

## PARTICLE PHYSICS OF BRANE WORLDS AND EXTRA DIMENSIONS

The possibility that we live in a higher-dimensional world with spatial dimensions greater than three started with the early work of Kaluza and Klein. However, in addressing experimental constraints, early model-builders were forced to compactify these extra dimensions to very tiny scales. With the development of brane-world scenarios, it became possible to consider novel compactifications which allow the extra dimensions to be large or to provide observable effects of these dimensions at experimentally accessible energy scales.

This book provides a comprehensive account of these recent developments, keeping the high-energy physics implications in focus. After a historical survey of the idea of extra dimensions, the book deals in detail with models of large extra dimensions, warped extra dimensions and other models such as universal extra dimensions. The theoretical and phenomenological implications are discussed in a pedagogical manner for both researchers and graduate students.

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Cambridge University Press

978-0-521-76856-6 - Particle Physics of Brane Worlds and Extra Dimensions

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<sup>†</sup> Available in paperback

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978-0-521-76856-6 - Particle Physics of Brane Worlds and Extra Dimensions

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University Printing House, Cambridge CB2 8BS, United Kingdom

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[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521768566](http://www.cambridge.org/9780521768566)

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First published 2016

Printed in the United Kingdom by Clays, St Ives plc

*A catalogue record for this publication is available from the British Library.*

*Library of Congress Cataloguing in Publication data*

Names: Raychaudhuri, Sreerup, author. | Sridhar, K., author.

Title: Particle physics of brane worlds and extra dimensions / Sreerup Raychaudhuri (Tata Institute of Fundamental Research, Mumbai), K. Sridhar (Tata Institute of Fundamental Research, Mumbai).

Description: Cambridge, United Kingdom ; New York, NY : Cambridge University Press, 2016. | © 2016 | Includes bibliographical references and index.

Identifiers: LCCN 2016011199 | ISBN 9780521768566 (alk. paper) | ISBN 052176856X (alk. paper)

Subjects: LCSH: Particles (Nuclear physics) | Branes. | String models. | Fourth dimension.

Classification: LCC QC793.2 .R39 2016 | DDC 539.7/2-dc23

LC record available at <http://lcn.loc.gov/2016011199>

ISBN 978-0-521-76856-6 Hardback

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*To Gita, Ira and Oindrila*

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## Preface

The gods love what is mysterious, and dislike what is evident.

*Brihadaranyaka Upanishad IV.2.2*

For 49 out of the 50 centuries of recorded history, the idea of extra dimensions of space was the province of the mystic, the crank and the trickster. For what could be more obvious than the fact that there are three dimensions – length, breadth and height – as every schoolchild knows? Take a brick, for example, which is so reassuringly solid, and seems so uncompromisingly three-dimensional. Yet, for the last century or so, modern atomic science has been telling us that the solid brick is 99.9999999999% empty space. What remains is some little bits of matter, flying around eternally, and held together by electromagnetic pulls and pushes of incredible strength. This may seem bizarre and unreal, but that's how it is.

We have learnt, therefore, that 'common sense' – or rather, the evidence of our five senses – is not a trustworthy guide to the world when we try to perceive it at length scales much below the size of an average human. Once we are trained to have an open mind in his way, there is no problem at all in conceiving of more dimensions of space – they just have to be curled up into tiny loops, or spheres or higher dimensional compact objects. If the size of these is below the resolution of the human eye (or any instruments which aid the human eye), these extra dimensions will be invisible, just as atoms and molecules were at the time they were conceived of.

Most scientists encounter curled-up dimensions without realising it, when they study the quantum physics of solids. There, one can set up a wave function of an electron in a solid by invoking a 'periodic boundary condition', which is another way of saying that the co-ordinates are curled up. In fact, a three-dimensional solid is actually treated in the textbooks as a 3-torus, which is a complicated topological object that cannot be drawn on paper. If we can get useful and experimentally verifiable results by assuming a block of copper, for example, to be a 3-torus, surely it is not too much of a flight of fancy to assume that the selfsame block also has other dimensions, similarly curled-up, which are too small to be seen?

This fascinating question was raised in the early half of the twentieth century, and it continues, after a hundred years, to excite the curiosity of the researcher. During this period, extra dimensions have come (and sometimes gone) in various

guises, first as a route to unification of forces, then as a support to string theories, later as a solution to the gauge-hierarchy problem and finally as an explanation of dark matter. As someone was heard to remark, extra dimensions have been used to do everything except make coffee!

The authors of this work knew, therefore, when undertaking to write a book on this subject, that it would not be an easy task. For the subject of extra dimensions, apart from having a long history, has grown today to span almost every branch of theoretical high-energy physics. Some of the developments are highly mathematical while others are purely experimental. We have, therefore, tried to create a work which reflects the flavour of our own laboured journey through this subject. Part of the development is pedagogic, and part is more like a monograph. We have deliberately chosen not to try to be comprehensive and cover everything in this area, for, apart from the problem of feasibility, this would swamp the reader with an intimidating amount of information.

This book is meant for graduate students and researchers in high-energy physics who wish to learn about extra dimensions and their impact on high-energy physics research in the last 15 years or so. After a brief warm-up, where the historical origins of extra dimensions are narrated, we introduce the early theories of Kaluza and Klein, and their immediate followers. This is followed by reviews of the Standard Model of particle physics, of string theory, and of effective theories, which form, so to say, the flesh that clothes the skeletal framework of extra dimensions. We then describe the three major paradigms developed over the last decade and a half, viz. large (flat) extra dimensions, universal (flat) extra dimensions and, finally, warped extra dimensions. In each case, we describe the theoretical model, show how to obtain the particle masses and interactions and then discuss how it may be possible to observe them, both in terrestrial experiments and in astrophysical observations, as the case may be.

Not suprisingly, this book has been a long time in the making. During this period, and before, we have been helped in various ways by so many people that it is a difficult task to remember all those whose names we need to include among our list of acknowledgements.

Primary thanks go to all our collaborators, Ben Allanach, Gautam Bhattacharyya, Biplob Bhattacharjee, Prasanta Kumar Das, Amit Chakraborty, Anindya Datta, Sukanta Datta, Dilip Kumar Ghosh, Rohini M. Godbole, Monoranjan Guchait, Ambreesh K. Gupta, Abhishek Iyer, Partha Konar, Anirban Kundu, Smaragda Lola, Uma Mahanta, Farvah Nazila Mahmoudi, Ushoshi Maitra, Namrata Manglani, Prakash Mathews, Naba K. Mondal, Biswarup Mukhopadhyaya, Ayon Patra, Poulouse Poulouse, Santosh Kumar Rai, V. Ravindran, Dhruv Ringe, Sophie Renner, Tousik Samui, Saswati Sarkar, Jordan P. Skittrall, W. L. van Neerven and Sudhir Vempati.

Roberto Contino, Dileep Jatkar and Gautam Mandal did a very careful reading of portions of the manuscript and helped us with their comments and advice.

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Thanks are also due to Abhishek Iyer and Rajan Pawar for help in making some of the figures.

We would like to thank the Hindustan Book Agency, New Delhi for permission to reproduce material which forms part of Chapter 6.

We are really grateful to our families for putting up with odd hours and abstracted behaviour for years on end.

We would like to thank Simon Capelin and his team at Cambridge University Press for all the encouragement and exemplary patience they have shown us during the long gestation period of this book.

Finally, to the prospective readers of this work we extend thanks in advance. Every time a reader gains something by reading this book, one little bit of our efforts will have been worthwhile.