

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

## MODERN INTRODUCTION TO SURFACE PLASMONS

Theory, *Mathematica* Modeling and Applications

Introducing graduate students in physics, optics, materials science and electrical engineering to surface plasmons, this book also covers guided modes at planar interfaces of metamaterials with negative refractive index.

The physics of localized and propagating surface plasmons on planar films, gratings, nanowires and nanoparticles is developed using both analytical and numerical techniques. Guided modes at the interfaces between materials with any combination of positive or negative permittivity and permeability are analyzed in a systematic manner. Applications of surface plasmon physics are described, including near-field transducers in heat-assisted magnetic recording and biosensors.

Resources at [www.cambridge.org/9780521767170](http://www.cambridge.org/9780521767170) include *Mathematica* code to generate figures from the book, color versions of many figures, and extended discussion of topics such as vector diffraction theory.

DROR SARID is Professor and former Director of the Optical Data Storage Center at the College of Optical Sciences, the University of Arizona. He participated in the development of the field of surface plasmons, identifying the long- and short-range surface plasmons and their important applications in science and technology.

WILLIAM CHALLENGER is a Research Scientist at Seagate Technology. He has worked on optical and magnetic data storage materials and systems, and various chemical and biological sensors employing surface plasmons and other evanescent wave optics.

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

# MODERN INTRODUCTION TO SURFACE PLASMONS

Theory, *Mathematica* Modeling and Applications

DROR SARID

*University of Arizona*

AND

WILLIAM A. CHALLENGER

*Seagate Technology*



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,  
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521767170](http://www.cambridge.org/9780521767170)

© D. Sarid and W. Challener 2010

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Sarid, Dror.

Modern introduction to surface plasmons : theory, mathematica modeling,  
and applications / Dror Sarid, William A. Challener.

p. cm.

ISBN 978-0-521-76717-0 (hardback)

1. Plasmons (Physics)
2. Plasma oscillations.
3. Surfaces (Physics)
4. Surfaces (Technology) – Analysis. I. Challener, William Albert. II. Title.  
QC176.8.P55S37 2010  
530.4'4–dc22  
2010000128

ISBN 978-0-521-76717-0 Hardback

Cambridge University Press has no responsibility for the persistence or  
accuracy of URLs for external or third-party internet websites referred to  
in this publication, and does not guarantee that any content on such  
websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling  
and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

---

To Lea, Rami, Uri, Karen and Danieli, and to Helen, Douglas and Gregory

Contents

|   |  |                |
|---|--|----------------|
|   | <i>Preface</i>                                 | <i>page xi</i> |
| 1 | Introduction                                   | 1              |
| 2 | Electromagnetics of planar surface waves       | 4              |
|   | 2.1 Introduction                               | 4              |
|   | 2.2 Topics in electromagnetic theory           | 5              |
|   | 2.3 Media type notation                        | 14             |
|   | 2.4 Mode and symmetry notation                 | 18             |
|   | 2.5 Wave vector notation                       | 19             |
|   | 2.6 Single-interface TE mode fields            | 20             |
|   | 2.7 Single-interface TM mode fields            | 22             |
|   | 2.8 Single-interface generalized fields        | 24             |
|   | 2.9 Double-interface TE mode fields            | 25             |
|   | 2.10 Double-interface TM mode fields           | 27             |
|   | 2.11 Double-interface generalized fields       | 30             |
|   | 2.12 Wave impedance                            | 31             |
|   | 2.13 Single-interface mode solution            | 33             |
|   | 2.14 Double-interface mode solution            | 35             |
|   | 2.15 Poynting vector                           | 40             |
|   | 2.16 Prism coupling                            | 46             |
|   | 2.17 Reflectivity and Goos–Hänchen shift       | 50             |
|   | 2.18 Summary                                   | 54             |
|   | 2.19 Exercises                                 | 55             |
|   | References                                     | 55             |
| 3 | Single-interface modes in the microwave regime | 56             |
|   | 3.1 Introduction                               | 56             |
|   | 3.2 Dispersion of $\epsilon_m$ and $\mu_m$     | 57             |
|   | 3.3 Single-interface lossless-mode solutions   | 60             |
|   | 3.4 Lossy modes in the Otto configuration      | 64             |

