PART I

Strategic interactions as games

INTRODUCTION

Social interaction is essential to human life. How do people choose what to do when they encounter one another? And how do organizations, firms or countries interact? Game theory is a modeling tool designed to represent and analyze such strategic interaction.

The first part of this book is devoted to introducing the basic building blocks of game theory. The parties to the interaction are called *players*, the courses of actions available to them are their *strategies*, and the *payoffs* of each player from the various profiles of strategies (of *all* players) represent the way each player ranks the possible outcomes of the interaction from her own individual point of view.

Chapter 1 will be devoted to the definition of these concepts, and their illustration with a preliminary example. Chapter 2 will expand on these modeling considerations in concrete real-world examples. The first of these will be a historical military episode in the Middle East. Additional examples will concern competition over promotion in the workplace, and the design of incentives for teamwork. The considerations elaborated in the modeling process will set the stage for analyzing and predicting the outcomes of such strategic encounters in the chapters that follow.

1 Strategic form games

Man is a social animal. Our notions about the world and ourselves are indissolubly linked with the way we acquired those notions and experienced them together with our family and friends in kindergarten and at school and work. Some of our social encounters get us into confrontations with others as we seek to secure some gain for ourselves or improve our standing. Other encounters find us primarily cooperating with others in an effort to achieve a common goal. But all social encounters share one salient and important attribute: the action chosen by each of the participants in the encounter affects the other participants. That is what **game theory** is about.

Game theory deals with the behavior of individuals in a social environment where the actions of each individual affect the others. Game theory analyzes the considerations that rational participants entertain when deciding on their moves, and how such considerations affect the moves they choose to make.

The participants may be, for example, individual employees on the job, commercial firms in the economic market, or nations in the international arena. In order not to restrict the context of the study *ab initio*, such individuals are habitually referred to as "players" and the interaction between them is called a "game."

In everyday usage, the word "game" typically refers to children's games or sporting and social games such as football, chess or Monopoly. We derive direct enjoyment and entertainment value from the game and sometimes other added values, too. In game theory, the term "game" is assigned a more general meaning, to describe interactive encounters between several participants.

The borrowed use of the game concept in this context is based on the fact that every game has its predetermined rules. These rules define a permanent and built-in connection between causes and effects. In modeling a social situation in game theory, one seeks to single out such built-in connections from the vast plethora of the details of the event. As in a game in the everyday meaning of the word, the motivating forces are the actions or the **strategies** that participants can employ, and the results are the effects of the action of each of the participating **players** on each and every one of them.

In reality, this influence is of course multifaceted and highly diversified: it may take the form of the material resources at the participant's disposal, her emotional sense, her social status, what she expects of herself and what others expect from her, and so forth.

Despite this complexity, we will assume that each participant is able to rate the outcome of the encounter (which depends on the actions of *all* participants) in accordance with her own order of priorities. We will express this scale of preferences with the aid of numbers, which we will call **payoffs** or levels of **utility**. Thus the highest payoff will be ascribed to the outcome that is the most desirable for that individual, and the lowest payoff will be ascribed to the result that is worst for her.

Consider the following example.

1.1 Representation in negotiations over a business partnership

A manufacturing firm and a marketing firm are negotiating the formation of a business partnership. They estimate that the partnership will yield a joint profit of $1 million. If the companies' chief executive officers (CEOs) negotiate directly with one another, they will agree to share the anticipated profit equally between the two companies – $500,000 for each. However, a CEO hiring a lawyer to support her in the negotiations will be able to increase her share in the joint profit by $100,000 at the expense of the second company (by virtue of a more painstaking and sophisticated wording of the clauses of the contract). The cost of such legal representation is $50,000. If both directors hire lawyers, the profit formulated in the contract will be divided equally between the two companies.

In this example, the players are the company CEOs.

Each CEO has two possible negotiation strategies: to hire legal representation or to conduct the negotiations herself. The following matrix describes each company's financial profit (in thousands of dollars) in accordance with the strategies chosen by the CEOs.

