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978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

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## STRATEGIC FINANCIAL PLANNING OVER THE LIFECYCLE

This book on personal financial planning and wealth management employs the lifecycle model of financial economics. The central idea of “consumption smoothing” is used to connect chapters and topics such as saving and investment, debt management, risk management, and retirement planning. The first part of the book is nontechnical and aimed at a wide audience with no special technical background. The second part of the book provides a rigorous presentation of the lifecycle model from first principles using the calculus of variations. The accompanying website is found at [http://www.yorku.ca/milevsky/?page\\_id=185](http://www.yorku.ca/milevsky/?page_id=185).

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Frontmatter

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Frontmatter

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# Strategic Financial Planning over the Lifecycle

*A Conceptual Approach to Personal Risk Management*

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Frontmatter

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*To our wives . . .*

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Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

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*... As far as I am aware, no one has challenged the view that if people were capable of it, they ought to plan their consumption, saving, and retirement according to the principles enunciated by Modigliani and Brumberg in the 1950s...*

**Angus S. Deaton**, Princeton University, 2005

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Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

## Contents

<i>Acknowledgments</i>	<i>page</i> xiii
1 Introduction and Motivation	1
2 Mathematical Preliminaries – Working with Interest Rates	4
2.1 Interest Rates	4
2.1.1 Effective Annual Rate (EAR) and Annual Percentage Rate (APR)	5
2.2 Time Value of Money – Definitions	7
2.3 Present Value and Future Value of a Single Cash Flow	8
2.4 Present Value of an Annuity with Constant Payments	9
2.4.1 Ordinary Annuity	9
2.4.2 Annuity Due	11
2.5 Future Value of an Annuity with Constant Payments	12
2.5.1 Ordinary Annuity	12
2.5.2 Annuity Due	13
2.6 Present Value of an Annuity Whose Payments Change at a Constant Rate	14
2.6.1 Constant-Growth Ordinary Annuity	14
2.6.2 Constant-Growth Annuity Due	18
2.7 Future Value of an Annuity Whose Payments Change at a Constant Rate	19
2.7.1 Constant-Growth Ordinary Annuity	19
2.7.2 Constant-Growth Annuity Due	20
2.8 A Preliminary Discussion of the Effects of Inflation	21
2.9 Appendix: Geometric Progressions and Annuity Relations	23
2.10 Questions and Assignments	26

Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

viii

*Contents*

3	Personal Balance Sheet and Human Capital	28
3.1	Personal Balance Sheets	28
3.1.1	Financial Assets	30
3.1.2	Gross Human Capital	31
3.1.3	Explicit Liabilities	32
3.1.4	Implicit Liabilities	32
3.1.5	Economic Net Worth	32
3.2	Estimation of Gross Human Capital	33
3.2.1	What Is Proper Discount Rate for Gross Human Capital?	36
3.3	Estimation of Implicit Liabilities	37
3.4	An Example of Human Capital Decisions: Investing in Education	40
3.4.1	Some Caveats	42
3.5	Questions and Assignments	42
4	Consumption Smoothing and Optimal Savings	46
4.1	Consumption Smoothing	46
4.2	Assumptions for Consumption Smoothing	49
4.3	Solving for Smoothness	51
4.3.1	Effects of Changes in Other Parameter Values	65
4.4	Consumption Smoothing vs. Other Approaches	70
4.4.1	The Fixed-Percentage Savings Rules	70
4.4.2	The 70% Income Replacement Rule	73
4.5	Practical Considerations	75
4.6	Questions and Assignments	76
5	Debts, Loans, and Mortgages [Canadian Content]	79
5.1	Mortgage Financing	79
5.1.1	Mortgage Amortization	81
5.1.2	Mortgage Insurance	84
5.1.3	Mortgage Refinancing	88
5.1.4	Fixed-Rate vs. Floating-Rate Loans	92
5.2	Borrowing to Invest	95
5.3	Debt Consolidation	98
5.4	Is Debt Good or Bad?	99
5.5	Questions and Assignments	100
6	Personal Income Taxes [Canadian Content]	103
6.1	The Logic of Income Tax Calculations	103



Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

<i>Contents</i>		ix
6.1.1	Federal Income Taxes	105
6.1.2	Provincial Income Taxes	109
6.1.3	Combining Federal and Provincial Income Taxes	111
6.2	Investment Income	114
6.2.1	Interest Income	115
6.2.2	Dividend Income	115
6.2.3	Capital Gains/Losses	119
6.3	Income Tax Planning	120
6.3.1	Income Splitting	121
6.3.2	Tax-Deferred Investment Vehicles	124
6.3.3	Tax-Free Savings Accounts (TFSA's)	131
6.3.4	RRSPs vs. TFSA's	132
6.4	Questions and Assignments	134
7	Risk, Utility, and Insurance	136
7.1	The Concepts of Risk and Risk Preferences	136
7.1.1	Definition of Risk	136
7.1.2	Risk Preferences	137
7.2	The Concept of Insurance	138
7.3	The Demand for Insurance	141
7.3.1	When Should People Buy Insurance?	141
7.3.2	How Much Should You Pay for Insurance?	142
7.3.3	The Effect of Current Wealth on Insurance Decisions	147
7.3.4	The Effect of Risk-Aversion Parameter on Insurance Decisions	149
7.3.5	Generalizations	151
7.4	Insurance and Consumption Smoothing	152
7.5	Questions and Assignments	155
8	Mortality Risk and Life Insurance	157
8.1	Who Needs Life Insurance?	157
8.2	How Much Coverage Do You Need?	158
8.2.1	Human-Capital Approach	158
8.2.2	The Expense (Needs) Approach	160
8.3	Types of Life Insurance	161
8.3.1	Term Life Insurance	162
8.3.2	Cash-Value Life Insurance	179
8.4	Questions and Assignments	182

Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

x

*Contents*

9	Investment and Diversification	183
9.1	Investment Decisions and Consumption	183
9.2	Major Asset Classes	186
9.2.1	Cash Equivalents	186
9.2.2	Fixed Income (Bonds)	187
9.2.3	Equities	188
9.2.4	Commodities	189
9.2.5	Real Estate	190
9.3	Historical Returns	191
9.3.1	Arithmetic Average vs. Geometric Average	193
9.3.2	Correlations of Returns	194
9.4	Asset Allocation and Diversification	194
9.5	Mutual Funds and Exchange-Traded Funds	197
9.5.1	Mutual Funds	197
9.5.2	Exchange-Traded Funds (ETFs)	201
9.6	Summary	201
9.7	Questions and Assignments	202
10	The Mathematics of Portfolio Diversification	204
10.1	Impressive Investment Performance: Real or Fake	204
10.2	Portfolio Diversification	206
10.2.1	Preliminaries	206
10.2.2	From Bernoulli to Binomial	209
10.2.3	The Central Limit Theorem	211
10.3	Portfolio Construction	214
10.3.1	Efficient Frontier for Two Assets	217
10.3.2	Efficient Frontier for More Than Two Assets	219
10.4	Summary	221
10.5	Questions and Assignments	222
11	Housing Decisions	224
11.1	Buying or Renting?	224
11.1.1	From a Purely Financial Perspective	224
11.1.2	Other Aspects of the Decision	232
11.1.3	Our Recommendation	234
11.2	Questions and Assignments	236
12	Pensions and Retirement [Canadian Content]	238
12.1	Longevity Risk	238
12.2	Retirement Plans that Provide Lifelong Income	243

Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

<i>Contents</i>		xi
12.2.1	Canada Pension Plan (CPP) and Old-Age Security Pension (OAS)	243
12.2.2	Defined-Benefit Pension Plan	244
12.2.3	Life Annuity	248
12.3	Optimal Retirement Spending Rate in the Presence of Longevity Risk	254
12.3.1	Retirement Spending Rates	255
12.3.2	Numerical Example	256
12.3.3	Including Pension Annuities	258
12.4	Summary	262
12.5	Questions and Assignments	264
13	Advanced Material: Part I. Continuous Time and the Calculus of Variations	266
13.1	Wages and Salary in Continuous Time	266
13.2	Human Capital Values in Continuous Time	267
13.3	A Different Route: Same Destination	270
13.4	Does Human Capital Grow or Shrink over Time?	272
13.5	Implicit Liabilities in Continuous Time	273
13.6	Consumption Smoothing: A Second Look	273
13.7	Calculus of Variations	279
13.8	Smooth Consumption: A Final Look	281
13.9	Life and Death in Continuous Time	283
13.10	The Problem of Retirement Income	291
13.10.1	The Euler-Lagrange Equation	293
13.10.2	Wealth Depletion	293
13.10.3	Classifying the Different Retirement Trajectories	295
13.10.4	Explicit Solution: Exponential Remaining Lifetime	297
13.10.5	Explicit Solution: Gompertz Mortality	299
13.11	Back to the Working Years	302
13.11.1	Patient Individual $k \leq \hat{k} < g$	303
13.11.2	Relatively Impatient Individual $k \leq g \leq \hat{k}$	305
13.11.3	Impatient Individual $g < k \leq \hat{k}$	308
13.12	Conclusion	309
13.13	Questions and Assignments	310
13.14	Appendix	313

Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

xii

*Contents*

14	Advanced Material: Part II. Stochastic Optimal Control and the HJB Equation	315
14.1	Dynamic Asset Allocation and Optimal Consumption	315
14.2	Asset Price Model	316
14.3	A Self-Financed Portfolio	317
14.4	The Asset-Allocation Problem	318
14.5	The Asset-Allocation-Consumption Problem	321
14.6	The Asset-Allocation-Consumption Problem with Deterministic Wage Income	323
14.7	An Advanced Stochastic Lifecycle Model	324
14.7.1	Model Specifications	325
14.7.2	HJB Equation for the Lifecycle Model	328
14.7.3	Closed-Form Solution	329
14.8	Questions and Assignments	337
14.9	Technical Appendix	338
14.9.1	Elementary Stochastic Calculus in Finance	338
14.9.2	Stochastic Dynamic Optimization	341
14.9.3	Finite Difference Method for Solving the HJB Equation	342
15	Concluding Thoughts and Next Steps	345
15.1	Portfolio Selection with Human Capital	346
15.2	Asset Location vs. Asset Allocation	347
15.3	Home Mortgage Financing	348
15.4	Life and Health Insurance	348
15.5	Employee Compensation and Proper Incentives	349
15.6	Choice of Defined Benefit vs. Defined Contribution Pensions	349
15.7	Starting a Retirement Pension Annuity	350
15.8	Estate Planning and Tax Arbitrage	350
15.9	The Real Options in Your Life	350
	<i>Bibliography</i>	353
	<i>Index</i>	357

Cambridge University Press

978-0-521-76456-8 - Strategic Financial Planning over the Lifecycle: A Conceptual Approach to Personal Risk Management

Narat Charupat, Huaxiong Huang and Moshe A. Milevsky

Frontmatter

[More information](#)

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Frontmatter

[More information](#)

---