

Prospect Theory

Prospect Theory: For Risk and Ambiguity provides the first comprehensive and accessible textbook treatment of the way decisions are made both when we have the statistical probabilities associated with uncertain future events (risk) and when we lack them (ambiguity). The book presents models, primarily prospect theory, that are both tractable and psychologically realistic. A method of presentation is chosen that makes the empirical meaning of each theoretical model completely transparent.

Prospect theory has many applications in a wide variety of disciplines. The material in the book has been carefully organized so as to allow readers to select pathways relevant to their own interests. With numerous exercises and worked examples, the book is ideally suited to the needs of students taking courses in decision theory in economics, mathematics, finance, psychology, management science, health, computer science, Bayesian statistics, and engineering.

A companion website with exercises, additional assignments, and solutions is available at www.cambridge.org/wakker.

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Preface

This book is the culmination of 14 years of teaching. In the 15th year, when for the first time I did not feel like rereading or rewriting, the time had come to publish it. The book received helpful comments from Han Bleichrodt, Arie de Wild, Itzhak Gilboa, Glenn Harrison, Amit Kothiyal, Gijs van de Kuilen, Georg Weizsäcker, and many students during the past 14 years. Thorough comments from Rich Gonzalez and Vitalie Spinu are especially acknowledged. I am most indebted to Stefan Trautmann for the numerous improvements he suggested. This book has also benefited from many inspiring discussions with Craig Fox, with whom I share the privilege of having collaborated with Amos Tversky on uncertainty during the last years of his life.