RAY TRACING AND BEYOND

This complete introduction to the use of modern ray-tracing techniques in plasma physics describes the powerful mathematical methods generally applicable to vector wave equations in nonuniform media, and clearly demonstrates the application of these methods to simplify and solve important problems in plasma wave theory.

Key analytical concepts are carefully introduced as needed, encouraging the development of a visual intuition for the underlying methodology, with more advanced mathematical concepts succinctly explained in the appendices, and supporting MATLAB code available online. Covering variational principles, covariant formulations, caustics, tunneling, mode conversion, weak dissipation, wave emission from coherent sources, incoherent wave fields, and collective wave absorption and emission, all within an accessible framework using standard plasma physics notation, this is an invaluable resource for graduate students and researchers in plasma physics.

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"*Ray Tracing and Beyond* is an encyclopedic and scholarly work on the linear theory of dispersive vector waves, summarizing the powerful general theory developed over the careers of four leading practitioners and teachers in theoretical plasma physics. It seems destined to become a 'must-read' classic for graduate students and researchers, not only specialists in plasma physics (a field which involves a myriad of wave problems in nonuniform media) but also the many other physicists and applied mathematicians working on problems involving waves."

Robert L. Dewar, Australian National University

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Phase Space Methods in Plasma Wave Theory

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For Louise Kaufman
She walked in beauty

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Preface

Waves exist in a great variety of media (in all phases of matter) as well as a vacuum (in the case of electromagnetic waves). All simple waves, on the one hand, share some basic properties such as amplitude, frequency and period, wavelength, and wave velocity (both phase and group velocity). Waves in a turbulent medium, or waves generated by random sources, on the other hand, are more appropriately described in terms of probability distributions of amplitude, and spectral densities in frequency and wavelength. In this book, we focus primarily upon *coherent* waves that are *locally plane wave* in character. That is, they have a well-defined amplitude, phase, and polarization at most (but not all) points. The regions where this local plane-wave approximation breaks down are important, and the development of appropriate local methods to deal with them is an important topic of the book. We include a very short discussion of phase space approaches for *incoherent* waves, for completeness.

This is the first book to present modern ray-tracing theory and its application in plasma physics. The emphasis is on methods and concepts that are generally applicable, including methods for visualizing ray families in higher dimensions. A self-contained exposition is given of the mathematical foundations of ray-tracing theory for vector wave equations, based upon the Weyl theory of operator symbols. Variational principles are used throughout. These provide a means to derive a Lorentz-covariant ray theory, along with related conservation laws for energy, momentum, and wave action. Phase space variational principles are also used to provide a unified treatment of caustics, tunneling, mode conversion, and gyro-resonant wave–particle interactions.

Many examples are presented to show the power of these ideas to simplify and solve problems in plasma wave theory. Each chapter ends with a set of problems that allows the reader to explore the topics in more depth.

The major theme of the book concerns the use of *phase space* methods. Originally developed by Hamilton for the study of optics, these methods became a familiar tool in the study of classical *particle* motion, and they are part of the standard toolkit for any physicist. The use of phase space methods in the study of plasma *waves* and the Weyl symbol calculus – and the underlying group theoretical and geometrical ideas these methods are based upon – are more recent developments that are less well-known in the plasma physics community.

The theory of short-wavelength asymptotics has advanced significantly since the 1960s. There is now a large literature on the topic in mathematics and certain subfields of physics, such as atomic, molecular, and optical (AMO) physics, and nuclear physics. This revolution in understanding has produced a relatively minor impact upon ray tracing as practiced by most plasma physicists. There are several reasons for this.

First, the modern theory of short-wavelength asymptotics (which is called "semiclassical analysis" in the AMO literature) is couched in mathematical terms that are unfamiliar to scientists who are trained in traditional approaches to plasma wave theory. Traditional approaches tend to emphasize the particular, rather than the general. There is a large emphasis placed upon naming the multitudes of modes, and their classification. These ideas are important, but they can overwhelm the student and the researcher with details and obscure the underlying universal principles. Most students are introduced to plasma wave theory in the uniform setting where Fourier methods apply. They are presented with a survey of the various types of plasma waves. They then quickly skip over how Fourier methods must be modified in a nonuniform plasma. If they are lucky, they are given a superficial introduction to a form of ray-tracing theory that is one-dimensional and largely of nineteenth-century vintage. As a result, most plasma physicists are completely unaware of the revolution that has occurred in ray-tracing theory, and they are therefore poorly prepared to apply it to their own area of work.

