

## **Inertial MEMS**

### Principles and Practice

A practical and systematic overview of the design, fabrication, and testing of MEMS-based inertial sensors, this comprehensive and rigorous guide shows you how to analyze and transform application requirements into practical designs, and helps you to avoid potential pitfalls and to cut design time.

With this book you'll soon be up to speed on the relevant basics, including MEMS technologies, packaging, kinematics and mechanics, and transducers. You'll also get a thorough evaluation of different approaches and architectures for design and an overview of key aspects of testing and calibration.

Unique insights into the practical difficulties of making sensors for real-world applications make this up-to-date description of the state of the art in inertial MEMS an ideal resource for professional engineers in industry, managers, and application engineers, as well as for students looking for a complete introduction to the area.

VOLKER KEMPE has more than 40 years of experience in research and development in both academia and industry. He led the microelectronics engineering department at Austria Mikro Systems for over 10 years. In 2003 he co-founded, and became Vice President of, SensorDynamics AG, and his current interests focus on the functionality, technology, and application of inertial MEMS.

Cambridge University Press  
978-0-521-76658-6 - Inertial MEMS: Principles and Practice  
Volker Kempe  
Frontmatter  
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Volker Kempe  
*Sensor Dynamics AG, Austria*

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CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521766586](http://www.cambridge.org/9780521766586)

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First published 2011

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Kempe, Volker.

Inertial MEMS : principles and practice / Volker Kempe.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-521-76658-6 (hardback)

1. Microelectromechanical systems. 2. Inertial navigation systems. 3. BioMEMS. I. Title.

TK7875.K46 2011

629.04'5 - dc22 2010037668

ISBN 978-0-521-76658-6 Hardback

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## Preface

Inertial microelectromechanical sensors – commonly abbreviated to inertial MEMS – have a history of more than two decades of intense research, development, and commercialization. Sometimes unperceived, they left the shadow of military and space-related utilities and entered daily life hidden in products surrounding us. Cars with airbag-release sensors and electronic stability control have become a matter of course. Activity monitoring of pacemaker patients and stabilization of platforms such as transport robots and cameras are now improving our quality of life. The creation of easy-to-use human–machine interfaces has helped many people to conquer complicated equipment around us, not only computer games. The penetration of inertial MEMS – often merged with other sensor systems – into new application areas is a trend that is still gaining momentum.

The intention of this book is to reflect the interdisciplinary complexity of inertial MEMS. It will try to give a systematic survey of the design, fabrication, and performance evaluation of MEMS-based inertial sensors, with emphasis on the practical problems arising from the impact of technological imperfections and of often harsh environmental conditions. A product going to the market has to be guaranteed to have a certain level of reliability against failure throughout its lifetime.

The basic concepts and the theoretical background of inertial measurements will be presented. However, the book has evolved not from academic activity but rather from conceptual and development work within industry. It is intended to address the symbiosis of practice and theory. Consequently, the analysis and transformation of application requirements into design concepts plays a significant role. Considerable space is devoted to the analysis and modeling of parasitic effects, of shock and vibration robustness, of the stability of the main performance parameters and so on, since this is necessary for practical work.

The book contains nine chapters. Six of them – including the introduction – cover various aspects of MEMS, with a special focus on inertial MEMS. The first chapter describes the most important transducers and their properties. The second one is dedicated to non-inertial forces such as spring forces and damping forces that play a crucial role for designing inertial MEMS. The next two chapters cover the main MEMS technologies, including packaging, while the electronic interfaces are presented in a further chapter. These six chapters may be

interesting not only for people working with inertial MEMS but also for everybody who is looking for a general introduction into mechanical MEMS.

The following two main chapters are devoted to the two representatives of inertial MEMS – accelerometers and gyroscopes. Here the focus is on the basic principles, the methods and models to describe the dynamic behavior, and a comprehensive presentation of different approaches and architectures, including their pros and cons. A short overview on test and calibration is added as a separate chapter.

The book is written on an engineering level. Where possible, effects and processes are described analytically by mathematical models in order to impart a feeling for the order of magnitude of different effects.

The book should be useful not only for specialists developing, manufacturing, and using inertial sensors but also for people working in the application field, for product managers, and for sales people looking for background knowledge in their area. The book can serve as a starting point for further academic investigations, for instance in the area of shock-impact analysis of an entire packaged gyro, including the effect of signal processing.

In the experience of the author, many engineers, physicists, and mathematicians are thankful for an exact but comprehensible presentation of the complex and difficult world of MEMS-based inertial sensors, where the effects and models behind the practical problems are reflected without improper simplifications or phenomenological descriptions. The book is a modest attempt to meet some of these challenges. Having worked with many specialists in the production, testing, and design of inertial sensors, the author is convinced that the book can meet actual needs, and hopes to elicit the broad interest of practitioners and scientists in this area. For interested people, including students, the book may also serve as an introduction to the world of mechanical MEMS.

## Acknowledgments

I would like to express my gratitude to all my colleagues at SensorDynamics AG (Austria) and the Institut für Silizium Technologie (ISIT) of the Fraunhofer Society (Germany) for creating an atmosphere that has helped to solve the manifold problems of MEMS industrialization. I would like to thank the ‘Inertial Micro Sensor Systems’ team, with whom I have had the great privilege of working even during the childhood of the newly founded company SensorDynamics AG. This time was most fruitful, flooding us all with new problems and insights into how to solve them.

My thanks go to my colleagues from SensorDynamics for providing me with such necessary illustrative material for the book as SEM photographs and measurement plots. Gottfried Fraiss, Manfred Heller, Christian Rossadini, Jörg Schönbacher, Ute Stotter, and Johann Wagner prepared a lot of material from which I could select the most appropriate items. Gerd Radl and his team accompanied me during all my mistakes with new hardware and software.

Drago Strle from the University of Ljubljana deserves my special thanks for the close cooperation throughout all  $\Sigma\Delta$ -related issues. Peter Merz from the ISIT gave me invaluable feedback on all technology-related questions. Professor Karl Wohllhart from the University of Graz helped me to gain a deeper understanding of the kinematics of gyroscopes. Hanno Hammer took on the burden of proof-reading the first chapters and supplying me with valuable feedback.

Last but not least, warmest thanks go to my family for supporting me despite all the personal loads that each of us has had to carry. Julia and Oded helped me to resolve quickly the countless difficulties in adapting appropriate document-creating tools to my needs. Vera, Ian, and Marius gave valuable advice on English phrasing. My dear wife took on additional duties despite her own excessive workload in working with very ill patients. Thank you.

## Notation

1. A convention employed in this book is the slightly lax usage of “ $s$ ” as differential operator,  $s = d/dt$ , as argument of the Laplace transformation, and as argument of the Fourier transformation,  $s = j\omega$ . The case-dependent unambiguous or multivalent meaning is usually clear from the context. Accordingly, a filter function is described by  $f = f(s)$ , which means that in a transfer function this expression has to be interpreted as a Laplace or Fourier transformation and within a differential equation as a rational fraction of two polynomial differential operators. Correspondingly, a variable like  $x$  has to be treated as a representant in the time domain if  $s = d/dt$  is supposed, or as a Laplace/Fourier-transformed function if  $s$  is meant as the argument of such a transformation.
2. Unless stated otherwise, coordinate systems pertain to the platform carrying the inertial sensor. In this case the  $x$ - and  $y$ -axes lie in the plane of the platform, while  $z$  is the out-of-plane axis. Out-of-plane and  $z$ -axis orientation are used synonymously.