		Marketing firm CEO	
		Negotiating directly	Hiring legal representation
Manufacturing firm CEO	Negotiating directly	500, 500	400, 550
	Hiring legal representation	550, 400	450, 450

Each entry in the matrix shows the financial profit (net of the expenses of legal representation, if any) of each of the companies for the combination of strategies corresponding to that entry. The left-hand figure is the profit of the manufacturing company, and the right-hand figure is the profit of the marketing company. Let us assume, for example, that the director of the manufacturing company chooses to

hire legal representation (her strategy is shown in the second row), while the marketing company director chooses to conduct the negotiations herself (her strategy appears in the left-hand column). In the resulting contract, the manufacturing company will earn $600,000 so that its profit, net of legal representation, will amount to $550,000. Correspondingly, the marketing company's profit will be $400,000 (the right-hand figure in that entry).

We note that the figures appearing in the matrix of this game are the corporate profits, while the players in the game we have described are not the firms themselves but rather the firm CEOs. The CEOs do not pocket the entire profit that they obtained for the firm, and therefore the figures in the matrix do not reflect the monetary payments received by the players. The figures do, however, aptly describe the players' preferences, since each CEO shows preference for that combination of strategies that will yield a higher profit for her firm, over some other combination of strategies that will yield it a lower profit. This preference may derive from the CEO's sense of identification with her role, from incentives in the form of monetary rewards that may accrue to her if she succeeds in improving the firm's profitability, from a personal valuation she may obtain from the company's shareholders and from her colleagues, and so forth. The figures appearing in the matrix are accordingly called "payoffs" because the order in which they are arranged faithfully reflects preferences: if a particular combination of strategies leads to a particular outcome that a player deems preferable over some other result obtainable from a different combination of strategies, the payoff to the player in the game matrix from the first outcome is higher than the payoff to that player from the second result.

1.2 Definition of a strategic form game

Definition **A strategic form game** is defined by means of the following three components:

1. **The players** who take part in the game.
2. The set of **strategies** of each player.
3. The **payoff** to each player from every possible strategy profile of the players.

Let N be the number of players in the game, and denote by I the set of players. For player k in the set I of players, we will denote by X_k the set of strategies available to player k. A strategy x_k of player k is therefore an element in the set X_k of strategies.

In the example shown in section 1.1 above, the set of players consists of the two CEOs:

$$I = \{\text{manufacturing firm CEO},\ \text{marketing firm CEO}\}$$

The set of strategies of the CEO of the manufacturing firm is:

$$X_{\text{manufacturing firm CEO}} = \{\text{negotiating independently},\ \text{hiring legal representation}\}$$

Similarly, the set of strategies of the CEO of the marketing firm is:

$$X_{\text{marketing firm CEO}} = \{\textit{negotiating independently},\ \textit{hiring legal representation}\}$$

A profile of strategies of the players has the form $x = (x_1, \ldots, x_k, \ldots, x_N)$. That is, for every player k, $k \in I$, the profile of strategies specifies the strategy x_k of player k.

We will denote by X the set of all strategy profiles of the players:[1]

$$X = \Pi_{k \in I} X_k$$

Likewise, we will denote by

$$X_{-i} = \Pi_{j \neq i} X_j$$

the set of strategy profiles of all the players other than player i. Thus a strategy profile of all the players other than i takes the form

$$x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N) \in X_{-i} = \Pi_{j \neq i} X_j$$

and a strategy profile of all the players takes the form

$$x = (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_N) = (x_i, x_{-i}) \in X_i \times X_{-i}$$
$$= \Pi_{k \in I} X_k = X$$

In this form of writing, we have presented the strategy profile x in two different ways. At first we presented it explicitly as the combination $(\ldots x_k, \ldots)_{k \in I}$ of the strategies x_k chosen by the players k in the set I of players. Next, we presented it as the combination (x_i, x_{-i}) of the choice x_i of player i and the profile of choices x_{-i} of the other players. We will use the latter presentation whenever we wish to emphasize that a profile x of strategies is a combination of the choice x_i of player i, which depends solely on her own will, and of the choice profile x_{-i} of the other players, which does not depend on the will of player i. If i and j are the only two players in the game, then $X_{-i} = X_j$, i.e. the set of strategy profiles of all the players other than i is simply the set of strategies of the second player, j, and there is no difference between the two presentations.

