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978-0-521-76505-3 - Handbook of Hybrid Systems Control: Theory, Tools, Applications
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Part I

Theory

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Introduction to hybrid systems

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This chapter gives an informal introduction to hybrid dynamical systems and illustrates by simple examples the main phenomena that are encountered due to the interaction of continuous and discrete dynamics. References to numerous applications show the practical importance of hybrid systems theory.

Chapter contents

Handbook of Hybrid Systems Control: Theory, Tools, Applications, ed. Jan Lunze and Françoise Lamnabhi-Lagarrigue. Published by Cambridge University Press.
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1.1 What is a hybrid system?

Wherever continuous and discrete dynamics interact, hybrid systems arise. This is especially profound in many technological systems, in which logic decision making and embedded control actions are combined with continuous physical processes. To capture the evolution of these systems, mathematical models are needed that combine in one way or another the dynamics of the continuous parts of the system with the dynamics of the logic and discrete parts. These mathematical models come in all kinds of variations, but basically consist of some form of differential or difference equations on the one hand and automata or other discrete-event models on the other hand. The collection of analysis and synthesis techniques based on these models forms the research area of hybrid systems theory, which plays an important role in the multi-disciplinary design of many technological systems that surround us.

1.1.1 Three reasons to study hybrid systems

The reasons to study hybrid systems can be quite diverse. Here we will provide three sources of motivation, which are related to (i) the design of technological systems, (ii) networked control systems, and (iii) physical processes exhibiting non-smooth behavior.

Challenges of multi-disciplinary design When designing a technological system (Fig. 1.1) such as a wafer stepper, electron microscope, copier, robotic system, fast component mounter, medical system, etc., multiple disciplines need to make the overall design in close cooperation. For instance, the electronic design, mechanical design, and software design together have to result in a consistent, functioning machine. The designs are typically made in parallel by multiple groups of people, where the communication between these groups is often hampered by lack of common understanding and common models. The lack of common models complicates the making of cross-disciplinary design decisions that may have advantages for one discipline, but disadvantages for others. To make a good trade-off, the overall effect of such a design decision has to be evaluated as early as possible. As the complexity of a technological system with typically millions of lines of codes and tens of thousands of mechanical components gives rise to many cross-disciplinary design decisions, a framework is required that supports efficient evaluation of design decisions incorporating quantitative information and models from multiple disciplines.

> Hybrid systems theory studies the behavior of dynamical systems, including the technological systems mentioned above described by modeling formalisms that involve both continuous models such as differential or difference equations describing the physical and mechanical part, and discrete models such as finite-state machines or Petri nets that describe the software and logical behavior.

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Fig. 1.1 Example of a technological system with hybrid dynamics (courtesy of Daimler).

This theory is one of the few scientific research directions that aim at approaching the design problem of technological systems in a rigorous manner and at developing a complete design framework. As such, hybrid systems theory combines ideas originating in the computer science and the software engineering disciplines on one hand, and systems theory and control engineering on the other. This mixed character explains the terminology "hybrid systems," which was used in this context for the first time by Witsenhausen in 1966 [665].

Hybrid systems theory is a relatively young research field as opposed to the more conventional mono-disciplinary research areas such as mechanical, electrical, or software engineering. The urgent need for multi-disciplinary design and development methods for technological systems has spurred the growth of hybrid systems theory in recent years. However, due to the inherent complexity of hybrid systems, many issues still remain unsolved at present, at least at the scale needed for industrial applications. The current status of hybrid systems theory is surveyed in this handbook, which can be used as a starting point for future developments in this appealing and challenging research domain.

Adding communication: networked control systems Besides merging software (discrete) and physical (continuous) aspects of systems, another important aspect of many technological systems is communication. Within one single system, many subsystems interact through communication networks. For systems-of-systems the coordination plays an even larger role, resulting in extremely complicated networks of communication. One might think of examples such as automated highways [426] and air-traffic management [629]. As the many control, computation, communication, sensing, or actuation actions take place through shared network or processor resources, another dimension is added to the design of these systems. Within the context of these *networked control systems* (Chapter 15), the asynchronous and event-driven nature of the data transmission caused by varying delays, varying sampling intervals, package loss, etc., and the implementation of the networks and protocols

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complicate their analysis and design even further. Also in this domain hybrid systems theory plays an essential role as a foundation to understand the behavior of these complex systems.

