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Excerpt
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Introduction

In 1952 Pines and Bohm discussed a quantized bulk plasma oscillation of electrons in a metallic solid to explain the energy losses of fast electrons passing through metal foils [1]. They called this excitation a "plasmon." Today these excitations are often called "bulk plasmons" or "volume plasmons" to distinguish them from the topic of this book, namely surface plasmons. Although surface electromagnetic waves were first discussed by Zenneck and Sommerfeld [2, 3], Ritchie was the first person to use the term "surface plasmon" (SP) when in 1957 he extended the work of Pines and Bohm to include the interaction of the plasma oscillations with the surfaces of metal foils [4].

SPs are elementary excitations of solids that go by a variety of names in the technical literature. For simplicity in this book we shall always refer to them as SPs. However, the reader should be aware that the terms "surface plasmon polariton" (SPP) or alternately "plasmon surface polariton" (PSP) are used nearly as frequently as "surface plasmon" and have the advantage of emphasizing the connection of the electronic excitation in the solid to its associated electromagnetic field. SPs are also called "surface plasma waves" (SPWs), "surface plasma oscillations" (SPOs) and "surface electromagnetic waves" (SEWs) in the literature, and as in most other technical fields, the acronyms are used ubiquitously. Other terms related to SPs which we will discuss in the course of this book include "surface plasmon resonance" (SPR), "localized surface plasmons" (LSPs), "long-range surface plasmons" (LRSPs) and of course "short-range surface plasmons" (SRSPs).

There are a variety of simple definitions in the literature for SPs. Many of these are inadequate or incomplete. The "on" suffix emphasizes the fact that SPs have particle-like properties including specific energies and (for propagating modes) momenta, and strictly speaking should be considered in the context of quantum mechanics. In this spirit, one might define a SP as a quantized excitation at the interface between a material with a negative permittivity and free charge carriers (usually a metal) and a material with a positive permittivity which involves a collective oscillation of surface charge and behaves like a particle with a discrete energy and, in the case of propagating SPs, momentum. We will find, however, that most of the important properties of SPs can be satisfactorily described in a classical electromagnetic model, which is all that we will employ in this book. A SP may be defined classically as a fundamental electromagnetic mode of an interface between a material with a negative permittivity and a material with a positive permittivity having a well-defined frequency and which involves electronic surface-charge oscillation. It is, of course, relevant to ask whether or not a classical description of SPs is acceptable. Bohren and Huffman address this question for nanoparticles directly [5]. They state,

> "Surface modes in small particles are adequately and economically described in their essentials by simple classical theories. Even, however, in the classical description, quantum mechanics is lurking unobtrusively in the background; but it has all been rolled up into a handy, ready-to-use form: the dielectric function, which contains all the required information about the collective as well as the individual particle excitations. The effect of a boundary, which is, after all, a macroscopic concept, is taken care of by classical electrodynamics."

This statement can be extended to all of the systems we are considering, not just small particles. If the objects supporting SPs are large enough that they can be described by a dielectric function (permittivity), then the classical approach should generally be adequate. This will be the case if the mean free path of the conduction electrons is shorter than the characteristic dimensions of the objects in the SP system. In practice it is found that the bulk dielectric constant accurately describes objects with dimensions down to ∼10 nm, and that a size-dependent dielectric constant can be employed for objects with dimensions down to about 1–2 nm [6–8]. For a detailed discussion about size effects of the dielectric function for small metal clusters, see Refs. [9] and [10]. As discussed in the Preface, the equations in this text are derived from Maxwell's equations as expressed in the SI system of units.

This text is based on *Mathematica*. *Mathematica* was not simply used as a word processor for formatting mathematical equations, but was also used to generate numerous figures within the text. The *Mathematica* notebooks, which are included in the online supplementary materials at the web site www.cambridge.org/9780521767170, contain all of the *Mathematica* code, color figures and some additional text. The notebooks can be used to regenerate many of the figures. Moreover, the reader may easily modify parameters in the *Mathematica* notebook code and recompute the figure for perhaps a different wavelength range or different material, etc. In chapters that discuss material properties, the refractive indices for a wide variety of plasmonic, noble and transition metals are available for calculations in addition to those materials which are specifically used in the figures. Some examples of the algorithms that are included in the *Mathematica* notebooks are a simple theory of the interaction of light with cylindrical nanowires and nanotubes in Chapter 8, Mie theory for calculations with spherical nanoparticles and nanoshells in Chapter 9, and the theory of Chandezon for vector diffraction of light from gratings in Chapter 10. In general, the reader should open the *Mathematica* notebook for the chapter of interest (it is, of course, necessary to purchase and install *Mathematica* first) and at the very beginning of each notebook there is a section labelled "Code." The experienced *Mathematica* user knows to double click on the downward arrow of the rightmost bracket of this section in order to expand it. The first paragraph in the Code section describes the steps that the *Mathematica* user should employ to reproduce a figure in the text. The reader is strongly encouraged to take advantage of these *Mathematica* features to gain the full benefit of the text! The online supplementary materials also include a pdf version of the color figures and a description of the Chandezon vector diffraction theory.

