The Evolution of Logic

The Evolution of Logic examines the relations between logic and philosophy over the last 150 years. Logic underwent a major renaissance beginning in the nineteenth century. Cantor almost tamed the infinite, and Frege aimed to undercut Kant by reducing mathematics to logic. These achievements were threatened by the paradoxes, like Russell's. This ferment generated excellent philosophy (and mathematics) by excellent philosophers (and mathematicians) up to World War II. This book provides a selective, critical history of the collaboration between logic and philosophy during this period.

After World War II, mathematical logic became a recognized subdiscipline in mathematics departments, and consequently but unfortunately philosophers have lost touch with its monuments. This book aims to make four of them (consistency and independence of the continuum hypothesis, Post's problem, and Morley's theorem) more accessible to philosophers, making available the tools necessary for modern scholars of philosophy to renew a productive dialogue between logic and philosophy.

W. D. Hart is currently Professor of Philosophy at the University of Illinois at Chicago, where he chaired the philosophy department for twelve years. He has also taught at the University of Michigan, University College London, and the University of New Mexico. He is the author of *The Engines of the Soul*, now available in paperback (Cambridge, 2009), and the editor of *The Philosophy of Mathematics* and has published more than seventy articles and reviews in academic journals.

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For my wife, Faith

The Evolution of Logic

W. D. HART

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Contents

Preface

In high school I read a book from the school library called *Introduction to Mathematical Thought* by E. R. Stabler. Early on he described a pattern in which he displayed a conditional, its antecedent, and its consequent. I stared at this display, trying to get the point. Then the penny dropped: this is a way the mind *moves*, from the first two to the third. That was my first experience of explicit logic, and it was eye-opening.

I took my first logic course at university from W. V. Quine. He lectured by mumbling at his stack of three-by-five cards, but the textbook was the second edition of his *Methods of Logic*, which I still think is the best baby logic book I have seen. As I learned gradually who Quine was, I was too awestruck to approach him again. Instead, I did a lot of work with his former student Burt Dreben. Dreben described himself as a logical positivist, but he was really a philosophical nihilist. He once said in lecture, "Rubbish is rubbish, but the history of rubbish is scholarship." The rubbish was all of philosophy, and the scholarship was where he wanted me directed. He taught us a lot of logic and early analytic philosophy. He first taught me Gödel's incompleteness theorem from Rudolf Carnap's *The Logical Syntax of Language*, and he later taught it from Gödel's original paper. Both now seem to me perverse pedagogy, especially the first, but it reflected his historical taste.

I never succumbed to Dreben's philosophical nihilism. He once told me I had a twelfth-century mind, and though I have never been able to penetrate scholastic philosophy, I still understood what he meant and pretty much agreed with him. I got from him a taste for the history of modern logic and analytic philosophy, but I wanted to do philosophy too, so I wanted to argue about whether authors we were working on are right or wrong. The first five chapters that follow grew out of

the kind of training I got from Dreben. They go from Dedekind and Cantor up to Gödel's incompleteness theorem, but they are a selective history, and it is critical – I allow myself to do some philosophy as we go along.

I continued to work with Dreben as I moved on to graduate school, but that was when Hilary Putnam joined the faculty of Harvard. Putnam is no philosophical nihilist. He reminded me a bit of Russell earlier on: both were amazing sources of a wealth of new philosophical ideas. Perhaps one sometimes wanted to try to do a second draft oneself, but that could be very instructive. Putnam taught a wonderful course in Gödel's and Cohen's (then recent) results on the consistency and independence of Cantor's continuum hypothesis. That course marked me for life.

In my first job after graduate school, I met Fred Goldstein, who had come to Michigan from the logic program at Stanford. Somewhere along the way I had developed the notion that only people with theorems named after them were logicians, so Russell and Quine were not logicians. Fred saw how silly and self-destructive this notion was. He thought that modern logic was distinguished by certain methods. After Gödel's method of arithmetization, which was like basic literacy, these distinctive methods were constructible sets, forcing and generic sets, and the priority method. Fred said a logician knows at least some of these methods. Fred's idea was liberating, and since I had seen constructible sets and generic sets with Putnam, Fred gave me a yen to learn the priority method. I regret to say that Fred was soon diagnosed with leukemia and died very young.

At that same university, Michael Morley had been denied tenure a few years earlier. He had then gone on to prove Morley's theorem, which inaugurated the second great age of model theory. That story shows there can be triumph after rejection. I think Fred died too early to see that Morley's theorem belongs with Fred's three distinctive methods. It took a while for the recognition of it to begin to percolate out from the specialists. The point of Chapters 6 through 9 is to present versions of Fred's four monuments to philosophers.

In the early 1990s, Paul Guyer asked me to write the logic and philosophy volume for a series of books he was editing for Cambridge University Press. I was intrigued by the project and wrote a proposal, but I was then taking up administrative chores that made sustained thought difficult, so the project hung fire for a long time. Then, as I was giving up administration, Paul asked whether I was still interested in

the project. You and I can be grateful to Paul for what you hold in your hands. There can be intellectual life after administration. I am grateful to Beatrice Rehl of Cambridge University Press and especially to Russell Hahn for their skillful shepherding of my manuscript through production.

In the early stages I had invaluable advice and support from my friend and former student Matthew Moore, now at Brooklyn College. Another friend and former student (the first one to retire), Andrew Lugg, also gave me useful comments. I wish to thank my able colleagues and friends Daniel Sutherland and Walter Edelberg for their insightful discussions and suggestions. As I was writing, Daniel organized a reading group on the philosophy of mathematics. That group was kind enough to discuss some of the earlier chapters, and I am grateful for their criticism and help. I must confess that I lost track of which graduate students dropped in or out, but I must acknowledge Sean Paul Morris (who also read the page proofs) and my astonishing former student Mihai Ganea. And I am grateful to Robert Fischer for preparing the four diagrams for publication.

My wife, Faith, is not mathematical, and she regards philosophy, especially my twelfth-century version of it, as pretty bizarre. She once told me that I think numbers get up and dance around. She edits and designs books, mostly in the history of art. I write longhand, and she types and edits what I write, and doing that with symbols is a labor of love. She also prods me to keep me going. Marrying Faith was the best thing I ever did.