|      |   |     |
|------|---|-----|
| viii | <i>Contents</i>   |     |
|      | 3.5 Lossy modes in the Kretschmann configuration                        | 72  |
|      | 3.6 Summary   | 78  |
|      | 3.7 Exercises   | 78  |
|      | References  | 79  |
| 4    | Single-interface lossless modes in $\epsilon_r'-\mu_r'$ parameter space | 80  |
|      | 4.1 Introduction  | 80  |
|      | 4.2 System  | 80  |
|      | 4.3 Mode equation solutions   | 84  |
|      | 4.4 Fields and local power flow   | 87  |
|      | 4.5 Summary   | 93  |
|      | 4.6 Exercises   | 93  |
|      | References  | 93  |
| 5    | Double-interface lossless modes in $\epsilon_r'-\mu_r'$ parameter space | 94  |
|      | 5.1 Introduction  | 94  |
|      | 5.2 System  | 94  |
|      | 5.3 Mode equation solutions   | 99  |
|      | 5.4 Complete mode equation solutions: $d_g = 500$ nm                    | 100 |
|      | 5.5 Complete mode equation solutions: $d_g = 25$ nm                     | 110 |
|      | 5.6 Summary   | 119 |
|      | 5.7 Exercises   | 120 |
|      | References  | 120 |
| 6    | Single-interface surface plasmons                                       | 121 |
|      | 6.1 Introduction  | 121 |
|      | 6.2 System  | 121 |
|      | 6.3 Mode equation solutions   | 124 |
|      | 6.4 Fields and local power flow   | 127 |
|      | 6.5 Propagating electric fields   | 132 |
|      | 6.6 Surface charge density and fields                                   | 133 |
|      | 6.7 Modes in the Otto and Kretschmann configurations                    | 135 |
|      | 6.8 Summary   | 139 |
|      | 6.9 Exercises   | 140 |
|      | References  | 140 |
| 7    | Double-interface surface plasmons in symmetric guides                   | 141 |
|      | 7.1 Introduction  | 141 |
|      | 7.2 System  | 141 |
|      | 7.3 Mode equation solutions   | 144 |
|      | 7.4 Fields and local power flow   | 148 |
|      | 7.5 Fields and phasors  | 154 |
|      | 7.6 Surface charge density and fields                                   | 156 |
|      | 7.7 Modes in the general prism coupling configuration                   | 158 |

|      | <i>Contents</i>  | ix  |
|------|--|-----|
| 7.8  | Summary  | 162 |
| 7.9  | Exercises  | 162 |
|      | References   | 162 |
| 8    | Quasi-one-dimensional surface plasmons   | 164 |
| 8.1  | Introduction   | 164 |
| 8.2  | Propagating surface plasmons on metallic wires<br>of circular cross section    | 164 |
| 8.3  | Propagating surface plasmons on metallic wires<br>of noncircular cross section | 172 |
| 8.4  | Propagating surface plasmons on hollow cylindrical<br>waveguides               | 178 |
| 8.5  | Propagating surface plasmons on hollow cylindrical<br>shells                   | 182 |
| 8.6  | Excitation of surface plasmons on nanowires<br>with plane waves                | 188 |
| 8.7  | Nanowires with noncircular cross sections                                      | 196 |
| 8.8  | Summary  | 198 |
| 8.9  | Exercises  | 198 |
|      | References   | 199 |
| 9    | Localized surface plasmons   | 201 |
| 9.1  | Nanoparticles  | 201 |
| 9.2  | Nanoholes or nanovoids   | 221 |
| 9.3  | Nanoshells   | 224 |
| 9.4  | Other nanoparticle shapes  | 232 |
| 9.5  | Dual nanoparticles   | 243 |
| 9.6  | Summary  | 249 |
| 9.7  | Exercises  | 251 |
|      | References   | 251 |
| 10   | Techniques for exciting surface plasmons                                       | 256 |
| 10.1 | Introduction   | 256 |
| 10.2 | Otto configuration   | 257 |
| 10.3 | Kretschmann configuration  | 261 |
| 10.4 | Diffraction gratings and Wood's anomalies                                      | 263 |
| 10.5 | Surface roughness  | 274 |
| 10.6 | End-fire coupling  | 275 |
| 10.7 | Near-field launching   | 276 |
| 10.8 | Summary  | 280 |