[1] The symbol Π, the "Cartesian product" (after the French philosopher and mathematician Descartes), signifies "the set of all the possible combinations of elements from the sets."

It now remains to define the players' payoffs. As we have explained, a key feature of a game is that the payoff of each player depends on the strategic choices of all the players. Therefore, the payoff to each player is a function that associates a number to each strategy profile of the players. We will call this function the player's payoff function or utility function. Formally, the payoff function u_i of player i,

$$u_i : X \to \mathbb{R}$$

profile x the real number $u_i(x)$. (The letter $\mathbb{R}$ represents the set of real numbers.)

The payoff function u_i represents the player's preferences. That is,

$$u_i(x) > u_i(x')$$

if and only if i prefers the outcome obtained from the strategy profile x over the outcome obtained from the strategy profile x'.

In the particular case in which there are only two players, $I = \{1,2\}$, each of whom has a finite set of strategies, the payoff function may be described by means of a game matrix, as in the example in section 1.1 above. In this matrix, each row corresponds to one of the strategies of player 1 and each column corresponds to one of the strategies of player 2. In each entry of the matrix there appear two payoffs – the payoff of player 1 (the left-hand figure) and the payoff of player 2 (the right-hand figure). The payoffs in the entry of the matrix in row m and column n are:

$$u_1(m,n), u_2(m,n)$$

These are the payoffs to the players when player 1 chooses the strategy corresponding to row m and player 2 chooses the strategy corresponding to column n.

Game theory is not the only domain that has borrowed the term "game" from its day-to-day usage. The philosopher Ludwig Wittgenstein coined the notion "language-game" in his book *Philosophical Investigations* (1953). Wittgenstein rejects the view whereby every word in a language is a name or representation of something in the real world, and the understanding of the meaning of a word consists of knowing what object the word represents. Wittgenstein maintains that "to imagine a language is to imagine a form of life." According to him, the meaning of a word is the way it is used in the language, and the meaning of a name is clarified by pointing at its subject. Accordingly, it is the social "game" that establishes the meaning of words. Thus Wittgenstein too adopts the term "game" to describe a basic and fundamental phenomenon in human society.

2 Representing strategic interactions with games

In this chapter we will examine several examples that can be analyzed using game-theoretic tools. These examples will help to illustrate the considerations involved when social interactions or confrontations are represented by **strategic form games**. We will also discover what aspects cannot be represented by a strategic form game, and find out in what ways the game concept needs to be extended so as to realize more appropriate representations.

2.1 The background to the Six Day War

The Six Day War of June 5–10, 1967, between Israel and its neighboring Arab states Egypt, Jordan and Syria, was a key event in the evolution of the conflict in the Middle East. The strategic dilemmas faced by the belligerents constitute a prime example for game-theoretic analysis.

After Israel declared its independence on May 14, 1948, in accordance with the United Nations resolution from November 1947, its borders with Egypt, Jordan, Syria, and Lebanon were established via a war which lasted until March 1949. In 1956, in response to terrorist infiltrations from the Sinai Peninsula, Israel captured it from Egypt, but withdrew under international pressure and guarantees for shipping rights in the Red Sea, from the port of Eilat via the Straits of Tiran.

2.1.1 The circumstances on the eve of the war

In the year preceding the Six Day War, tension between Israel and Syria was on the rise, sparked by three principal causes. First, Syria was claiming additional rights over the Jordan River source waters, while planning to divert the Yarmuk River in such a way that a smaller share of its water would reach territory under Israeli control. Second, Israel and Syria had a dispute about the cultivation rights of the agricultural land in the demilitarized zone established between the two armies at the end of Israel's War of Independence. The third factor was the terror attacks initiated and perpetrated by Palestinian terrorist organizations (primarily the Fatah) under Syrian auspices, by infiltrating Israel from within Syrian territory.

