I

Inputs to the Standard Model

This book is about the Standard Model of elementary particle physics. If we set the beginning of the modern era of particle physics in 1947, the year the pion was discovered, then the ensuing years of research have revealed the existence of a consistent, self-contained layer of reality. The energy range which defines this layer of reality extends up to about 1 TeV or, in terms of length, down to distances of order 10^{-17} cm. The Standard Model is a field-theoretic description of strong and electroweak interactions at these energies. It requires the input of as many as 28 independent parameters.[1] These parameters are not explained by the Standard Model; their presence implies the need for an understanding of Nature at an even deeper level. Nonetheless, processes described by the Standard Model possess a remarkable insulation from signals of such New Physics. Although the strong interactions remain a calculational challenge, the Standard Model (generalized from its original form to include neutrino mass) would appear to have sufficient content to describe all existing data.[2] Thus far, it is a theoretical structure which has worked splendidly.

I–1 Quarks and leptons

The Standard Model is an $SU(3) \times SU(2) \times U(1)$ gauge theory which is spontaneously broken by the Higgs potential. Table I–1 displays mass determinations [RPP 12] of the Z^0 and $W^{\pm}$ gauge bosons, the Higgs boson H^0, and the existing mass limit on the photon γ.

In the Standard Model, the fundamental fermionic constitutents of matter are the quarks and the leptons. Quarks, but not leptons, engage in the strong interactions as a consequence of their color charge. Each quark and lepton has spin one-half.

[1] There are six lepton masses, six quark masses, three gauge coupling constants, three quark-mixing angles and one complex phase, three neutrino-mixing angles and as many as three complex phases, a Higgs mass and quartic coupling constant, and the *QCD* vacuum angle.

[2] Admittedly, at this time the sources of *dark matter* and of *dark energy* are unknown.

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Table I–1. *Boson masses.*

Particle	Mass (GeV/c^2)
γ	$< 1 \times 10^{-27}$
$W^\pm$	80.385 ± 0.015
Z^0	91.1876 ± 0.0021
H^0	126.0 ± 0.4

Collectively, they display conventional Fermi–Dirac statistics. No attempt is made in the Standard Model either to explain the variety and number of quarks and leptons or to compute any of their properties. That is, these particles are taken at this level as truly elementary. This is not unreasonable. There is no experimental evidence for quark or lepton compositeness, such as excited states or form factors associated with intrinsic structure.

Quarks

There are six quarks, which fall into two classes according to their electrical charge Q. The $u,\ c,\ t$ quarks have $Q = 2e/3$ and the $d,\ s,\ b$ quarks have $Q = -e/3$, where e is the electric charge of the proton. The $u,\ c,\ t$ and $d,\ s,\ b$ quarks are eigenstates of the hamiltonian ('mass eigenstates'). However, because they are believed to be permanently confined entities, some thought must go into properly defining quark mass. Indeed, several distinct definitions are commonly used. We defer a discussion of this issue and simply note that the values in Table I–2 provide

Table I–2. *The quarks.*

Flavor	Mass[a] (GeV/c^2)	Charge	I_3	S	C	B	T
u	$(2.55^{+0.75}_{-1.05}) \times 10^{-3}$	2e/3	1/2	0	0	0	0
d	$(5.04^{+0.96}_{-1.54}) \times 10^{-3}$	−e/3	−1/2	0	0	0	0
s	$0.105^{+0.025}_{-0.035}$	−e/3	0	−1	0	0	0
c	$1.27^{+0.07}_{-0.11}$	2e/3	0	0	1	0	0
b	$4.20^{+0.17}_{-0.07}$	−e/3	0	0	0	−1	0
t	173.4 ± 1.6	2e/3	0	0	0	0	1

[a]The t-quark mass is inferred from top quark events. All others are determined in $\overline{\text{MS}}$ renormalization (cf. Sect. II–1) at scales $m_{u,d,s}(2\ \text{GeV}/c^2)$, $m_c(m_c)$ and $m_b(m_b)$ respectively.

