

FIELD THEORIES OF CONDENSED MATTER PHYSICS

Presenting the physics of the most challenging problems in condensed matter using the conceptual framework of quantum field theory, this book is of great interest to physicists in condensed matter and high-energy and string theorists, as well as to mathematicians. Revised and updated, this second edition features new chapters on the renormalization group, the Luttinger liquid, gauge theory, topological fluids, topological insulators, and quantum entanglement.

The book begins with the basic concepts and tools, developing them gradually to bring readers to the issues currently faced at the frontiers of research, such as topological phases of matter, quantum and classical critical phenomena, quantum Hall effects, and superconductors. Other topics covered include one-dimensional strongly correlated systems, quantum ordered and disordered phases, topological structures in condensed matter and in field theory and fractional statistics.

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SECOND EDITION

EDUARDO FRADKIN

University of Illinois at Urbana-Champaign



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Preface to the second edition

I am extremely happy to, at long last, be able to present the second edition of this book. In spite of what I stated in the preface of the 1991 edition, I ended up not only writing a second edition but, in a sense, a new book. So one can say, once again, that we have met the enemy and it is us. I have been pleased that the 1991 edition of this book was appreciated by many people who found it useful and stimulating. I am really happy that my effort was not in vain.

My motivation for writing this book, in 1991 and now, was to present quantum field theory as a conceptual framework to understand problems in condensed matter physics that cannot be described perturbatively, and hence do not admit a straightforward reduction to some non-interacting problem. In essence, almost all interesting problems in condensed matter physics have this character. Two prime examples of problems of this type in condensed matter physics that developed in the late 1980s, and even more so in the 1990s, are the understanding of high-temperature superconductors and the quantum Hall effects. In both areas field theory played (and plays) a central role. If anything, the use of these ideas has become widespread and increasingly plays a key role. It was lucky that the first edition of this book appeared at just about the right time, even though this meant that I had to miss out on research that was and still is important. This was probably the only time that I was on time, as people who know me can relate. Much has happened since the first edition appeared in print. The problem of the quantum Hall effects has developed into a full-fledged framework to understand topological phases of matter. Although it is still an unsolved problem, the research in high-temperature superconductors (and similar problems) has motivated theorists to look for new ways to think of these problems, and the ideas of quantum field theory have played a central role. The concepts, and subtleties, of gauge theory have come to play a key role in many areas, particularly in frustrated quantum magnetism. The interactions between condensed matter and other areas of physics, particularly high-energy physics and string theory, have become more

important. Concepts in topology and other areas of mathematics rarely frequented by condensed matter physicists have also entered the field with full force. More recent developments have seen the incorporation of ideas of general relativity and quantum entanglement into the field.

These developments motivated me to work on a second edition of this book. I have to thank Simon Capelin, my editor from Cambridge University Press, who took the time to persuade me that this was not a foolish project. So, some time in 2007 (I think) I finally agreed to do it. Of course, this was a more complex project than I had expected (nothing new there!). For this reason it took until now, the Spring of 2012, for me to finish what I thought would take just one year (or so). I wish to thank Simon Capelin and the people at Cambridge University Press for working with me throughout this project.

This second edition contains essentially all that was included in the ten chapters of the first edition, with a substantial editing of misprints and “misprints.” However, it has grown to have seven more chapters to incorporate some important material that I left out in 1991 and to add new material to reflect some of the new developments. The result is that this is essentially a new book. I hope that in the process of writing this second edition I have not ruined what was good in the first one, and that the new material will be useful to a wide spectrum of people, not only in condensed matter. Although the book is significantly larger than its first edition, I had to leave out some really important material. In particular, I incorporated hardly any discussion of Fermi liquids, non-Fermi liquids (except for Luttinger liquids), and superconductors, among many important problems that are also of interest to me.

Several notable books that cover some parts of the material I cover have appeared in print since 1991, such as Xiao-Gang Wen’s *Quantum Field Theory of Many Body Systems* (published in 2003) and Subir Sachdev’s *Quantum Phase Transitions* (published in 1999). Other books that cover some aspects of the material are Assa Auerbach’s *Interacting Electrons and Magnetism* (published in 1994) and the book by A. Gogolin, A. Nersisyan, and A. Tsvelik, *Bosonization and Strongly Correlated Systems* (published in 2004), as well as the superb *Principles on Condensed Matter Physics* by Paul Chaikin and Tom Lubensky (published in 1995) and John Cardy’s *Scaling and Renormalization in Statistical Physics* (published in 1996).