This tension would sporadically erupt in the form of military skirmishes such as that in July 1966 when Israel launched an attack on the Yarmuk River waters diversion project, using aircraft and artillery. On April 7, 1967, an incident between Israel and Syria escalated into a day of combat when the Syrians fired on an Israeli tractor that had entered the demilitarized zone. Israel's refusal to withdraw the tractor prompted an exchange of fire between amored, artillery and aerial forces, culminating in the silencing of the Syrian positions and the downing by the Israel Air Force of six Syrian MiG airplanes. Two of the airplanes were shot down over the outskirts of Damascus, in full view of Syrian citizens attending the Baath Party anniversary celebrations that were taking place that day.

Yet the frequency of incursions from Syrian territory into Israel kept mounting. In the first half of May 1967, Israel sent Damascus some severe warnings. Speaking publicly, Prime Minister Levi Eshkol stated that Israel might resort to measures no less harsh than those taken on April 7, 1967. News agencies cited a senior Israeli source as saying that Israel might take restricted military action with the aim of overthrowing the military regime in Damascus if the incursions from Syrian territory into Israel were to continue.

Syria responded by appealing to Egypt for support, as part of the mutual defense pact the two countries had formed on November 4, 1966. In addition, Egypt was receiving (erroneous) intelligence reports from the Soviet Union to the effect that Israel was massing forces on her border with Syria. Egypt, which under President Gamal Abdel Nasser aspired to leadership of the Arab world, therefore decided to take action. On May 15, Egyptian troops ostentatiously passed through Cairo *en route* for the Sinai Peninsula. The next day, the Egyptian Chief of Staff notified the Officer in Command of the UN Forces in Sinai (which had been positioned there in 1957 as a barrier between the armies of Israel and Egypt) that they must vacate their positions immediately. On May 18, Egyptian forces took over the UN positions in Sharm-el-Sheikh that controlled the Straits of Tiran. On May 22, Nasser accused Israel of threatening to go to war, and announced that the Straits would henceforth be closed to Israeli ships making their way to the port of Eilat. On May 26, Nasser, in a public speech, alleged that the problem was not just Israel but also the United States and the western nations that supported Israel. He declared that if Israel attacked Syria and Egypt, the conflict would not be confined to the frontier regions only but would be an all-out war, in which Egypt's aim would be to destroy Israel. The Voice of Cairo radio station then started airing broadcasts featuring calls to wipe Israel off the face of the earth.

The opening of the Straits of Tiran to Israeli vessels under the supervision of the United Nations had been one of Israel's most important achievements in the 1956 Sinai Campaign, and Israel's traditional position was that the closure of the Straits by Egypt would constitute a *casus belli*. Nevertheless, Egypt was hoping that Israel would be afraid to open war simultaneously on two fronts.

2.1.2 Presentation of the circumstances as a game

Following the closure of the Straits, both countries had to simultaneously reach a decision as to how to act.

Thus, Israel faced two principal alternatives:

1. To fulfill its commitment to war following the closure of the Straits.
2. To threaten to go to war, but not to launch hostilities on its own initiative.

Egypt had three principal alternatives:

1. To launch a war on its own initiative.
2. To leave the situation unchanged.
3. To respond to international mediation efforts, to withdraw its troops from Sinai, and to restore the status quo ante.[1]

Question 2.1 Describe this historical situation as a game. Who are the players? What strategies do they have at their disposal? What are the payoffs in this game?

Answer According to the description, the players are Israel and Egypt, and the strategies available to each player are the alternatives outlined above.

What are the payoffs to the players? The payoffs represent the ranking, from the point of view of each player, of the six combinations of choices that are possible. We will now propose certain considerations that will lead to such a ranking.

We must not, however, lose sight of the fact that this is not necessarily the only ranking that is consistent with the historical data on the parties' positions. This is therefore an example of the manner in which using game theory to analyze a historical situation can sharpen the discussion on how to **interpret** historical events.

To our understanding, the leading considerations for describing the parties' preferences are as follows.

Israel's preferences

1. Had Egypt indeed intended to launch an all-out war, it would have been preferable for Israel to initiate the war itself, thus gaining whatever military advantages might be had from taking the initiative. Of all the strategy combinations leading to war, this is the one that is best for Israel, since the justification for

[1] In 1960, Egypt had put troops in Sinai without ordering the UN forces out, and had withdrawn them after a certain period.