Quantum physics and special relativity theory were two of the greatest breakthroughs in physics during the twentieth century and contributed to paradigm shifts in physics. This book combines these two discoveries to provide a complete description of the fundamentals of relativistic quantum physics, guiding the reader effortlessly from relativistic quantum mechanics to basic quantum field theory.

The book gives a thorough and detailed treatment of the subject, beginning with the classification of particles, the Klein–Gordon equation and the Dirac equation. It then moves on to the canonical quantization procedure of the Klein–Gordon, Dirac, and electromagnetic fields. Classical Yang–Mills theory, the LSZ formalism, perturbation theory and elementary processes in QED are introduced, and regularization, renormalization, and radiative corrections are explored. With exercises scattered through the text and problems at the end of most chapters, the book is ideal for advanced undergraduate and graduate students in theoretical physics.

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RELATIVISTIC QUANTUM PHYSICS
From Advanced Quantum Mechanics to Introductory Quantum Field Theory

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In memory of my father Dick
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Preface

This book is based on my lectures in the course ‘Relativistic Quantum Physics’ at the Royal Institute of Technology (KTH) in Stockholm, Sweden. These lectures have been given four times during the academic years 2006–2007, 2007–2008, 2008–2009, and 2009–2010. The main sources of inspiration for the lectures were the books A. Z. Capri, *Relativistic Quantum Mechanics and Introduction to Quantum Field Theory*, World Scientific (2002) and M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995), and indeed, this book serves as a textbook for relativistic quantum mechanics with continuation to basic quantum field theory. The book is mainly intended for final-year undergraduate students in physics or first-year graduate students in physics and/or theoretical physics, who want to learn relativistic quantum mechanics, the basics of quantum field theory, and the techniques of calculating cross-sections for elementary reactions in quantum electrodynamics. Thus, the book should be suitable for any course on relativistic quantum mechanics as well as it might be suitable for a beginners’ course on quantum field theory. In summary, the book is a self-contained technical treatment on relativistic quantum mechanics, introductory quantum field theory, and the step in between, i.e. it should fill the gap between advanced quantum mechanics and quantum field theory, which I have called relativistic quantum physics. It contains a thorough and detailed mathematical treatment of the subject with smaller exercises throughout the whole text and larger problems at the end of most chapters.

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