

Contents

<i>Preface</i>	<i>page</i> xiii
<i>Acknowledgements</i>	xix
1 Introduction	1
1.1 Fermat’s principle of stationary time	2
1.1.1 General comments	2
1.1.2 Uniform media	3
1.1.3 Snell’s Law	4
1.1.4 Distributed sources	5
1.1.5 Stationarity vs. minimization: the law of reflection	6
1.1.6 Smoothly varying media	9
1.2 Hamilton’s principle of stationary phase	11
1.2.1 Phase speed	12
1.2.2 Phase integrals and rays	14
1.3 Modern developments	18
1.3.1 Quantum mechanics and symbol calculus	18
1.3.2 Ray phase space and plasma wave theory	22
1.4 One-dimensional uniform plasma: Fourier methods	27
1.4.1 General linear wave equation: $D(-i\partial_x, i\partial_t)\psi = 0$	28
1.4.2 Dispersion function: $D(k, \omega)$	29
1.4.3 Modulated wave trains: group velocity and dispersion	31
1.4.4 Weak dissipation	34
1.4.5 Far field of dispersive wave equations	35
1.5 Multidimensional uniform plasma	38
1.6 One-dimensional nonuniform plasma: ray tracing	40
1.6.1 Eikonal equation for an EM wave	40
1.6.2 Wave-action conservation	42
1.6.3 Eikonal phase $\theta(x)$	43
	vii

viii	<i>Contents</i>	
	1.6.4 Amplitude $A(x)$	44
	1.6.5 Hamilton's equations for rays	45
	1.6.6 Example: reflection of an EM wave near the plasma edge	46
	1.7 Two-dimensional nonuniform plasma: multidimensional ray tracing	47
	1.7.1 Eikonal equation for an EM wave	48
	1.7.2 Wave-action conservation	48
	1.7.3 Eikonal phase $\theta(x,y)$ and Lagrange manifolds	49
	1.7.4 Hamilton's equations for rays	49
	Problems	51
	References	55
2	Some preliminaries	62
	2.1 Variational formulations of wave equations	62
	2.2 Reduced variational principle for a scalar wave equation	63
	2.2.1 Eikonal equation for the phase	64
	2.2.2 Noether symmetry and wave-action conservation	64
	2.3 Weyl symbol calculus	66
	2.3.1 Symbols in one spatial dimension	66
	2.3.2 Symbols in multiple dimensions	72
	2.3.3 Symbols for multicomponent linear wave equations	74
	2.3.4 Symbols for operator products: the Moyal series	74
	Problems	76
	References	78
3	Eikonal approximation	80
	3.1 Eikonal approximation: x -space viewpoint	81
	3.2 Eikonal approximation: phase space viewpoint	84
	3.2.1 Lifts and projections	89
	3.2.2 Matching to boundary conditions	92
	3.2.3 Higher-order phase corrections	94
	3.2.4 Action transport using the focusing tensor	95
	3.2.5 Pulling it all together	97
	3.2.6 Frequency-modulated waves	102
	3.2.7 Eikonal waves in a time-dependent background plasma	104
	3.2.8 Symmetries	105
	3.2.9 Curvilinear coordinates	108
	3.3 Covariant formulations	111
	3.3.1 Lorentz-covariant eikonal theory	111
	3.3.2 Energy-momentum conservation laws	119

	<i>Contents</i>	ix
3.4	Fully covariant ray theory in phase space	121
3.5	Special topics	128
3.5.1	Weak dissipation	129
3.5.2	Waveguides	132
3.5.3	Boundaries	134
3.5.4	Wave emission from a coherent source	139
3.5.5	Incoherent waves and the wave kinetic equation	142
	Problems	146
	References	151
4	Visualization and wave-field construction	154
4.1	Visualization in higher dimensions	155
4.1.1	Poincaré surface of section	155
4.1.2	Global visualization methods	157
4.2	Construction of wave fields using ray-tracing results	170
4.2.1	Example: electron dynamics in parallel electric and magnetic fields	173
4.2.2	Example: lower hybrid cutoff model	173
	References	182
5	Phase space theory of caustics	183
5.1	Conceptual discussion	187
5.1.1	Caustics in one dimension: the fold	187
5.1.2	Caustics in multiple dimensions	191
5.2	Mathematical details	193
5.2.1	Fourier transform of an eikonal wave field	194
5.2.2	Eikonal theory in k -space	196
5.3	One-dimensional case	198
5.3.1	Summary of eikonal results in x and k	198
5.3.2	The caustic region in x : Airy's equation	200
5.3.3	The normal form for a generic fold caustic	205
5.3.4	Caustics in vector wave equations	210
5.4	Caustics in n dimensions	212
	Problems	218
	References	226
6	Mode conversion and tunneling	228
6.1	Introduction	228
6.2	Tunneling	242
6.3	Mode conversion in one spatial dimension	247