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Electromagnetics of planar surface waves

2.1 Introduction

This chapter presents the electromagnetic theory that describes the main characteristics of surface electromagnetic modes in general and surface plasmons (SPs) in particular that propagate along single- and double-interface planar guiding structures. We begin with an introduction to electromagnetic theory that discusses Maxwell's equations, the constitutive equations and the boundary conditions. Next, Maxwell's equations in terms of time-harmonic fields, electric and magnetic fields in terms of each other, and the resultant wave equations are presented. Group velocity and phase velocity, surface charge at a metal/dielectric interface and the perfect electric conductor conclude this introduction. Following this introduction are sections that describe the properties of electromagnetic modes that single- and double-interface planar guiding structures can support in terms of the media they are composed of. These media will be presented in terms of their permittivity and permeability whose real part can be either positive or negative. A new formalism will be developed to treat such media in the context of natural materials such as metals and dielectrics and in terms of a collection of subwavelength nanostructures dubbed metamaterials. Finally, the power flow along and across the guiding structures is presented, and the reflectivity from the base of a coupling prism and the accompanied Goos–Hänchen shift are treated. The material covered in this chapter draws heavily from Refs. [1] to [3] for the theory of electromagnetic fields and from Refs. [4] and [5] for the theory of optical waveguides. The theory of metamaterials and their applications as guiding media makes use of Refs. [6] to [12] where citations to a vast body of literature can be found. The concept of Poynting vectors and energy flow in general and in metamaterials in particular is adapted from Refs. [13] to [15].

2.2 Topics in electromagnetic theory

2.2.1 Maxwell's equations

The electromagnetic fields in empty space are given in terms of two vectors, $\boldsymbol{E}$ and $\boldsymbol{B}$, called the electric vector and magnetic induction, respectively. The presence of matter in the space occupied by these vector fields requires three more vectors, $\boldsymbol{D}$, $\boldsymbol{H}$ and $\boldsymbol{j}$, called electric displacement, magnetic vector and free electric current density, respectively. Each one of these five vectors, whose components are described in terms of the Cartesian unit vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$, can be complex, which means that they have a phase relative to each other as well as to the components of the other vectors. The space- and time-dependence of these five vectors are prescribed by Maxwell's vector and scalar equations. The two vector equations, in terms of the curl ($\nabla\times$) operator and the partial time derivative ($\partial/\partial t$), are given by

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{2.1}$$

and

$$\nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{j}. \tag{2.2}$$

The two scalar equations are given in terms of the divergence ($\nabla\cdot$) operator by

$$\nabla \cdot \boldsymbol{D} = \rho \tag{2.3}$$

and

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2.4}$$

where ρ denotes free electric charge density.

2.2.2 Constitutive equations

The presence of matter modifies the electromagnetic fields that are described by three constitutive (material) equations. For linear media, these equations take the form

$$\boldsymbol{D} = \epsilon_0 \, \epsilon_r \, \boldsymbol{E}, \tag{2.5}$$

$$\boldsymbol{B} = \mu_0 \, \mu_r \, \boldsymbol{H}, \tag{2.6}$$

and

$$\boldsymbol{j} = \sigma \, \boldsymbol{E}. \tag{2.7}$$

Here, ϵ_r, μ_r are the relative (electric) permittivity, relative (magnetic) permeability and specific conductivity, respectively, which are in general tensors: ϵ_0 and μ_0 are the permittivity and permeability of free space, and σ is the specific conductivity. Except for ϵ_0, whenever ϵ and μ have subscripts they denote relative values, while otherwise $\epsilon = \epsilon_0 \epsilon_r$ and $\mu = \mu_0\mu_r$. Note that ϵ is also called the dielectric constant, or dielectric function. Note also that from here on, the real and imaginary parts of ϵ_r, μ_r and any other parameter will be marked by a prime or double prime, respectively. Throughout this book we consider only "simple" materials; namely, those that are linear, isotropic and homogeneous (LIH), for which ϵ and μ are scalars. Although such an assumption is not strictly valid for metamaterials, we will still use it because it simplifies the treatment of their optical response.