|            |   |     |
|------------|---|-----|
| x          | <i>Contents</i>   |     |
| 10.9       | Appendix: description of grating code (See the<br>online supplemental material at <a href="http://www.cambridge.org/9780521767170">www.cambridge.org/9780521767170</a> .) | 280 |
| 10.10      | Exercises   | 280 |
|            | References  | 281 |
| 11         | Plasmonic materials   | 283 |
| 11.1       | Introduction  | 283 |
| 11.2       | Real metals   | 283 |
| 11.3       | Drude metals  | 293 |
| 11.4       | Summary   | 302 |
| 11.5       | Exercises   | 302 |
|            | References  | 303 |
| 12         | Applications  | 305 |
| 12.1       | Introduction  | 305 |
| 12.2       | Measuring the optical constants of metals   | 305 |
| 12.3       | Chemical and biological sensors   | 309 |
| 12.4       | Near-field microscopy   | 315 |
| 12.5       | Surface-enhanced Raman spectroscopy   | 320 |
| 12.6       | Nonlinear optics  | 322 |
| 12.7       | Heat-assisted magnetic recording  | 324 |
| 12.8       | Nanophotonics   | 328 |
| 12.9       | Cancer detection and treatment  | 340 |
| 12.10      | Other applications  | 341 |
| 12.11      | Summary   | 343 |
| 12.12      | Exercises   | 344 |
|            | References  | 344 |
| Appendix A |   | 349 |
| A.1        | Finite-difference time-domain method  | 349 |
| A.2        | Poynting vector and local power flow  | 361 |
|            | References  | 367 |
|            | <i>Index</i>  | 369 |



Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

## Preface

When deciding how to organize a book on surface plasmons, it seemed natural to consider the dimensionality of the surfaces on which they exist. On planar surfaces, which include both semi-infinite surfaces as well as multilayer thin films, there is a rich body of phenomena related to propagating surface plasmons. The same is true for surfaces of nanoparticles having a rich variety of phenomena for localized surface plasmons. Surfaces of nanowires and nanogrooves lie in between these two regimes, and these surfaces support both propagating and nonpropagating surface plasmons. In this book, therefore, we have initially categorized the chapters by surface dimensionality, trying to point out both the differences and similarities of the surface plasmon phenomena in these three regimes.

This book does not hesitate to include mathematical derivations of the equations that describe the basic surface-plasmon properties. After all, it was our desire to base the book on *Mathematica* precisely so that these equations could be explored in detail. Our derivations of the properties of surface plasmons are based on Maxwell's equations in SI units. In Chapter 2, Maxwell's equations are introduced for dense media, i.e., media which can be described by frequency dependent permittivity, permeability and conductivity. Because interfaces are essential to surface plasmons, the electromagnetic boundary conditions are required. Practically all of the results in this book are based on time-harmonic fields that can be most simply represented in complex notation. Unfortunately, in the literature there is no standard definition for the complex functional dependence on time of the electric and magnetic fields. We choose a time dependence of  $\exp(-i\omega t)$ , which has the advantage of making both real and imaginary parts of the complex optical refractive indices positive numbers as they are generally given in standard handbooks. Other properties of waves, including their group velocity, phase velocity, impedance and Poynting vectors are also derived in Chapter 2.

At optical frequencies (near IR and visible) it has been standard practice until recently to automatically set the permeability equal to unity. With the discovery

of metamaterials and the predictions of potentially amazing properties like perfect lenses and invisibility cloaks, it is no longer adequate or safe to do so. In Chapters 2 to 7, the physics of surface waves propagating along single and double interfaces are carefully examined for all combinations of materials with both positive and negative permittivity and positive and negative permeability. As a result, unfamiliar modes such as surface magnons, which depend upon negative permeability, are analyzed in addition to those of surface plasmons. It transpires that it is important to define the refractive index of a medium,  $n$ , as the product of the square roots of the relative permittivity and permeability,  $\sqrt{\epsilon_r}\sqrt{\mu_r}$ , rather than the square root of their product. A new formalism is presented in which the media of single- and double-interface structures are characterized in terms of an  $\epsilon'_r - \mu'_r$  parameter space, represented as a vector in polar coordinates, where the prime denotes the real part. This formalism also uses a medium with a double positive set  $(\epsilon'_r, \mu'_r)$  to generate the other three sets of media,  $(\epsilon'_r, -\mu'_r)$ ,  $(-\epsilon'_r, \mu'_r)$  and  $(-\epsilon'_r, -\mu'_r)$ . The properties of guided modes propagating along single- and double-interface structures, obtained by using this formalism, are then discussed in detail in these chapters. With the single- and double-interface model, it is also straightforward to understand the manner in which prism coupling via attenuated total reflection is used to launch surface plasmons on metallic surfaces and what effect the prism has upon the properties of the surface plasmon, such as propagation distance and line width.