Table I–3. *The leptons.*

Flavor	Mass (GeV/c^2)	Charge	L_e	L_μ	L_τ
ν_e	$< 0.2 \times 10^{-8}$	0	1	0	0
e	$5.10998928(11) \times 10^{-4}$	−e	1	0	0
ν_μ	$< 1.9 \times 10^{-4}$	0	0	1	0
μ	0.1056583715(35)	−e	0	1	0
ν_τ	< 0.0182	0	0	0	1
τ	1.77682(16)	−e	0	0	1

an overview of the quark mass spectrum. A useful benchmark for quark masses is the energy scale Λ_{QCD}($\simeq$ several hundred MeV) associated with the confinement phenomenon. Relative to Λ_{QCD}, the $u,\ d,\ s$ quarks are light, the $b,\ t$ quarks are heavy, and the c quark has intermediate mass. The dynamical behavior of light quarks is described by the chiral symmetry of massless particles (cf. Chap. VI) whereas heavy quarks are constrained by the so-called Heavy Quark Effective Theory (cf. Sect. XIII–3). Each quark is said to constitute a separate *flavor*, i.e. six quark flavors exist in Nature. The $s,\ c,\ b,\ t$ quarks carry respectively the quantum numbers of strangeness (S), charm (C), bottomness (B), and topness (T). The $u,\ d$ quarks obey an $SU(2)$ symmetry (isospin) and are distinguished by the three-component of isospin (I_3). The flavor quantum numbers of each quark are displayed in Table I–2.

Leptons

There are six leptons which fall into two categories according to their electrical charge. The charged leptons $e,\ \mu,\ \tau$ have $Q = -e$ and the neutrinos $\nu_e,\ \nu_\mu,\ \nu_\tau$ have $Q = 0$. Leptons are also classified in terms of three lepton types: electron (ν_e, e), muon (ν_μ, μ), and tau (ν_τ, τ). This follows from the structure of the charged weak interactions (cf. Sect. II–3) in which these charged-lepton/neutrino pairs are coupled to $W^\pm$ gauge bosons. Associated with each lepton type is a lepton number L_e, L_μ, L_τ. Table I–3 summarizes lepton properties.

At this time, there is only incomplete knowledge of neutrino masses. Information on the mass parameters $m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}$ is obtained from their presence in various weak transition amplitudes. For example, the single beta decay experiment ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \overline{\nu}_e$ is sensitive to the mass m_{ν_e}. In like manner, one constrains the masses m_{ν_μ} and m_{ν_τ} in processes such as $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\tau^- \rightarrow 2\pi^- + \pi^+ + \nu_\tau$ respectively. Existing bounds on these masses are displayed in Table I–3.

It is known experimentally that upon creation the neutrinos $\{\nu_\alpha\} \equiv (\nu_e.\nu_\mu, \nu_\tau)$ will not propagate indefinitely but will instead mix with each other. This means that the basis of states $\{\nu_\alpha\}$ cannot be eigenstates of the hamiltonian. Diagonalization of the leptonic hamiltonian is carried out in Sect. VI–2 and yields the basis $\{\nu_i\} \equiv \{\nu_1, \nu_2, \nu_3\}$ of mass eigenstates. Information on the neutrino mass eigenvalues m_1, m_2, m_3 is obtained from neutrino oscillation experiments and cosmological studies. Oscillation experiments (cf. Sects. VI–3,VI–4) are sensitive to squared-mass differences.[3] Throughout the book, we adhere to the following relations,

$$\text{definition: } \Delta m_{ij}^2 \equiv m_i^2 - m_j^2, \qquad \text{convention: } m_2 > m_1. \tag{1.1}$$

From the compilation in [RPP 12], the squared-mass difference $|\Delta m_{32}^2|$ deduced from the study of atmospheric and accelerator neutrinos gives

$$|\Delta m_{32}^2| = 2.32^{+0.12}_{-0.08} \times 10^{-3}\ \text{eV}^2, \tag{1.2a}$$

whereas data from solar and reactor neutrinos imply a squared-mass difference roughly 31 times smaller,

$$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5}\ \text{eV}^2. \tag{1.2b}$$

Thus the neutrinos ν_1 and ν_2 form a quasi-doublet. One speaks of a *normal* or *inverted* neutrino mass spectrum, respectively, for the cases[4]

$$\text{normal: } m_3 > m_{1,2}, \qquad \text{inverted: } m_{1,2} > m_3. \tag{1.2c}$$