2.2.3 Boundary conditions

To obtain a full description of an electromagnetic field, we must supplement the four Maxwell equations and the three constitutive equations with four continuity equations. This third group of equations, called the boundary conditions, imposes restrictions on the electromagnetic fields at an abrupt interface separating two media. Let $\hat{\boldsymbol{n}}_{12}$ denote a unit vector pointing from media 1 to media 2 that is perpendicular to an infinitesimal area of this interface. Elementary considerations dictate the existence of two vector equations,

$$\hat{\boldsymbol{n}}_{12} \times \left(\boldsymbol{E}^{(2)} - \boldsymbol{E}^{(1)}\right) = 0 \tag{2.8}$$

and

$$\hat{\boldsymbol{n}}_{12} \times \left(\boldsymbol{H}^{(2)} - \boldsymbol{H}^{(1)}\right) = \hat{\boldsymbol{j}}. \tag{2.9}$$

Here, the tangential component of $\boldsymbol{E}$ is continuous across this interface, while the tangential component of $\boldsymbol{H}$ equals the surface electric current density, $\hat{\boldsymbol{j}}$, across this interface. Also dictated are two scalar equations,

$$\hat{\boldsymbol{n}}_{12} \cdot \left(\boldsymbol{D}^{(2)} - \boldsymbol{D}^{(1)}\right) = \hat{\rho} \tag{2.10}$$

and

$$\hat{\boldsymbol{n}}_{12} \cdot \left(\boldsymbol{B}^{(2)} - \boldsymbol{B}^{(1)}\right) = 0, \tag{2.11}$$

where the subscripts $i = 1$ and 2 refer to each of the bounding media. Equations (2.10) and (2.11) show that the normal component of $\boldsymbol{D}$ equals the surface charge density, $\hat{\rho}$, across the interface, while the normal component of $\boldsymbol{B}$ is continuous across this interface. Note that the most frequently used boundary conditions relate to $\boldsymbol{E}$ and $\boldsymbol{H}$ which will also be referred to as the (vector) electric and (vector) magnetic fields.

2.2.4 Maxwell's equations in terms of time-harmonic fields

Let $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$ and $\boldsymbol{B}$ be time-harmonic propagating fields, denoted generally by $\boldsymbol{F}$. $\boldsymbol{F}$ can be decomposed into a time-independent part, $\boldsymbol{F}_0$, multiplied by the time-harmonic function $e^{i\omega t} = \cos(\omega t) + i\,\sin(\omega t)$, where f, $\omega = 2\pi f$ and t denote frequency, angular frequency and time, respectively, and $i = \sqrt{-1}$. In the next section we treat a propagating wave in terms of $\boldsymbol{F}$ such that

$$\boldsymbol{F} = \boldsymbol{F}_0\, f e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}. \tag{2.12}$$

Here, $\boldsymbol{F}_0 = \boldsymbol{F}_0(\boldsymbol{r})$ is a space-dependent and time-independent vector field, $\boldsymbol{r}$ a position vector and $\boldsymbol{k}$ a complex wave vector perpendicular to the plane of constant phase of a propagating field. Note that the real and imaginary parts of $\boldsymbol{k}$ will be denoted by $\boldsymbol{k}'$ and $\boldsymbol{k}''$, respectively. Let $\boldsymbol{k}$ have three Cartesian components given by

$$\boldsymbol{k} = k_x\,\hat{\boldsymbol{x}} + k_y\,\hat{\boldsymbol{y}} + k_z\,\hat{\boldsymbol{z}}, \tag{2.13}$$

such that the vector field $\boldsymbol{F}$, when propagating along the $\hat{\boldsymbol{k}}$-direction, can be written explicitly as

$$\boldsymbol{F} = \boldsymbol{F}_0\, e^{i\left(k_x{}'x+k_y{}'y+k_z{}'z-\omega t\right)}\, e^{-k_x{}''x}\, e^{-k_y{}''y}\, e^{-k_z{}''z}. \tag{2.14}$$

Here, the real (primed) and imaginary (double-primed) parts in the exponents represent the propagating and decaying parts of the wave, respectively. It will be convenient to express the curl of $\boldsymbol{F}$ using the determinant form, namely

$$\nabla \times \boldsymbol{F} \equiv \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}. \tag{2.15}$$