In the remaining chapters, the discussion is narrowed to surface plasmons alone (positive  $\mu_r$ ), both propagating and localized modes. Quasi-one-dimensional surfaces, nanowires and nanogrooves, are discussed in Chapter 8 and quasi-zero-dimensional surfaces, nanoparticles and nanovoids, are discussed in Chapter 9. Interactions among neighboring nanoparticles are also considered. Although the Otto and Kretschmann prism-coupling configurations were analyzed in Chapter 2, they are briefly reconsidered and compared to other techniques for launching surface plasmons in Chapter 10. In particular, the Chandezon technique for computing vector diffraction in a semi-analytical way is implemented to discuss the ability of gratings to couple optical energy into surface plasmons. A detailed analysis of this technique is described in the online supplemental materials for this book found at the web site [www.cambridge.org/9780521767170](http://www.cambridge.org/9780521767170). Newer techniques, that make use of near-field interactions to excite surface plasmons, are also described.

The text would not be complete without a discussion of plasmonic materials. There are relatively few metals that are plasmonic at optical frequencies and it is not surprising that both gold and silver are so frequently used in surface-plasmon calculations and devices. The relationship between the complex permittivity of a material and its ability to exhibit surface-plasmon phenomena is considered

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

## Preface

xiii

in Chapter 11. The Drude dielectric function, as a phenomenological model for metals, is also considered in this chapter. Chapter 12 is a survey of various actual and potential applications of surface plasmons. This marvelous effect has already proven itself in the form of label-free biosensing for pharmaceutical development and medical diagnostics. It may soon find even larger applications in nanophotonics and magnetic data storage.

The finite difference time domain (FDTD) technique – a numerical method for computing the response of materials to incident electromagnetic fields when the geometry is too complex for analytical techniques – is described in the Appendix. Although FDTD is not implemented within *Mathematica* (it would take forever to run even simple calculations), it has been used to model some of the examples that are considered within the text and it is shown to deliver highly accurate results. A short discussion of the connection between the Poynting vector and the local power flow is also included in the Appendix.

Most chapters conclude with several exercises that are meant to stimulate further thought about the properties of surface plasmons that could not be covered in detail in the text, and are well worth the time and effort to study. Generally, the *Mathematica* routines that are included with the online supplementary materials are employed to solve these exercises.

Every chapter also has a reference section. The field of surface plasmons has grown so much over the last two decades that no one text can do an adequate job of covering it. The aim of this book is to provide a sufficient level of understanding of surface-plasmon physics so that the reader can both begin to design his, or her, own research program and also be prepared to tackle the scientific literature on this subject. There are literally thousands of journal articles related to surface plasmons. We have tried to cite many of the more important articles, including some which at this point are several decades old or older, for a more historical context, and these should give the reader a good start in further investigations, but there are also many important articles that we did not include or, unfortunately, overlooked.

This book, which represents the product of many months of collaborative work, was on the whole a very enjoyable experience. Obviously, most of the results described in the text are not original to us. Nevertheless, we have striven to make sure of the accuracy of the equations, derivations, *Mathematica* implementations and descriptions of experimental results, and any errors that remain are solely our responsibility.

We would like to express our appreciation for the kind support and encouragement provided by Seagate Technology during the writing of the book. This book could not have been written without the many contributions of the students, post-docs, collaborators and granting agencies, cited in Dror Sarid's (one of the author's) papers related to short- and long-range surface plasmons. Many thanks are also due

Cambridge University Press

978-0-521-76717-0 - Modern Introduction to Surface Plasmons: Theory, Mathematica Modeling and Applications

Dror Sarid and William A. Challener

Frontmatter

[More information](#)

---

xiv

*Preface*

to Professor Richard W. Ziolkowski for helpful discussions involving metamaterials, and to Tammy Orr and Juliet A. Hughes for their able help in editing chapters of this book. Bill Challener (one of the authors) would also like to thank several of his colleagues who have shared their expertise with him in both the theory and applications of surface plasmons, including Dr. Edward Gage, Dr. Amit Itagi, Dr. Chubing Peng, Dr. Timothy Rausch and Dr. Zhongping Yang.

*Dror Sarid*

*Bill Challener*

Tucson, Arizona

Eden Prairie, Minnesota

July, 2009