Since the largest neutrino mass $m_{\rm lgst}$, be it m_2 or m_3, cannot be lighter than the mass splitting of Eq. (1.2), we have the bound $m_{\rm lgst} > 0.049$ eV. Finally, a combination of cosmological inputs can be employed to bound the neutrino mass sum $\sum_{i=1}^3 m_i$, the precise bound depending on the chosen input data set. In one example [deP *et al*. 12], photometric redshifts measured from a large galaxy sample, cosmic microwave background (CMB) data and a recent determination of the Hubble parameter are used to obtain the bound

$$m_1 + m_2 + m_3 < 0.26\ \text{eV}, \tag{1.3a}$$

whereas data from the CMB combined with that from baryon acoustic oscillations yields [Ad *et al.* (Planck collab.) 13]

$$m_1 + m_2 + m_3 < 0.23\ \text{eV}. \tag{1.3b}$$

A further discussion of the neutrino mass spectrum appears in Sect. VI–4.

[3] Only two of the mass differences can be independent, so $\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$.

[4] There is also the possibility of a *quasi-degenerate* neutrino mass spectrum ($m_1 \simeq m_2 \simeq m_3$), which can be thought of as a limiting case of both the normal and inverted cases in which the individual masses are sufficiently large to dwarf the $|\Delta m_{32}^2|$ splitting.

Quark and lepton numbers

Individual quark and lepton numbers are known to be not conserved, but for different reasons and with different levels of nonconservation. Individual quark number is not conserved in the Standard Model due to the charged weak interactions (cf. Sect. II–3). Indeed, quark transitions of the type $q_i \rightarrow q_j + W^{\pm}$ induce the decays of most meson and baryon states and have led to the phenomenology of *Flavor Physics*. Individual lepton number is not conserved, as evidenced by the observed $\nu_\alpha \leftrightarrow \nu_\beta$ $(\alpha, \beta = e, \mu, \tau)$ oscillations. This source of this phenomenon is associated with nonzero neutrino masses. There is currently no additional evidence for the violation of individual lepton number despite increasingly sensitive limits such as the branching fraction $\mathrm{B}_{\mu^- \to e^- e^- e^+} < 1.0 \times 10^{-12}$.

Existing data are consistent with conservation of total quark and total lepton number, e.g. the proton lifetime bound $\tau_p > 2.1 \times 10^{29}$ yr [RPP 12] and the nuclear half-life limit $t_{1/2}^{0\nu\beta\beta}[^{136}\mathrm{Xe}] > 1.6 \times 10^{25}$ yr [Ac *et al.* (EXO-200 collab.) 11]. These conservation laws are empirical. They are not required as a consequence of any known dynamical principle and in fact are expected to be violated by certain nonperturbative effects within the Standard Model (associated with quantum tunneling between topologically inequivalent vacua – see Sect. III–6).

I–2 Chiral fermions

Consider a world in which quarks and leptons have no mass at all. At first, this would appear to be a surprising supposition. To an experimentalist, mass is the most palpable property a particle has. It is why, say, a muon behaves differently from an electron in the laboratory. Nonetheless, the massless limit is where the Standard Model begins.

The massless limit

Let $\psi(x)$ be a solution to the Dirac equation for a massless particle,

$$i \not\partial \psi = 0. \tag{2.1}$$

We can multiply this equation from the left by γ_5 and use the anticommutativity of γ_5 with γ^μ to obtain another solution,

$$i \not\partial \gamma_5 \psi = 0. \tag{2.2}$$

We superpose these solutions to form the combinations

$$\psi_L = \frac{1}{2}(1 + \gamma_5)\psi, \qquad \psi_R = \frac{1}{2}(1 - \gamma_5)\psi, \tag{2.3}$$

where '1' represents the unit 4×4 matrix. The quantities ψ_L and ψ_R are solutions of definite *chirality* (i.e. handedness). For a massless particle moving with precise momentum, these solutions correspond respectively to the spin being anti-aligned (left-handed) and aligned (right-handed) relative to the momentum. In other words, chirality coincides with helicity for zero-mass particles. The matrices $\Gamma_{\substack{L\\R}} = (1 \pm \gamma_5)/2$ are chirality projection operators. They obey the usual projection operator conditions under addition,

$$\Gamma_L + \Gamma_R = 1, \tag{2.4}$$

and under multiplication,

$$\Gamma_L \Gamma_L = \Gamma_L, \qquad \Gamma_R \Gamma_R = \Gamma_R, \qquad \Gamma_L \Gamma_R = \Gamma_R \Gamma_L = 0. \tag{2.5}$$

In the massless limit, a particle's chirality is a Lorentz-invariant concept. For example, a particle which is left-handed to one observer will appear left-handed to all observers. Thus chirality is a natural label to use for massless fermions, and a collection of such particles may be characterized according to the separate numbers of left-handed and right-handed particles.