Physical processes modeled as hybrid systems In the technological and networked systems mentioned above, the digital and logic (embedded control) aspects are typically brought in by design in order to control the physics and mechanics of the system. However, hybrid system theory is not only useful within these domains. Many physical processes, exhibiting both fast and slow changing behaviors, can often be well described by using (simple) hybrid models. For instance, in non-smooth mechanics [123], the evolution of impacting rigid bodies can be captured in hybrid models. Indeed, as the impacts occur at a much smaller time scale than the unconstrained motion, the behavior can be described well by introducing discrete events and actions in a smooth model. The bouncing ball presented in Example 2.4 is a simple demonstration of this. Also the vector fields defining the behavior of the system might be different over time as they depend crucially on the fact whether a contact is active or not. The dynamics of a robot arm moving freely in space is completely different from the situation in which it is striking the surface of an object. Other examples in mechanics with hybrid behavior include motion systems with friction models that distinguish between stick and slip modes, backlash in gears, and dead zones in cog wheels.

Examples are not only found in the mechanical domain. Nowadays, switches such as thyristors and diodes are used in electrical networks for a wide variety of applications in both power engineering and signal processing. Examples include switched-capacitor filters, modulators, analog-to-digital converters, power converters, and choppers. In the ideal case, diodes are considered as elements with two (discrete) modes: the blocking mode and the conducting mode. Mode transitions for diodes are governed by state events, where currents or voltages change their sign. This indicates that hybrid modeling and analysis offer an attractive perspective on these switched circuits [306]. The DC-DC converter discussed in Section 1.3.3 forms a simple example of this.

Also many biological and chemical systems can often be efficiently described by hybrid models. For example, simulating moving bed processes, which are a special kind of chromatographic separation processes, have to be switched regularly among different structures in order to avoid that the separation process will eventually stop. Like in DC-DC converters, the switching is an integral part of the physical principle utilized in such processes. For the analysis of these systems and for control design, the model has to be switched accordingly, which demonstrates the necessity to extend continuous models towards hybrid models.

1.1.2 Behavior of hybrid systems

The previous section indicates that multi-disciplinary design of technological systems and the study of several non-smooth physical processes require the understanding of

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the complex interaction between discrete dynamics and continuous dynamics. To provide some insight in this interaction, let us consider the following example.

Example 1.1 *Thermostat*

As a textbook example of a simple hybrid system consider the regulation of the temperature in a house. In a simplified description, the heating system is assumed either to work at its maximum power or to be turned off completely. This is a system that can operate in two modes: "on" and "off." In each mode of operation (given by the discrete state $q \in \{\text{on}, \text{off}\}$) the evolution of the temperature T can be described by a different differential equation. This is illustrated in Fig. 1.2 in which each mode corresponds to a node of a directed graph, while the edges indicate the possible discrete state transitions. As such, this system has a hybrid state (q, T) consisting of a discrete state q taking the discrete values "on" and "off" and a continuous state T taking values in the real numbers.

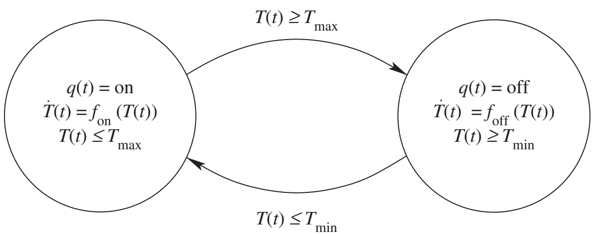

Fig. 1.2 Model of a temperature control system.

Clearly, the value of the discrete state q affects the evolution of the continuous state T as a different vector field is active in each mode. Vice versa, the switching between the two modes of operations is controlled by a logical device (the embedded controller) called the *thermostat* and depends on the value of the continuous state T. The mode is changed from "on" to "off" whenever the temperature T reaches the value $T_{\max}$ (determined by the desired temperature). Vice versa, when the temperature T reaches a minimum value $T_{\min}$, the heating is switched "on."

This example already shows some of the main features of hybrid systems:

- The thermostat is a hybrid system, because its state consists of a discrete state q and a continuous state T.
- The continuous behavior of the system depends on the discrete state, i.e. depending on whether the mode is "on" or "off" a different dynamics $\dot{T}(t) = f_{\text{on}}(T(t))$ or $\dot{T}(t) = f_{\text{off}}(T(t))$, respectively, governs the evolution of the temperature T.
- The changes of the discrete state q are determined by the continuous state T and different conditions on T might trigger the change of the discrete state (e.g. when the discrete state is "on," $T = T_{\max}$ triggers the mode change, while $T = T_{\min}$ triggers the change when the discrete state is "off.") □

Although the thermostat example is rather simple, it already contains some of the basic ingredients that are needed to properly model hybrid systems. A proper modeling format must involve (at least) the description of the evolution of both

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continuous-valued signals (temperatures, positions, velocities, currents, voltages, etc.) and discrete-valued signals (operation mode, position of switch, alarm on or off, etc.) over time and their mutual influence, see Fig. 1.3 for an abstraction of this perspective.

