STRUCTURES IN THE UNIVERSE BY EXACT METHODS

Formation, Evolution, Interactions

As the structures in our Universe are mapped out on ever larger scales, and with increasing detail, the use of inhomogeneous models is becoming an essential tool for analysing and understanding them.

This book reviews a number of important developments in the application of inhomogeneous solutions of Einstein's field equations to cosmology. It shows how inhomogeneous models can be used to study the evolution of structures such as galaxy clusters and galaxies with central black holes, and to account for cosmological observations such as supernova dimming, the cosmic microwave background, baryon acoustic oscillations or the dependence of the Hubble parameter on redshift within classical general relativity.

Whatever 'dark matter' and 'dark energy' turn out to be, inhomogeneities exist on many scales and need to be investigated by all appropriate methods. This book will be of great value to astrophysicists and cosmologists, from graduate students to academic researchers.

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Formation, Evolution, Interactions

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Foreword

Cosmology, from its very birth, has been a science beset by insufficient data and ill-founded tenets. Faced with extreme difficulties in collecting observational data, it used to rely on simplifying working assumptions. Those assumptions had the tendency to evolve into dogmas or into elaborate theoretical constructs, immersed in which their practitioners all too easily forgot about these shaky foundations. It is enough to recall the original cosmological paper of Einstein from 1917, who, swayed by prevailing beliefs, was absolutely sure that the Universe must be spatially uniform and unchanging in time – so much so that he preferred to modify his freshly created theory rather than say that it contradicts the astronomical dogma. Another instructive example is the steady state theory. It had been very much in vogue for about 20 years, before it was proved wrong by a single discovery, that of the cosmic microwave background (CMB) radiation. It was glamorous and successful, in the eyes of its proponents, even though it relied on an assumption that is drastically at variance with laboratory physics (continuous creation of matter particles out of nothing).

In spite of such cautionary examples, this tendency continues. Today we are told that the Universe is perfectly homogeneous 'at sufficiently large scales', although the actual size of those 'scales' is not better defined than as 'a few' hundred Mpc. We are told that, consequently, the Friedmann–Lemaître–Robertson–Walker (FLRW) models[1] are the perfect models of the Universe that do not need any modification other than the perturbations needed to account for the formation of structures. By sticking to the FLRW models we are led to believe that more than 95% of the energy density of our Universe consists of entities that no-one has yet seen in any laboratory – aptly called 'dark matter' and 'dark energy'. We are told that the Universe had to go through a phase of exponential expansion called inflation, to 'explain' its extreme homogeneity today. Almost everybody happily believes this, forgetting that the theory postulating this

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[1] The term 'FLRW models' will be used to denote the general Robertson–Walker class of metrics, with an unspecified scale factor, and solutions of Einstein's equations within this class with a given equation of state $p(\rho)$. The term 'Friedmann models' will denote the dust ($p = 0$) Robertson–Walker models, also when $\Lambda \neq 0$.

phenomenon (I) uses scalar fields which up to now have never been either observed or theoretically characterised; (II) cannot explain how this exponential expansion stopped; (III) has relied on explicit models that are homogeneous from the very beginning, so the process of homogenisation has in fact never been demonstrated; (IV) requires that the duration of the inflationary phase is fine tuned so that the initial variations in the energy density of the inflating region are smoothed out at the end.

Our book aims at showing that good old experimentally verified gravitation theory has not yet been taken to its limits in application to cosmology. Well-established physics can explain several of the phenomena observed by cosmologists without introducing highly speculative elements, like dark matter, dark energy, exponential expansion at densities never attained in any experiment, and the like. We are aware that at some point the Einstein theory may need modification or generalisation. However, we believe that we should undertake such changes only when we really hit a brick wall – i.e. encounter difficulties that cannot be overcome in any other way. As long as the existing theory can explain things, modifications should be laid aside. This has been a good practice in physics before the modern methods of marketing entered our science. This is not to say speculative models should not be explored, rather that they should not displace well-established physics until their theoretical and observational foundations are properly established.

We expect our readers to be well versed in the relativity theory. Also, our book is not intended to be a textbook on relativistic cosmology – we assume quite a bit of prior expertise also in this field. Readers who need an introduction to these subjects will have to consult other sources. There are many good textbooks on relativity on the market, and also several textbooks on cosmology, although most of the latter are strongly biased toward the modern ideas which we criticise. Readers in need of a quick introduction may wish to use the textbook by Plebański and Krasiński (2006) – it may be an easier reading because it has a similar style, the same conventions and the same notation as the present book, and parts of the introductory material here are discussed there at more length. In particular, this applies to the Robertson–Walker models that are not given any introduction here.

This book is mostly based on our own investigations, but we did pay attention to related work by other authors. However, we have included only the applications of exact solutions. Although the averaging methods seem to be very promising in dealing with inhomogeneities, the averaging procedure still lacks a generally accepted methodology. Many interesting aspects of averaging are not uniquely defined and need to be compared with exact solutions. So, while we are looking at this subject with sympathy, we must refer our readers to works by other authors, such as those reported in the articles by

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Buchert (2000, 2001); Buchert and Carfora (2002, 2003); Wiltshire (2007a,b), and in the reviews by Räsänen (2006b) and Buchert (2008), and references cited therein.

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