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(CSIC)

CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521768139

First published 2004
Second edition 2015

Printed in the United Kingdom by CPI Group Ltd, Croydon CR0 4YY

A catalog record for this publication is available from the British Library

ISBN 978-0-521-76813-9 Hardback

To Marimar, Diego, and Tomás, the sweet strings that tie me to the real world

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Preface to the second edition

In spite (or because) of its relentless progress, science is a perpetually unfinished work and so must be a description of any field of research at a given time. The first edition of this book tried to review the foundations and main achievements of the field that we called *semiclassical string gravity* covering the basics of general relativity, supergravity, and superstring theory[1] aiming to provide a complete and self-consistent introduction to the effective field theory description and the black-hole and black-brane solutions of the latter (ten-dimensional supergravity and some of its compactifications). However, many interesting topics and results had to be omitted then due to lack of space and many others have emerged in the following years and I started feeling quite soon that the book was not complete and the goals I had set forth had not been reached.

Of course, for the aforementioned reasons, it is intrinsically impossible to give a complete and final description of this field in the absolute sense, but I think (the reader will be the judge) that the inclusion of a reasonable number of new topics was necessary and will make the book much more useful. The second edition is the result of trying to cover that necessity while preserving the self-consistency of the book by adding background and complementary material.

The two main gaps I have tried to close are the lack of a complete discussion of the black-hole attractor mechanism and a description of the classification/characterization of the supersymmetric solutions of general (matter-coupled) four-dimensional supergravities.[2] These two subjects are linked by the original discovery of the attractor mechanism in supersymmetric extremal black-hole solutions of $N = 2, d = 4$ supergravity coupled to vector supermultiplets.

[1] This field, lying at the triple intersection of gravity, supergravity, and superstring theory, could well be named by the acronym *GRASS*.

[2] There are gaps in many other directions that could have been completed as well. For instance, a chapter on higher-derivative modifications of GR ($f(R)$ theories in particular), a deeper discussion on the definition of conserved charges in gauge theories (including gravity and supergravity) and the relation with the symmetry groups of given boundary conditions (for Kerr/CFT duality purposes), an introduction to AdS/CFT correspondence, the inclusion of asymptotically AdS and stationary solutions etc. could have been found useful by many readers. The final choice is quite subjective and associated to the author's own taste and limits.

A self-consistent description of these two subjects has required, first, the addition of several new chapters (Chapters 6–8) on matter-coupled $N = 1$ to $N = 8$ four-dimensional supergravities, including detailed descriptions of the gaugings of the $N = 1$ and $N = 2$ theories. Due to the relation via KK dimensional reduction between $N = 1, d = 5$ coupled to vector multiplets and the cubic models of $N = 2, d = 4$ supergravity, a chapter on the former (Chapter 9) has also been included, and the dimensional reduction has been performed in Chapter 15. Again, several appendices (Appendices E–J) describing the geometries of the scalar manifolds of these supergravities and the gauging of their isometries have been added for the sake of self-consistency. Furthermore, since the description of those supergravities makes heavy use of the results by Gaillard and Zumino on the general duality symmetries of (the equations of motion of) four-dimensional field theories, a section (Section 2.6) has been added describing them and their extension to higher dimensions.

With this background at hand we have been able to address the classification/characterization of the supersymmetric solutions of those supergravity theories using the Killing spinor bilinear method in Chapter 18, extending the results on the maximally supersymmetric ones of the first edition, and we have applied it in Chapter 19 to the construction of general families of supersymmetric black-hole solutions including multi-black-hole solutions and five-dimensional supersymmetric black rings.

The attractor mechanism has been explained in Chapter 27 in the framework of the Ferrara–Gibbons–Kallosh formalism and its (spacetime and worldvolume) higher-dimensional extension. Finally, the H-FGK formalism connects the results on supersymmetric black-hole solutions of Chapter 19 with the results of the FGK formalism.

There many other minor additions: an introduction to the embedding tensor formalism (Section 2.7), a review of non-linear electric–magnetic duality within Section 2.6, the algebra of four- and five-dimensional spinor bilinears (Section D.3), etc.