Second, if students explore ray tracing in the *mathematics* literature, they will find that there is a relatively limited range of examples studied, such as the single-particle Schrödinger equation, or the wave equation with a spatially dependent wave speed. Plasma wave equations have a much richer variety. Plasma wave equations include phenomena that the mathematical literature overlooks, such as wave–particle resonance, gyroresonance, and finite-temperature effects. Dissipation is often ignored in the mathematical literature, as is wave emission, matching to boundary conditions, and Lorentz covariance. All of these topics are important in plasma applications, and all are covered in this book.

Third, instead of introducing students to these important theoretical ideas in plasma wave theory, there has been a growing emphasis on teaching full-wave simulation methods. While computational tools for the study of waves are very important (and will continue to grow in importance), the lack of coverage of modern eikonal theory leaves many students and researchers without a firm grasp of its use in wave problems. Ray-tracing codes, which do not include the modern improvements mentioned above, are commonly used for "quick-and-dirty" calculations by experimentalists. This is done for the simple reason that following rays provides insight and promotes physical intuition in practical situations. But by following a collection of rays independently, as is commonly done in such calculations, one is really treating the wave field as *incoherent* without regard to proper matching to boundary values. This also makes it impossible to correctly compute the wave amplitude. A coherent wave field, properly matched to given boundary values, must be synthesized using a family of rays that have a well-defined set of properties. With some effort (for example by properly dealing with mode conversion), the calculation can be kept "quick" but "cleaned up" so it can be used to accurately compute the phase, amplitude, and polarization along each ray, thereby providing the possibility for a full construction of the wave field.

A major focus of the book is the investigation of the processes by which waves, or waves and particles, interact with each other, so that they may exchange energy, momentum, and wave action. These basic processes arise in various settings, and a unified treatment is possible. Examples include mode conversion, wave absorption, and emission by resonant particles in nonuniform plasma. In order for such conversion to take place, certain resonance conditions involving the participating waves, or waves and particles, must be met. These conditions are described in terms of the dynamics of Hamiltonian orbits in ray phase space, and particle orbits in particle phase space. While the focus will be on cold-plasma models for pedagogic purposes, we also include a discussion of finite-temperature effects for completeness (Chapter 7). The effects of finite temperature are studied using Case–van Kampen methods, with the theory adapted to the ray phase space setting.

A reader already well-versed in plasma physics should find the examples very familiar, though the method of analysis pursued in later chapters is likely to be new, as well as our emphasis on the use of variational principles. A brief derivation of the cold-plasma fluid model is included in Appendix A for those readers who are not plasma physicists. More detailed discussions of the physical assumptions underlying the models can be found in the references cited.

Chapter 1 begins with some introductory comments and examples. A brief historical survey is presented to provide context and to highlight some of the more recent developments in ray tracing. The historical survey here is highly selective, and the narrative provides a first introduction to topics such as the use of variational principles to derive wave equations and their conservation laws, Hamilton's ray theory, Weyl's theory of operator symbols, and the much more recent use of these foundational ideas in plasma theory. A brief introduction to eikonal theory for a scalar wave equation is then presented, starting with a quick review of waves in uniform plasma, Fourier methods, and the concepts of phase velocity, group velocity, dispersion, and diffraction. Eikonal methods are then introduced in order to study waves in nonuniform plasma. The treatment here is very traditional, using asymptotic series and brute force calculation.

In Chapter 2, we introduce two important tools that will be used throughout the later chapters: variational methods for wave equations and the Weyl symbol calculus. The use of a variational principle allows a unified treatment of later topics, and provides an elegant derivation of the wave-action conservation law using Noether's theorem. (Further discussion of variational methods and Noether's theorem is provided in Appendix B.) The Weyl theory of operator symbols underlies everything done later in the book.[1] This theory was first developed in the context of quantum mechanics, but the methods are completely general and they provide a systematic means for deriving wave equations that are local in both x and k. These methods are needed for dealing with caustics, tunneling, and mode conversion, all of which involve a breakdown of the eikonal approximation that is *local in ray phase space*.

In Chapter 3, we begin our discussion of eikonal theory for vector wave equations in earnest. In this chapter, we transition by stages from the more familiar x-space discussion of eikonal theory to a covariant phase space treatment. In many settings, the covariant formulation is not needed, and we strive to keep things as concrete as possible. Therefore, in this book we tend to use a preferred laboratory frame and work with physical fields (*e.g.* $\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$), rather than allowing arbitrary frames and using four-vector notation and the vector potential $A_\mu(x)$. However, covariant formulations are needed in some astrophysical and space plasma applications, so we include them even if they are not a major focus of the book. The conservation of energy, momentum, and wave action is covered in this chapter.