x	<i>Contents</i>	
	6.3.1 Derivation of the 2×2 local wave equation	247
	6.3.2 Solution of the 2×2 local wave equation	252
6.4	Examples	258
	6.4.1 Budden model as a double conversion	259
	6.4.2 Modular conversion in magnetohelioseismology	261
	6.4.3 Mode conversion in the Gulf of Guinea	263
	6.4.4 Modular approach to iterated mode conversion	269
	6.4.5 Higher-order effects in one-dimensional conversion models	273
6.5	Mode conversion in multiple dimensions	276
	6.5.1 Derivation of the 2×2 local wave equation	276
	6.5.2 The 2×2 normal form	279
6.6	Mode conversion in a numerical ray-tracing algorithm: RAYCON	283
6.7	Example: Ray splitting in rf heating of tokamak plasma	295
6.8	Iterated conversion in a cavity	301
6.9	Wave emission as a resonance crossing	303
	6.9.1 Coherent sources	304
	6.9.2 Incoherent sources	308
	Problems	310
	Suggested further reading	322
	References	323
7	Gyroresonant wave conversion	327
	7.1 Introduction	327
	7.1.1 General comments	329
	7.1.2 Example: Gyroballistic waves in one spatial dimension	331
	7.1.3 Minority gyroresonance and mode conversion	333
	7.2 Resonance crossing in one spatial dimension: cold-plasma model	335
	7.3 Finite-temperature effects in minority gyroresonance	348
	7.3.1 Local solutions near resonance crossing for finite temperature	359
	7.3.2 Solving for the Bernstein wave	373
	7.3.3 Bateman–Kruskal methods	379
	Problems	385
	References	392
Appendix A	Cold-plasma models for the plasma dielectric tensor	394
	A.1 Multifluid cold-plasma models	395
	A.2 Unmagnetized plasma	397

<i>Contents</i>	xi
A.3 Magnetized plasma	399
A.3.1 $\mathbf{k} \parallel \mathbf{B}_0$	400
A.3.2 $\mathbf{k} \nparallel \mathbf{B}_0$	401
A.4 Dissipation and the Kramers–Kronig relations	403
Problems	404
References	405
Appendix B Review of variational principles	406
B.1 Functional derivatives	406
B.2 Conservation laws of energy, momentum, and action for wave equations	408
B.2.1 Energy-momentum conservation laws	408
B.2.2 Wave-action conservation	409
References	411
Appendix C Potpourri of other useful mathematical ideas	412
C.1 Stationary phase methods	412
C.1.1 The one-dimensional case	412
C.1.2 Stationary phase methods in multidimensions	416
C.2 Some useful facts about operators and bilinear forms	421
Problem	424
References	424
Appendix D Heisenberg–Weyl group and the theory of operator symbols	426
D.1 Introductory comments	426
D.2 Groups, group algebras, and convolutions on groups	427
D.3 Linear representations of groups	430
D.3.1 Lie groups and Lie algebras	434
D.4 Finite representations of Heisenberg–Weyl	436
D.4.1 The translation group on n points	436
D.4.2 The finite Heisenberg–Weyl group	439
D.5 Continuous representations	442
D.6 The regular representation	445
D.7 The primary representation	445
D.8 Reduction to the Schrödinger representation	446
D.8.1 Reduction via a projection operator	446
D.8.2 Reduction via restriction to an invariant subspace	446
D.9 The Weyl symbol calculus	447
References	452

xii	<i>Contents</i>	
Appendix E	Canonical transformations and metaplectic transforms	453
E.1	Examples	453
E.2	Two-dimensional phase spaces	457
E.2.1	General canonical transformations	457
E.2.2	Metaplectic transforms	459
E.3	Multiple dimensions	466
E.3.1	Canonical transformations	466
E.3.2	Lagrange manifolds	467
E.3.3	Metaplectic transforms	469
E.4	Canonical coordinates for the 2×2 normal form	471
References		476
Appendix F	Normal forms	479
F.1	The normal form concept	479
F.2	The normal form for quadratic ray Hamiltonians	482
F.3	The normal form for 2×2 vector wave equations	488
F.3.1	The Braam–Duistermaat normal forms	497
F.3.2	The general case	497
References		499
Appendix G	General solutions for multidimensional conversion	500
G.1	Introductory comments	500
G.2	Summary of the basis functions used	500
G.3	General solutions	504
G.4	Matching to incoming and outgoing fields	506
Reference		510
	<i>Glossary of mathematical symbols</i>	511
	<i>Author index</i>	514
	<i>Subject index</i>	517