With the addition of all this new and highly correlated material, the organization of the book has become quite non-linear. For instance, general duality (Gaillard–Zumino) symmetries (Section 2.6) are described long before the simplest electric–magnetic duality transformations are introduced (Section 12.7). These non-linearities have no easy and economical solution, but, hopefully, they can be sorted out thanks to the cross-references provided in the main text. The index should also be helpful to those searching for specific theories, solutions, and results.

Since the publication of the first edition, several excellent books on gravity [1284, 557], supergravity [564], and superstrings [111, 860, 1248] have appeared. They deal with the basics of gravity, supergravity, and superstrings in much more depth, but I think the interdisciplinary topics studied in this book (whose contents do not fit in a nutshell, not even in a coconut shell!) provide a useful complement not specifically covered by any of them.

Just as new material had to be added to this edition, I must also add the names of people to whom I am grateful as a scientist, as a person, or both. First and foremost, I have to thank my family (Marimar, Tomás, and Diego) for their understanding and support, because nothing would have been possible without them. My students Jorge Bellorín, Pablo Bueno, Wissam Chemissany, Mechthild Hübscher, Carlos Shahbazi, and Simone Sorgato, and young collaborators Pietro Galli, José Juan Fernández-Melgarejo, Jelle Hartong, Jan Perz, Diederick Roest (now not so young!), and Silvia Vaulà helped and pushed me into

new directions and taught me many things which are now in this book. I have also learned many new things from Eric Bergshoeff, Renata Kallosh, and Roberto Emparan that have found a place here. Their support, as well as that of Enrique Álvarez, Luis Álvarez-Gaumé, José Adolfo de Azcárraga, Igor Bandos, Yolanda Lozano, and Emilio Torrente-Luján, has been essential.

My long-time collaborator Patrick Meessen deserves a special mention, and he has my long-lasting gratitude for his many direct and indirect contributions to this book, for the time and energy spent in our common projects, and for his friendship. Joaquim Gomis believed in this project and shared with me his courage and wisdom. I have learned many useful things from him *¡Moltes gràcies Quim!*

The hospitality and financial support of the CERN Theory Division and the Instituto Balseiro in Bariloche have provided the calm and positive working environment that I badly needed to conclude the book. Thank you very much.

I would also like to thank Irene Pizzie for her thorough review of the manuscript. She has eliminated most inconsistencies and has made the book much more readable. Whatever defects remain are my sole responsibility.

Finally, I must thank Simon Capelin from Cambridge University Press for suggesting, encouraging, and allowing me to write this second edition to my entire satisfaction (so I am the only one to blame for its shortcomings), showing he has boundless patience.

Comments and notifications of misprints can be sent to the e-mail address `Tomas.Ortin@csic.es`. The *errata* will be posted in `http://ramon.ift.uam-csic.es/prc/misprints.html`.

Preface to the first edition

String theory has lived for the past few years during a golden era in which a tremendous upsurge of new ideas, techniques, and results has proliferated. In what form they will contribute to our collective enterprise (theoretical physics) only time can tell, but it is clear that many of them have started to have an impact on closely related areas of physics and mathematics, and, even if string theory does not reach its ultimate goal of becoming a theory of everything, it will have played a crucial, inspiring role.

There are many interesting things that have been learned and achieved in this field that we feel can (and perhaps should) be taught to graduate students. However, we have found that this is impossible without the introduction of many ideas, techniques, and results that are not normally taught together in standard courses on general relativity, field theory, or string theory, but which have become everyday tools for researchers in this field: black holes, strings, membranes, solitons, instantons, unbroken supersymmetry, Hawking radiation .... They can, of course, be found in various textbooks and research papers, presented from various viewpoints, but not in a single reference with a consistent organization of the ideas (not to mention a consistent notation).