It is simple to incorporate chirality into a lagrangian formalism. The lagrangian for a massless noninteracting fermion is

$$\mathcal{L} = i \overline{\psi} \not{\partial} \psi, \tag{2.6}$$

or in terms of chiral fields,

$$\mathcal{L} = \mathcal{L}_L + \mathcal{L}_R, \tag{2.7}$$

where

$$\mathcal{L}_{L,R} = i \overline{\psi}_{L,R} \not{\partial} \psi_{L,R}. \tag{2.8}$$

The lagrangians $\mathcal{L}_{L,R}$ are invariant under the global chiral phase transformations

$$\psi_{L,R}(x) \to \exp(-i\alpha_{L,R}) \psi_{L,R}(x), \tag{2.9}$$

where the phases $\alpha_{L,R}$ are constant and real-valued but otherwise arbitrary. Anticipating the discussion of Noether's theorem in Sect. I–4, we can associate conserved particle-number current densities $J^\mu_{L,R}$,

$$J^\mu_{L,R} = \overline{\psi}_{L,R} \gamma^\mu \psi_{L,R} \qquad (\partial_\mu J^\mu_{L,R} = 0), \tag{2.10}$$

with this invariance. From these chiral current densities, we can construct the vector current $V^\mu(x)$,

$$V^\mu = J^\mu_L + J^\mu_R \tag{2.11}$$

and the axial-vector current $A^\mu(x)$,

$$A^\mu = J_L^\mu - J_R^\mu. \tag{2.12}$$

Chiral charges $Q_{L,R}$ are defined as spatial integrals of the chiral charge densities,

$$Q_{L,R}(t) = \int d^3x \; J^0_{L,R}(x), \tag{2.13}$$

and represent the number operators for the chiral fields $\psi_{L,R}$. They are time-independent if the chiral currents are conserved. One can similarly define the vector charge Q and the axial-vector charge Q_5,

$$Q(t) = \int d^3x \; V^0(x), \qquad Q_5(t) = \int d^3x \; A^0(x). \tag{2.14}$$

The vector charge Q is the total number operator,

$$Q = Q_R + Q_L, \tag{2.15}$$

whereas the axial-vector charge is the number operator for the difference

$$Q_5 = Q_L - Q_R. \tag{2.16}$$

The vector charge Q and axial-vector charge Q_5 simply count the sum and difference, respectively, of the left-handed and right-handed particles.

Parity, time reversal, and charge conjugation

The field transformations of Eq. (2.9) involve parameters $\alpha_{L,R}$ which can take on a continuum of values. In addition to such continuous field mappings, one often encounters a variety of *discrete* transformations as well. Let us consider the operations of parity

$$x = (x^0, \mathbf{x}) \to x_P = (x^0, -\mathbf{x}), \tag{2.17}$$

and of time reversal

$$x = (x^0, \mathbf{x}) \to x_T = (-x^0, \mathbf{x}), \tag{2.18}$$

as defined by their effects on spacetime coordinates. The effect of discrete transformations on a fermion field $\psi(x)$ will be implemented by a unitary operator P for parity and an antiunitary operator T for time reversal. In the representation of Dirac matrices used in this book, we have

$$P\psi(x)P^{-1} = \gamma^0\psi(x_P), \qquad T\psi(x)T^{-1} = i\gamma^1\gamma^3\psi(x_T). \tag{2.19}$$

An additional operation typically considered in conjunction with parity and time reversal is that of charge conjugation, the mapping of matter into antimatter,

$$C\psi(x)C^{-1} = i\gamma^2\gamma^0\overline{\psi}^T(x), \tag{2.20}$$

Table I–4. *Response of Dirac bilinears to discrete mappings.*

C	P	T
$S(x)$	$S(x_P)$	$S(x_T)$
$P(x)$	$-P(x_P)$	$-P(x_T)$
$-J^\mu(x)$	$J_\mu(x_P)$	$J_\mu(x_T)$
$J_5^\mu(x)$	$-J_{5\mu}(x_P)$	$J_{5\mu}(x_T)$
$-T^{\mu\nu}(x)$	$T_{\mu\nu}(x_P)$	$-T_{\mu\nu}(x_T)$

where $\overline{\psi}_\beta^T \equiv \psi_\alpha^\dagger \gamma_{\alpha\beta}^0$ $(\alpha, \beta = 1, \ldots, 4)$. The spacetime coordinates of field $\psi(x)$ are unaffected by charge conjugation.

