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978-0-521-76848-1 - A Guide to Monte Carlo Simulations in Statistical Physics, Third Edition

David P. Landau and Kurt Binder

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