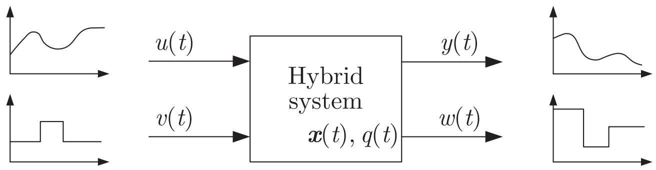

Fig. 1.3 Hybrid dynamical system.

The system depicted in Fig. 1.3 has six types of signals:

- $y(t)$ is a continuous output signal;
- $w(t)$ is a discrete output signal;
- $\boldsymbol{x}(t)$ is a continuous (n-dimensional) state vector;
- $q(t)$ is a discrete state;
- $u(t)$ is a continuous input signal;
- $v(t)$ is a discrete input signal.

The input and output signals may be scalar or vector-valued, but for explaining the main idea of hybrid systems this distinction is not important.

Whereas the discrete signals (such as the "on" and "off" modes of the thermostat example) are typically piecewise constant, the continuous signals may change their value continuously or discontinuously. In the thermostat example the continuous signal representing the temperature is only changing continuously. There are no jumps (discontinuities) in the temperature. The state of the hybrid system is described by the pair $(\boldsymbol{x}, q)$ consisting of the continuous state vector $\boldsymbol{x}$ and the discrete state q. An important characteristic of hybrid systems lies in the fact that this pair influences the future behavior of the system. Moreover, the evolution of the system may also be influenced by a continuous as well as a discrete input, which are denoted by u and v, respectively, and one may receive some information on the hybrid state $(\boldsymbol{x}, q)$ from the discrete and continuous outputs w and y, respectively.

Figure 1.4 displays the typical behavior of an autonomous hybrid system (i.e. a hybrid system without an input), where the scalar continuous state x and the discrete state q are identical to the outputs. It shows that the evolution consists of smooth phases in which the discrete state remains constant and the continuous state changes continuously. At the transition times t_1, t_2, t_3, ... the discrete state changes from its current value to a new value. Simultaneously, the continuous state may jump as shown in the figure for the time t_1. At time t_1 the state changed abruptly from $x(t_1^-)$ to $x(t_1^+)$, where $x(t_1^-)$ and $x(t_1^+)$ denote the (limit) values of x just before and just after the state jump, respectively. It is important to realize that the transition times are

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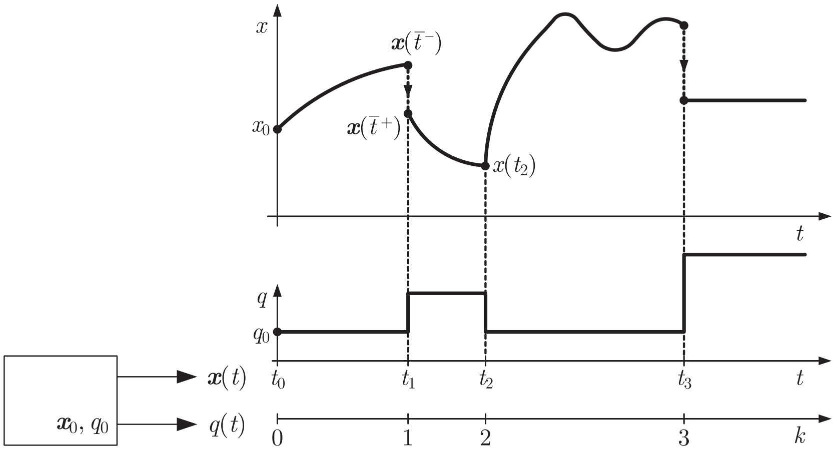

Fig. 1.4 Behavior of an autonomous hybrid system.

not necessarily prescribed by some clock (time events), but usually depend on both the discrete and the continuous state. For instance, in the thermostat example these transition times were determined by the temperature T reaching the values $T_{\min}$ or $T_{\max}$ (state events). In summary:

> The trajectories of hybrid systems are partitioned into several time intervals. At the interval borders, the discrete state changes and/or jumps of the continuous state occur, whereas within all intervals the continuous signals change smoothly and the discrete state remains constant.

For a hybrid system with inputs, the behavior also depends upon the input signals. In this case the time instant at which the discrete state jumps, the new discrete and continuous states that are assumed afterwards as well as the continuous state evolution are all affected by these inputs.