In the study of discrete transformations, the response of the normal-ordered Dirac bilinears

$$\begin{aligned} S(x) &= :\overline{\psi}(x)\psi(x): & P(x) &= :\overline{\psi}(x)\gamma_5\psi(x): \\ J^\mu(x) &= :\overline{\psi}(x)\gamma^\mu\psi(x): & J_5^\mu(x) &= :\overline{\psi}(x)\gamma^\mu\gamma_5\psi(x): \\ T^{\mu\nu}(x) &= :\overline{\psi}(x)\sigma^{\mu\nu}\psi(x): \end{aligned} \tag{2.21}$$

is of special importance to physical applications. Their transformation properties appear in Table I–4. Close attention should be paid there to the location of the indices in these relations. Another example of a field's response to these discrete transformations is that of the photon $A^\mu(x)$,

$$\begin{aligned} C\ A^\mu(x)\ C^{-1}\ c &= -A^\mu(x), & P\ A^\mu(x)\ P^{-1} &= \ A_\mu(x_P), \\ T\ A^\mu(x)\ T^{-1}\ c &= \ A_\mu(x_T). \end{aligned} \tag{2.22}$$

Beginning with the discussion of Noether's theorem in Sect. 1–4, we shall explore the topic of invariance throughout much of this book. It suffices to note here that the Standard Model, being a theory whose dynamical content is expressed in terms of hermitian, Lorentz-invariant lagrangians of local quantum fields, is guaranteed to be invariant under the combined operation *CPT*. Interestingly, however, these discrete transformations are individually symmetry operations only of the strong and electromagnetic interactions, but not of the full electroweak sector. We see already the possibility for such behavior in the occurrence of chiral fermions $\psi_{L,R}$, since parity maps the fields $\psi_{L,R}$ into each other,

$$\psi_{L,R} \to P\ \psi_{L,R}(x)\ P^{-1} = \gamma^0 \psi_{R,L}(x_P). \tag{2.23}$$

Thus any effect, like the weak interaction, which treats left-handed and right-handed fermions differently, will lead inevitably to parity-violating phenomena.

I–3 Fermion mass

Although the discussion of chiral fermions is cast in the limit of zero mass, fermions in Nature do in fact have nonzero mass and we must account for this. In a lagrangian, a mass term will appear as a hermitian, Lorentz-invariant bilinear in the fields. For fermion fields, these conditions allow realizations referred to as *Dirac* mass and *Majorana* mass.[5]

Dirac mass

The Dirac mass term for fermion fields $\psi_{L,R}$ involves the bilinear coupling of fields with opposite chirality

$$-\mathcal{L}_D = m_D[\,\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\,] = m_D\,\overline{\psi}\psi \tag{3.1}$$

where $\psi \equiv \psi_L + \psi_R$ and m_D is the Dirac mass. The Dirac mass term is invariant under the phase transformation $\psi(x) \rightarrow \exp(-i\alpha)\psi(x)$ and thus does not upset conservation of the vector current $V^\mu = \overline{\psi}\gamma^\mu\psi$ and the corresponding number fermion operator Q of Eq. (2.15). All fields in the Standard Model, save possibly for the neutrinos, have Dirac masses obtained from their interaction with the Higgs field (cf. Sects. II–3, II–4). Although right-handed neutrinos have no couplings to the Standard Model gauge bosons, there is no principle prohibiting their interaction with the Higgs field and thus generating neutrino Dirac masses in the same manner as the other particles.