Chapter 4 discusses visualization and wave-field construction. The modern theory of ray tracing is a geometrical theory. Geometrical theories appeal to the visual intuition. A visual picture can help guide us through a calculational thicket by providing a map. Mixing metaphors, the goal of this chapter is to help bring visual intuition onto the battlefield as a valued ally along with the more analytical methods of calculation. This chapter also provides examples of the construction of wave fields from ray-based data. This section connects us directly back to the original motivation of eikonal theory: to find solutions of wave equations.

The next three chapters concern situations where the eikonal approximation breaks down in various ways.[2]

[1] The theory of operator symbols is based upon the representation theory for the Heisenberg–Weyl group. A short, self-contained, introduction to this topic is provided in Appendix D.

[2] By an *eikonal solution*, we mean a wave field that has well-defined phase, amplitude, and polarization:

$$\psi(\mathbf{x}, t) = A(\mathbf{x}, t)\exp[i\theta(\mathbf{x}, t)]\hat{\mathbf{e}}(\mathbf{x}, t).$$

Chapter 5 presents the phase space theory of caustics. These involve a breakdown of the eikonal approximation in x-space, but not in k-space. Hence, caustics are dealt with by moving between the x- and k-representations, solving for the local wave behavior near the caustic in the k-representation, then matching to the incoming and outgoing fields in the x-representation. This strategy works because of certain fundamental results from the theory of phase integrals; hence, we provide an extensive discussion of *stationary phase methods* in Appendix C. Chapter 5 involves the first encounter with what are called *normal form methods*, which also play an important role in the chapters that follow.[3]

Chapter 6 treats tunneling and mode conversion. These phenomena are due to a resonance *in ray phase space*. In the case of mode conversion, two distinct types of eikonal waves, associated with *two* polarizations, have dispersion functions that are nearly degenerate, meaning that – for a given wave frequency ω – at some point $\mathbf{x}$ these two wave types have nearly equal values of $\mathbf{k}$. This causes a breakdown in the eikonal approximation, which is not valid *in any representation* near the mode conversion point. Weyl methods are used, in tandem with variational principles, to derive the appropriate local wave equation, which is a 2×2 vector wave equation involving the polarizations of the two wave types undergoing conversion.[4] This local wave equation is then solved and matched to incoming and outgoing eikonal fields.[5]

Chapter 7 discusses the phase space theory of gyroresonant wave conversion. This phenomenon presents a significant challenge for full-wave simulation because of the wide range of spatial scales involved. The inclusion of finite-temperature effects makes the problem even more challenging numerically. A phase space theory, however, allows us to treat the problem in modular fashion, and to use matched asymptotics to construct a complete solution throughout the resonance region. The calculation is the most technical in the book, but self-contained, and it illustrates the power of phase space ideas.

Examples are scattered throughout the book. They are drawn from a wide range of applications in plasma physics and beyond. The phase space theory of Budden-type resonances is covered in great detail at various places in the book, in one dimension and tokamak geometry, for cold and warm plasma. The ray-tracing algorithm RAYCON is described. This is the first ray-tracing algorithm that can deal with ray splitting due to mode conversion in tokamak geometry. In addition, mode conversion in magnetohelioseismology and equatorial ocean waves is briefly covered

[3] In addition to the discussion of normal forms for phase integrals in Appendix C, a general introduction to those aspects of the theory of normal forms needed elsewhere in this book is provided in Appendix F.
[4] *Tunneling* involves only one polarization, and hence can be reduced to a scalar problem.
[5] The normal form theory for the local 2×2 wave equation is presented in Appendix F, and the general solution of the local wave equation is presented in Appendix G.

to illustrate the wide applicability of the methods. Many of the more technical details, and additional mathematical topics, have been relegated to an extended set of appendices.

The power of the phase space viewpoint will become apparent as these various topics are developed. The discussions in the main part of the book are kept relatively brief, with an emphasis on the concepts. To avoid bogging down the discussion, many technical details are either presented in the appendices, or developed as exercises for the reader. An extensive list of citations is provided for readers who wish to learn even more. Those who are new to these ideas are strongly encouraged to attempt the problems, as the only way to learn is by doing. The many figures provided in the text are a key part of the discussion; they help to develop geometrical intuition. In particular, the MATLAB code RAYCON – which was used to generate the figures for ray tracing in tokamaks in Section 6.6 – is available online as a supplement to the text, and the reader is encouraged to use this code as well.

Our goal is to make the material accessible and useful to a wide audience. We have written the material for graduate students in plasma physics and related fields, which should also make it accessible to researchers in these fields as well. We assume that the reader is mathematically sophisticated, but that the primary interest of the reader lies in understanding how to apply these methods to real physical problems. Comments and suggestions are welcome.

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