1.1.3 Hybrid dynamical phenomena

Appropriate models for hybrid systems are often obtained by adding new dynamical phenomena to the classical description formats of the mono-disciplinary research areas. Indeed, continuous models represented by differential or difference equations, as adopted by the dynamics and control community, have to be extended to be suitable for describing hybrid systems. On the other hand, the discrete models used in computer science, such as automata or finite-state machines, need to be extended by concepts like time, clocks, and continuous evolution in order to capture the mixed discrete and continuous evolution in hybrid systems. The hybrid system models explained in Part I of this handbook combine both ideas. Here we will describe

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the phenomena one has to add to the continuous models based on the differential equations

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)). \tag{1.1}$$

Roughly speaking, as also argued in the previous discussion, four new phenomena that are typical for hybrid systems are required to extend the dynamics of purely continuous systems as in (1.1):

- autonomous switching of the dynamics;
- autonomous state jumps;
- controlled switching of the dynamics;
- controlled state jumps.

These phenomena are first explained for autonomous hybrid systems.

Autonomous switching of the dynamics This reflects the fact that the vector field $\boldsymbol{f}$ that occurs in (1.1) is changed discontinuously. The switching may be invoked by a clock if the vector field $\boldsymbol{f}$ depends explicitly on the time t:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t).$$

For instance, if periodic switching between two different modes of operation is used with period $2T$, we would have

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t) := \begin{cases} f_1(\boldsymbol{x}(t)), & \text{if } t \in [2kT, (2k+1)T) \text{ for some } k \in \mathbb{N}, \\ f_2(\boldsymbol{x}(t)), & \text{if } t \in [(2k+1)T, (2k+2)T) \text{ for some } k \in \mathbb{N}. \end{cases}$$

This is an example of *time-driven switching*.

The switching can also be invoked when the continuous state $\boldsymbol{x}$ reaches some *switching set* $\mathcal{S}$. As the situation $\boldsymbol{x}(t) \in \mathcal{S}$ is considered to be a state event, this kind of switching is said to be *event-driven*. The thermostat example provided an illustration of event-driven switching as the transition from the "on" mode to the "off" mode was triggered by the temperature reaching the value $T_{\max}$.

The following example also illustrates event-driven switching.

Example 1.2 *Hybrid tank system*

The tank systems shown in Fig. 1.5 illustrate two situations in which the dynamics of a process changes in dependence upon the state (liquid level). The tank in the left part of the figure is filled by the pump, which is assumed to deliver a constant flow Q_{P}, and emptied by two outlet pipes, whose outflows $Q_1(t)$ and $Q_2(t)$ depend upon the level $h(t)$. As the flow $Q_2(t)$ vanishes if the liquid level is below the threshold h_{p} given by the position of the upper pipe, the dynamical properties of the tank change if the level $h(t)$ exceeds this threshold.

The dependence of the vector field upon the state can be simply written down. For $h(t) < h_{\mathrm{v}}$, the differential equation is given by

$$\dot{h}(t) = \frac{1}{A}(Q_{\mathrm{P}} - \sqrt{2gh(t)}) = f_1(h(t)),$$

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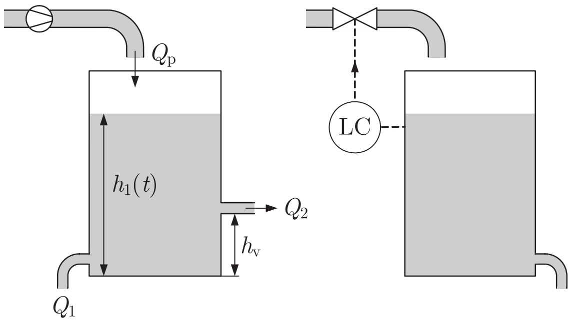

Fig. 1.5 Hybrid tank systems.

where g denotes the gravity constant. For $h(t) \geq h_v$ this equation changes into

$$\dot{h}(t) = \frac{1}{A}(Q_P - \sqrt{2gh(t)} - \sqrt{2g(h(t) - h_v)}) = f_2(h(t)).$$

Hence, the model can be written as

$$\dot{h}(t) = \begin{cases} f_1(h(t)) \text{ if } h(t) < h_v, \\ f_2(h(t)) \text{ if } h(t) \geq h_v, \end{cases}$$

which shows that the vector field switches between two different functions f_1 and f_2 in dependende upon the state $h(t)$ with the switching surface

$$\mathcal{S} = \{h \in \mathbb{R} \,|\, h = h_v\}.$$

Now assume that the pump is switched on and off at different time instances t_1 and t_2. Then the function f occurring in the differential equation changes at these time points but this switching does not depend upon the state $h(t)$, but is time-driven.

The tank in the right part of Fig. 1.5 illustrates that autonomous switching is a typical phenomenon introduced by safety measures. In the tank system the level controller is equipped with a safety switch-off. If the liquid level is below the corresponding threshold, the dynamics is given by the controlled tank. If the level exceeds the threshold, the pump is switched off, which brings about a corresponding switching of the differential equation of the tank. □

Switching among different dynamics has important consequences for the behavior of the hybrid system. For instance, Example 2.3 shows that switching between two linear stable vector fields can result in an unstable overall system.

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