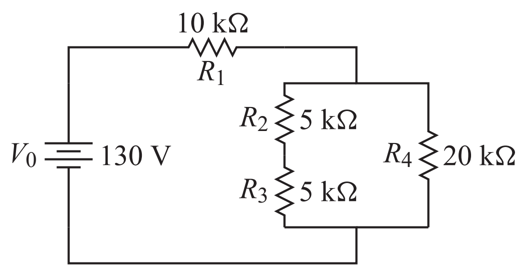

Figure 1.9 Example resistor circuit.

it, which is the same as the current supplied by the battery. Thus, if we can get the current supplied by the battery we can solve the problem. To get the battery current, we combine all our resistors into one equivalent resistor. The implementation of this strategy goes as follows.

1. Combine the two 5 kΩ series resistors into a 10 kΩ resistor.
2. This 10 kΩ resistor is then in parallel with the 20 kΩ resistor. Combining these we get (using Eq. (1.16))

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10\ \text{k}\Omega)(20\ \text{k}\Omega)}{10\ \text{k}\Omega + 20\ \text{k}\Omega} = 6.67\ \text{k}\Omega. \tag{1.17}$$

3. This 6.67 kΩ resistor is then in series with a 10 kΩ resistor, giving a total equivalent circuit resistance $R_{eq} = 16.67$ kΩ.
4. The current supplied by the battery is then

$$I = \frac{V_0}{R_{eq}} = \frac{130\ \text{V}}{16.67 \times 10^3\ \Omega} = 7.8 \times 10^{-3}\ \text{A} = 7.8\ \text{mA}. \tag{1.18}$$

5. KVL then gives 130 V − (7.8 mA)(10 kΩ) − V_{20k} = 0. Solving this gives $V_{20k} = 52$ V.
6. Ohm's Law then gives $I_{20k} = \frac{52\ \text{V}}{20\ \text{k}\Omega} = 2.6$ mA, which is the solution to the first part of our problem. As a check, it is comforting to note that this current is less than the total battery current, as it must be. The remainder goes through the two 5 kΩ resistors.
7. The power consumed by the 20 kΩ resistor is $P = I^2 R = (2.6 \times 10^{-3}\ \text{A})^2 (2 \times 10^4\ \Omega) = 0.135$ W. This is too much for a $\frac{1}{8}$ W resistor, so we must use at least a $\frac{1}{4}$ W resistor.

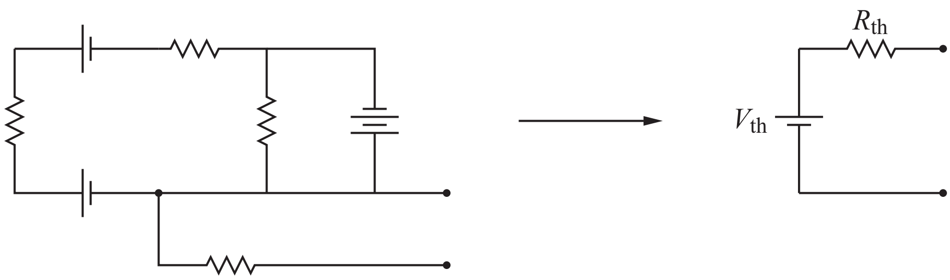

Figure 1.10 Representation of Thevenin's theorem.

1.2.1.3 Thevenin's theorem and Norton's theorem

The third of our equivalent circuit laws, Thevenin's theorem, is a more general result that actually includes the first two laws as special cases. The theorem states that any two-terminal network of sources and resistors can be replaced by a series combination of a single resistor R_{th} and voltage source V_{th}. This is represented by the example in Fig. 1.10. The sources can include both voltage and current sources (the current source is described below). A more general version of the theorem replaces the word *resistor* with *impedance*, a concept we will develop in Chapter 2.

The point of Thevenin's theorem is that when we connect a component to the terminals, it is much easier to analyze the circuit on the right than the circuit on the left. But there is no free lunch – we must first determine the values of V_{th} and R_{th}.

V_{th} is the voltage across the circuit terminals when nothing is connected to the terminals. This is clear from the equivalent circuit: if nothing is connected to the terminals, then no current flows in the circuit and there is no voltage drop across R_{th}. The voltage across the terminals is thus the same as V_{th}. In practice, the voltage across the terminals must be calculated by analyzing the original circuit.

There are two methods for calculating R_{th}; you can use whichever is easiest. In the first method, you start by short circuiting all the voltage sources and open circuiting all the current sources in the original circuit. This means that you replace the voltage sources by a wire and disconnect the current sources. Now only resistors are left in the circuit. These are then combined into one resistor using the resistor equivalent circuit laws. This one resistor then gives the value of R_{th}. In the second method, we calculate the current that would flow in the circuit if we shorted (placed a wire across) the terminals. Call this the short circuit current I_{sc}. Then from the Thevenin equivalent circuit it is clear that $R_{th} = \frac{V_{th}}{I_{sc}}$.

There is also a similar result known as Norton's theorem. This theorem states that any two-terminal network of sources and resistors can be replaced by a parallel