Majorana mass

A Majorana mass term is one which violates fermion number by coupling two fermions (or two antifermions). In the Majorana construction, use is made of the *charge-conjugate* fields,

$$\psi^c \equiv C\gamma^0\psi^*, \qquad (\psi_{L,R})^c = (\Gamma_{L,R}\psi)^c, \tag{3.2}$$

where C is the charge-conjugation operator, obeying

$$C = -C^{-1} = -C^\dagger = -C^T. \tag{3.3}$$

In the Dirac representation of gamma matrices (cf. App. C), one has $C = i\gamma^2\gamma^0$. Some useful identities involving ψ^c include

[5] We suppress spacetime dependence of the fields in this section.

$$\begin{aligned}
\overline{(\psi_i^c)}\,\psi_j &= \psi_i^T\,C\,\psi_j, & \overline{\psi_i}\,\psi_j^c &= -\psi_i^{*T}\,C\,\psi_j^*,\\
\left(\overline{(\psi_i^c)}\,\psi_j\right)^\dagger &= \overline{\psi_j}\,\psi_i^c, & \left(\psi_i^T\,C\,\psi_j\right)^\dagger &= -\psi_j^{*T}\,C\,\psi_i^*,\\
\overline{(\psi_i^c)}\,\psi_j^c &= \overline{\psi_j}\,\psi_i, & \overline{(\psi_i^c)}\,\gamma^\mu\,\psi_j^c &= -\overline{\psi_j}\,\gamma^\mu\,\psi_i,\\
\overline{(\psi_R^c)}\,\psi_L &= 0, & \overline{(\psi_R^c)}\,\gamma^\mu\,\psi_R &= 0.
\end{aligned} \tag{3.4}$$

The two identities in the bottom line follow from $\Gamma_R C \Gamma_L = 0$.

The possibility of a Majorana mass term follows from the fact that a combination of two fermion fields $\psi^T C\psi$ is an invariant under Lorentz transformations. Two equivalent expressions for a Majorana mass term involving chiral fields $\psi_{L,R}$ are[6]

$$\begin{aligned}
-\mathcal{L}_M &= \frac{m_{L,R}}{2}\left[\overline{(\psi_{L,R})^c}\,\psi_{L,R} + \overline{\psi_{L,R}}\,(\psi_{L,R})^c\right]\\
&= \frac{m_{L,R}}{2}\left[(\psi_{L,R})^T C\psi_{L,R} - (\psi_{L,R}^*)^T C\psi_{L,R}^*\right].
\end{aligned} \tag{3.5}$$

Because the cross combination $(\psi_R)^T C\psi_L = 0$, the Majorana mass terms involves either two left-chiral fields or two right-chiral fields, and the left-chiral and right-chiral masses are independent. Treating ψ and ψ^* as independent variables, the resulting equations of motion are

$$i\partial\!\!\!/\,\psi_R - m_R\,\psi_R^c = 0, \qquad i\partial\!\!\!/\,\psi_R^c - m_R\,\psi_R = 0, \tag{3.6}$$

with a similar set of equations for ψ_L. Iteration of these coupled equations shows that m_R indeed behaves as a mass.

A Majorana mass term clearly does not conserve fermion number and mixes the particle with its antiparticle. Indeed, a Majorana fermion can be identified with its own antiparticle. This can be seen, using ψ_R as an example, by rewriting the lagrangian in terms of the self-conjugate field

$$\psi_M = \frac{1}{\sqrt{2}}\left[\psi_R + \psi_R^c\right], \tag{3.7}$$

which, given the equations of motion above, will clearly satisfy the Dirac equation. The total Majorana lagrangian can be simply rewritten in terms of this self-conjugate field as

$$\begin{aligned}
\mathcal{L}_{KE}^{(\mathrm{R})} + \mathcal{L}_M^{(\mathrm{R})} &= \overline{\psi}_R i\partial\!\!\!/\,\psi_R - \frac{m_R}{2}\left[\overline{(\psi_R)^c}\,\psi_R + \overline{\psi_R}\,(\psi_R)^c\right]\\
&= \overline{\psi}_M i\partial\!\!\!/\,\psi_M - m_R\overline{\psi}_M\,\psi_M\\
&= \psi_M^T C i\partial\!\!\!/\,\psi_M - m_R\psi_M^T C\,\psi_M,
\end{aligned} \tag{3.8}$$

[6] The factor of $1/2$ with the Majorana mass parameters $m_{L,R}^{(\mathrm{M})}$ compensates for a factor of 2 encountered in taking the matrix element of the Majorana mass term.