The components of the determinant are

$$(\nabla \times \boldsymbol{F})_x \equiv \hat{\boldsymbol{x}}\left(\frac{\partial}{\partial y}F_z - \frac{\partial}{\partial z}F_y\right), \tag{2.16}$$

$$(\nabla \times \boldsymbol{F})_y \equiv -\hat{\boldsymbol{y}}\left(\frac{\partial}{\partial x}F_z - \frac{\partial}{\partial z}F_x\right) \tag{2.17}$$

and

$$(\nabla \times \boldsymbol{F})_z \equiv \hat{\boldsymbol{z}}\left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right). \tag{2.18}$$

If $\boldsymbol{F}_0$ is not only frequency independent but also space-independent, then the two Maxwell vector equations, Eqs. (2.1) and (2.2), can be written, respectively, as

$$\boldsymbol{k} \times \boldsymbol{E} - \omega\,\boldsymbol{B} = 0 \tag{2.19}$$

and

$$i\,\boldsymbol{k} \times \boldsymbol{H} + i\,\omega\,\epsilon\,\boldsymbol{E} = \sigma\,\boldsymbol{E}, \tag{2.20}$$

which can also be written as

$$\boldsymbol{k} \times \boldsymbol{B} + \mu\,(\omega\,\epsilon_0\,\epsilon_r + i\,\sigma)\,\boldsymbol{E} = 0. \tag{2.21}$$

We can define a generalized form of relative permittivity, $\hat{\epsilon_r}$, where the electric conductivity is absorbed into the conventional definition of the permittivity $\hat{\epsilon_r}$,

$$\hat{\epsilon_r} = \epsilon_r + i\,\sigma\,\big/(\epsilon_0\,\omega)\;, \tag{2.22}$$

such that

$$\boldsymbol{k} \times \boldsymbol{B} + \omega\epsilon_0\hat{\epsilon_r}\,\mu\,\boldsymbol{E} = 0. \tag{2.23}$$

From now on, for simplicity, we will omit the hat above ϵ_r. The two Maxwell scalar equations, Eqs. (2.3) and (2.4), can also be expressed in terms of time-harmonic functions by

$$\boldsymbol{k} \cdot \boldsymbol{E} = 0 \tag{2.24}$$

and

$$\boldsymbol{k} \cdot \boldsymbol{B} = 0. \tag{2.25}$$

We can rewrite Eq. (2.19), assuming a plane-parallel wave propagating in the $\hat{\boldsymbol{k}}$ direction where $\boldsymbol{E} \perp \boldsymbol{H} \perp \boldsymbol{k}$, as

$$\boldsymbol{H} = \frac{k}{\omega\,\mu}\hat{\boldsymbol{k}} \times \boldsymbol{E}, \tag{2.26}$$

where $\hat{\boldsymbol{k}}$ is a unit vector in the $\boldsymbol{k}$ direction. Using $\lambda f = v = c/(\sqrt{\epsilon_r}\sqrt{\mu_r})$, where c and v are the speed of light in free space and in the medium in which the wave propagates, respectively, gives

$$\boldsymbol{H} = \sqrt{\frac{\epsilon}{\mu}}\,\hat{\boldsymbol{k}} \times \boldsymbol{E}. \tag{2.27}$$

Note that we have explicitly used $\sqrt{\epsilon_r}\sqrt{\mu_r}$ rather than $\sqrt{\epsilon_r\,\mu_r}$ as will be explained at a later stage.

2.2.5 *Electric and magnetic fields in terms of each other*

The determinant representation of the curl of $\boldsymbol{E}$ is

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = i\,\mu\,\omega\,\boldsymbol{H}. \tag{2.28}$$

It yields the three components of $\boldsymbol{H}$ in terms of the partial derivatives of $\boldsymbol{E}$,

$$H_x = \frac{-i}{\mu\,\omega}\left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right), \tag{2.29}$$

$$H_y = \frac{-i}{\mu\,\omega}\left(\frac{\partial}{\partial x}E_z - \frac{\partial}{\partial z}E_x\right) \tag{2.30}$$

and

$$H_z = \frac{-i}{\mu\,\omega}\left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right). \tag{2.31}$$

