

GEOMETRIC AND TOPOLOGICAL METHODS FOR QUANTUM FIELD THEORY

Aimed at graduate students in physics and mathematics, this book provides an introduction to recent developments in several active topics at the interfaces between algebra, geometry, topology and quantum field theory.

The first part of the book begins with an account of important results in geometric topology. It investigates the differential equation aspects of quantum cohomology, before moving on to noncommutative geometry. This is followed by a further exploration of quantum field theory and gauge theory, describing AdS/CFT correspondence, and the functional renormalization group approach to quantum gravity. The second part covers a wide spectrum of topics on the borderline of mathematics and physics, ranging from orbifolds to quantum indistinguishability and involving a manifold of mathematical tools borrowed from geometry, algebra and analysis.

Each chapter presents introductory material before moving on to more advanced results. The chapters are self-contained and can be read independently of the rest.

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This volume is dedicated to Julius Wess and Krzysztof Wojciechowski.

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Introduction

This volume offers an introduction to some recent developments in several active topics at the interfaces between algebra, geometry, topology and quantum field theory. It is based on lectures and short communications delivered during the summer school ‘Geometric and Topological Methods for Quantum Field Theory’ held in Villa de Leyva, Colombia, in July 2007.

The invited lectures, aimed at graduate students in physics or mathematics, start with introductory material before presenting more advanced results. Each lecture is self-contained and can be read independently of the rest.

The volume begins with an introductory course by Paul Kirk on the history and problems of geometric topology, which explains how ideas coming from physics have had an impact on low-dimensional topology in the last 20 years. In the second lecture, Martin Guest discusses differential equation aspects of quantum cohomology, as part of a framework which accommodates the KdV equations and other well-known integrable systems.

We are then led into the realm of noncommutative geometry with a lecture by Claire Debord and Jean-Marie Lescure, who present a proof of Atiyah and Singer’s index theorem using groupoids and KK -theory, which they then generalize to the case of conical pseudomanifolds.

The remaining lectures take us to the world of quantum field theory, starting with a lecture by Alessandra Frabetti, who presents the Connes–Kreimer algebra for renormalization and its associated proalgebraic group of formal series after having reviewed the Dyson–Schwinger equations for Green’s functions and the renormalization procedure for graphs. We then step into gauge theory with José Gracia-Bondía’s lecture, which sheds light on BRS invariance of gauge theories using Utiyama’s general gauge theory. David Berenstein then gives a short but gentle introduction to the rather sophisticated ideas of AdS/CFT correspondence. In the last lecture, Martin Reuter and Frank Saueressig survey the background material underlying the functional renormalization group approach to quantum gravity.

The invited lectures are followed by six short communications on a wide spectrum of topics on the borderline between mathematics and physics, ranging from orbifolds to quantum indistinguishability and involving a multitude of mathematical tools borrowed from geometry, algebra and analysis.

We hope that these contributions will give – as much as the school itself seems to have given – young students the desire to pursue what might be their first acquaintance with some of the problems on the boundary between mathematics and physics presented here. On the other hand, we hope that the more advanced reader will find some pleasure in reading about different outlooks on related topics and seeing how the well-known geometric tools prove to be useful in some areas of quantum field theory.

We are indebted to various organizations for their financial support for this school. Let us first of all thank the Universidad de los Andes, which has been supporting this and many other schools of this kind that we have been organizing in Colombia since 1999. We are also deeply grateful to the ICTP in Trieste, for its constant financial support over the years and specifically for this school. We also thank the IMU for its support. We are also greatly indebted to other organizations – such as CLAF in Brazil, and Colciencias, ICETEX and ICFES in Colombia – which also contributed in a substantial way to the financial support needed for this school.

Special thanks to Sergio Adarve, Alexander Cardona and Andrés Reyes (Universidad de los Andes), coorganizers of the school, who dedicated time and energy to make this school possible in a country like Colombia where many difficulties are bound to arise along the way due to social, political and economic problems.

We are also very grateful to Marta Kovacsics, Ana Cristina García and Melissa Caro for their help in various essential tasks needed for the successful development of the school. Without the people named here, all of whom helped in the organization in some way or another, before, during and after the school, this scientific event would not have left such vivid memories in the lecturers' and participants' minds. Last but not least, thanks to all the participants who gave us all, lecturers and editors, the impulse to prepare this volume through the enthusiasm they showed during the school, and special thanks to all the contributors and referees for their participation in the realization of these proceedings.

Hernán Ocampo, Eddy Pariguán and Sylvie Paycha