These are the main reasons for the existence of this book, which tries to fill this gap by covering a wide range of topics related, in one way or another, to what we may call *semiclassical string gravity*. The selection of material is according to the author's taste and personal preferences with the aim of self-consistency and the ultimate goal of creating a basic, pedagogical, reference work in which all the results are written in a consistent set of notations and conventions. Some of the material is new and cannot be found elsewhere.

Precisely because of the blend of topics we have touched upon, although a great deal of background material is (briefly) reviewed here, this cannot be considered a textbook on general relativity, supergravity, or string theory. Nevertheless, some chapters can be used in graduate courses on these matters, either providing material for a few lectures on a selected topic or combined (as the author has done with the first part, which is self-contained) into an advanced (and a bit eclectic) course on gravity.

It has not been too difficult to order logically the broad range of topics that had to be discussed, however. We can view string theory as the summit of a pyramid whose building blocks are the theories, results, and data that become more and more fundamental and basic the more we approach the base of the pyramid. At the very bottom (Part I) one can find tools

such as differential geometry and the use of symmetry in physics and fundamental theories of gravity such as general relativity and extensions to accommodate fermions such as the CSK theory and supergravity. The rest of the book is supported by it. In particular, we can see string theory as the culmination of long-term efforts to construct a theory of quantum gravity for a spin-2 particle (the graviton), and our approach to general relativity as the only self-consistent classical field theory of the graviton is intended to set the ground for this view.

Part II investigates the consequences, results, and extensions of general relativity through some of its simplest and most remarkable solutions, which can be regarded as point-particle like: the Schwarzschild and Reissner–Nordström solutions, gravitational waves, and the Taub–NUT solution. In the course of this study we introduce the reader to black holes, "no-hair theorems," black-hole thermodynamics, Hawking radiation, gravitational instantons, charge quantization, electric–magnetic duality, the Witten effect, etc. We will also explain the essentials of dimensional reduction and will obtain black-hole solutions of the dimensionally reduced theory. To finish Part II we introduce the reader to the idea and implications of residual supersymmetry. We will review all our results on black-hole thermodynamics and other black-hole properties in the light of unbroken supersymmetry.

Part III introduces strings and the string effective action as a particular extension of general relativity and supergravity. String dualities and extended objects will be studied from the string-effective-action (spacetime) point of view, making use of the results of Parts I and II and paying special attention to the relation between worldvolume and spacetime phenomena. This part, and the book, closes with an introduction to the calculation of black-hole entropies using string theory.

During these years, I have received the support of many people to whom this book, and I personally, owe much: Enrique Álvarez, Luis Álvarez-Gaumé, and my long-time collaborators Eric Bergshoeff and Renata Kallosh encouraged me and gave me the opportunity to learn from them. My students Natxo Alonso-Alberca, Ernesto Lozano-Tellechea, and Patrick Meessen used and checked many versions of the manuscript they used to call the *PRC*. Their help and friendship in these years has been invaluable. Roberto Emparan, José Miguel Figueroa-O'Farrill, Yolanda Lozano, Javier Más, Alfonso Vázquez-Ramallo, and Miguel Ángel Vázquez-Mozo read several versions of the manuscript and gave me many valuable comments and advice, which contributed to improving it. I am indebted to Arthur Greenspoon for making an extremely thorough final revision of the manuscript.

Nothing would have been possible without Marimar's continuous and enduring support.

If, in spite of all this help, the book has any shortcomings, the responsibility is entirely mine. Comments and notifications of misprints can be sent to the e-mail address `tomas.ortin@uam.es`. The *errata* will be posted in `http://gesalerico.ft.uam.es/prc/misprints.html`.

This book started as a written version of a review talk on string black holes prepared for the first String Theory Meeting of the Benasque Center for Theoretical Physics, back in 1996; parts of it made a first public appearance in a condensed form as lectures for the charming Escuela de Relatividad, Campos y Cosmología "La Hechicera" organized by the Universidad de Los Andes (Mérida, Venezuela); and it was finished during a long-term visit to the CERN Theory Division. I would like to thank the organizers and members of these institutions for their invitations, hospitality, and economic support.