We can repeat the same procedure for the curl of $\boldsymbol{H}$,

$$\nabla \times \boldsymbol{H} \equiv \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = -i\,\epsilon\,\omega\,\boldsymbol{E}, \tag{2.32}$$

from which the three components of $\boldsymbol{E}$ are derived,

$$E_x \equiv \frac{i}{\epsilon\,\omega}\left(\frac{\partial}{\partial y}H_z - \frac{\partial}{\partial z}H_y\right), \tag{2.33}$$

$$E_y = \frac{i}{\epsilon\,\omega}\left(\frac{\partial}{\partial x}H_z - \frac{\partial}{\partial z}H_x\right) \tag{2.34}$$

and

$$E_z = \frac{i}{\epsilon\,\omega}\left(\frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x\right). \tag{2.35}$$

We will use Eqs. (2.29) to (2.31) and (2.33) to (2.35) extensively when solving for the electromagnetic modes that propagate along interfaces that separate two or more media.

2.2.6 Wave equations and the appearance of a refractive index

We now introduce a wave equation that describes the propagation of an electromagnetic wave in terms of its electric and magnetic fields. Let us start by applying vector calculus to Eqs. (2.1), (2.2), (2.5) and (2.6), assuming that we deal with a simple material and with harmonic fields. Eliminating $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{H}$ yields the second-order differential equation in $\boldsymbol{E}$

$$\nabla^2\,\boldsymbol{E} - \epsilon\,\mu\frac{\partial^2}{\partial t^2}\boldsymbol{E} = \nabla^2\,\boldsymbol{E} - \frac{n}{c}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = 0, \tag{2.36}$$

with an identical equation where $\boldsymbol{H}$ replaces $\boldsymbol{E}$. These two equations are called wave equations because they connect the second-order spatial derivative of a field with its second-order temporal derivative. Note that the parameter n in Eq. (2.36), which is called the refractive index, is usually given as the square root of the product of ϵ_r and μ_r. This is fine for materials whose $\epsilon_r{}'$ and $\mu_r{}'$ are not both negative. However, this is not the case for metamaterials for which both can be negative across a frequency range. To accomodate such a case we will from now on broaden the concept of the refractive index and define it as $n = \sqrt{\epsilon_r}\sqrt{\mu_r}$. Thus, n is positive for positive $\epsilon_r{}'$ and $\mu_r{}'$ and negative for negative $\epsilon_r{}'$ and $\mu_r{}'$. If only one is negative, the two definitions are identical. Note that this is our first encounter of a parameter composed of a square root of two other parameters. We will encounter other such cases as we go along. Note that the extension to complex values of ϵ_r, μ_r and n is not straightforward, because the sign of the imaginary part of the refractive index is associated with a decaying or growing field, so that energy conservation has to be taken into account. The solution of Eq. (2.36), whose general form is given by Eq. (2.12), yields

$$\boldsymbol{E} = \boldsymbol{E}_0\, e^{i\left(n\, k_0\, \hat{\boldsymbol{k}}\cdot\boldsymbol{x} - \omega\, t\right)}. \tag{2.37}$$

Equation (2.37) describes a wave that propagates with a velocity v given by

$$v = c/n, \tag{2.38}$$

where $c = 1/\sqrt{\epsilon_0\, \mu_0}$ is the speed of light in free space.

2.2.7 Group velocity and phase velocity

Consider now a scalar wave packet, $E(t, z)$, consisting of a superposition of scalar plane-parallel harmonic waves, propagating in the z-direction in a simple medium, having a Gaussian envelope. The packet as a function of t and z is given by

$$E(t,\; z) = \int_{-\infty}^{\infty} E_0(\omega)\epsilon^{i[k(\omega)z-\omega\, t]} d\,\omega, \tag{2.39}$$

and shown in Fig. 2.1 as a function of t at a fixed position z.

The k-vector associated with this wave packet, $k(\omega)$, can be expanded in a Taylor series at $\bar{\omega}$, yielding to second order

$$k(\omega) = k\,(\bar{\omega}) + \frac{d\,k(\omega)}{d\,\omega}\Delta\omega. \tag{2.40}$$

Here $\bar{\omega}$ is the mean angular frequency of the wave packet and $\Delta\omega$ is defined by

$$\Delta\omega = \omega - \bar{\omega}. \tag{2.41}$$

Equation (2.39) can now be written as

$$E(t, z) = \epsilon^{i[k(\bar{\omega})z-\bar{\omega}\, t]} \int_{-\infty}^{\infty} E_0(\omega)\, e^{-i\,\Delta\omega\left(t-\frac{d\,k(\omega)}{d\,\omega}|_{\bar{\omega}}z\right)}\, d\,\omega. \tag{2.42}$$