I am deeply indebted to many people whose work has influenced my views. I have to particularly thank Steve Kivelson for his long-term friendship and collaboration, which has had a strong impact on my work, as reflected here. I also thank my collaborators in many projects, some of which are reflected here, Chetan Nayak, Claudio Chamon, Paul Fendley, Shivaji Sondhi, Joel Moore, and Fidel Schaposnik. I am also indebted to Lenny Susskind and Steve Shenker, who played a great role during my formative years as a theorist and whose outlook has strongly influenced these pages. I also thank my former students Ana López, Christopher Mudry,

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Antonio Castro Neto, Eun-Ah Kim, Michael Lawler, Kai Sun, and Benjamin Hsu, whose work is also reflected here. I am also indebted to my colleagues Mike Stone and Rob Leigh, with whom I collaborated in several projects and had countless stimulating discussions. Their work has strongly influenced my own. I also wish to thank Taylor Hughes and Shinsei Ryu for explaining their work (and others) on topological insulators, and motivating me to think on these problems. I am also grateful to Pouyan Ghaemi for reading the chapter on topological insulators and catching several misprints, and to Rodrigo Soto Garrido and to Ponnuraj Krishnakumar for proofreading the entire book and for their great help in generating the skyrmion figures for the cover.

I must also acknowledge the constant and permanent support of the Department of Physics of the University of Illinois, and my colleagues in our department. Some of the material presented here was also used in several special-topics courses I taught in Urbana over the years. I am particularly grateful to Professor Dale van Harlingen, our Department Head, for his constant support. I also wish to thank the many people who over the years have pointed out to me several conceptual issues present in the first edition as well as numerous misprints. I hope the editing of the second edition is substantially better than that of the first. I also wish to thank the National Science Foundation, which supported my research for many years.

This second edition, much like the first, could not have existed without the emotional support and love of Claudia, my wife and lifetime companion. Our children have fortunately (for them) been spared this second edition, which also could not have existed without my father constantly asking when I was going to be done with it.

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Preface to the first edition

This volume is an outgrowth of the course “Physics of Strongly Correlated Systems” which I taught at the University of Illinois at Urbana-Champaign during the Fall of 1989. The goal of my course was to present the field-theoretic picture of the most interesting problems in Condensed Matter Physics, in particular those relevant to high-temperature superconductors. The content of the first six chapters is roughly what I covered in that class. The remaining four chapters were developed after January 1, 1990. Thus, that material is largely the culprit for this book being one year late! During 1990 I had to constantly struggle between finalizing the book and doing research that I just could not pass on. The result is that the book is one year late and I was late on every single paper that I thought was important! Thus, I have to agree with the opinion voiced so many times by other people who made the same mistake I did and say, don’t ever write a book! Nevertheless, although the experience had its moments of satisfaction, none was like today’s when I am finally done with it.

This book exists because of the physics I learned from so many people, but it is only a pale reflection of what I learned from them. I must thank my colleague Michael Stone, from whom I have learned so much. I am also indebted to Steven Kivelson, Fidel Schaposnik, and Xiao-Gang Wen, who not only informed me on many of the subjects which are discussed here but, also, more importantly, did not get too angry with me for not writing the papers I still owe them.

This book would not have existed either without the extraordinary help of Christopher Mudry, Carlos Cassanello, and Ana López, who took time off their research to help me with this crazy project. They have done an incredible job in reading the manuscript, finding my many mistakes (not just typos!), making very useful comments, and helping me with the editing of the final version. I am particularly indebted to Christopher, who made very important remarks and comments concerning the presentation of very many subjects discussed here. He also generated the figures. Mrs. Phyllis Shelton-Ball typeset the first six chapters. My wife,

Claudia, made this project possible by learning \LaTeX at great speed and typesetting the last four chapters, correcting some of my very boring and awkward writing style.

This book was also made possible by the love and help of my children Ana, Andrés, and Alejandro, who had to live with a father who became a ghost for a while. Ana and Andrés helped in the proofreading, and took care of their little brother, who helped by keeping everybody happy.

Finally, I must acknowledge the support of the Department of Physics and the Center for Advanced Study of the University of Illinois. The help and understanding of the staff at Addison Wesley is also gratefully acknowledged.

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