NONLINEAR RESONANCE ANALYSIS
Theory, Computation, Applications

Nonlinear resonance analysis is a unique mathematical tool that can be used to study resonances in relation to, but independently of, any single area of application. This is the first book to present the theory of nonlinear resonances as a new scientific field, with its own theory, computational methods, applications, and open questions.

The book includes several worked examples, mostly taken from fluid dynamics, to explain the concepts discussed. Each chapter demonstrates how nonlinear resonance analysis can be applied to real systems, including large-scale phenomena in the Earth's atmosphere and novel wave turbulent regimes, and explains a range of laboratory experiments.

The book also contains a detailed description of the latest computer software in the field. It is suitable for graduate students and researchers in nonlinear science and wave turbulence, along with fluid mechanics and number theory. Color versions of a selection of the figures are available at www.cambridge.org/9780521763608.

ELENA KARTASHOVA is a Professor at the Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Linz. Her research interests include number theory, integrable dynamical systems, mathematical physics and their application in various areas, from fluid mechanics and meteorology to engineering. She was awarded the V. I. Vernadsky medal by the Russian Academy of Natural Sciences in 2009.

NONLINEAR RESONANCE ANALYSIS

Theory, Computation, Applications

ELENA KARTASHOVA
RISC, J. Kepler University, Linz, Austria

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521763608

First published 2011

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data

Kartashova, Elena, 1956–
Nonlinear resonance analysis : theory, computation, applications / Elena Kartashova.
p. cm.
ISBN 978-0-521-76360-8 (Hardback)
1. Differential equations, Nonlinear–Numerical solutions. 2. Fourier analysis. 3. Resonance.
4. Mathematical analysis. I. Title.
QA372.K247 2010
532′.05270151864–dc22
2010027604

ISBN 978-0-521-76360-8 Hardback

Additional resources for this publication at www.cambridge.org/9780521763608

Cambridge University Press
978-0-521-76360-8 - Nonlinear Resonance Analysis: Theory, Computation, Applications
Elena Kartashova
Frontmatter
More information

In memory of my father, Lozanovsky Alexander Leonidovich

Cambridge University Press
978-0-521-76360-8 - Nonlinear Resonance Analysis: Theory, Computation, Applications
Elena Kartashova
Frontmatter
More information

Contents

Preface

Description of the universe in the scientific paradigm is based on conceptions of action and reaction. The main question then is: What sort of reaction should be expected to this or that action? Qualitatively, it looks logical to expect a bigger reaction to a bigger action, and this is mostly the case. But nature is not to be put into the Procrustes bed of our logical schemes, and a remarkable exception exists – the phenomenon of resonance. Resonance was first described by Galileo Galilei in 1638: "one can confer motion upon even a heavy pendulum which is at rest by simply blowing against it; by repeating these blasts with a frequency which is the same as that of the pendulum one can impart considerable motion" [73].

Nowadays resonance is generally regarded as a red thread that runs through almost every branch of physics; without resonance we would not have radio, television, music, etc. Resonance causes an object to oscillate; sometimes the oscillation is easy to see (vibration of a guitar string), but sometimes this is impossible without measuring instruments (electrons in an electrical circuit). Soldiers are commanded to break step while marching over a bridge, otherwise the bridge may collapse.

Probably the most well-documented example of the resonance of a bridge is given by Tacoma Narrows Bridge, which was the third longest suspension bridge in the world in 1940. On the morning of November 7, 1940, the four-month old Tacoma Narrows Bridge began to oscillate dangerously up and down, tore itself apart, and collapsed over a period of about one hour. Though designed for winds of 120 mph, a wind of only 42 mph destroyed it. Experts agreed that somehow the wind caused the bridge to resonate and, nowadays, wind tunnel testing of bridge designs is mandatory. The very fortuitous fact for the history of science is that one professor of engineering decided to have a walk along the beach at the time when the oscillations began. He ran home, took a camera, and began to take a picture every five seconds. The pictures survived and have been turned into a video movie [227] showing the last few minutes before the catastrophe.

Other famous examples are the experiments of Tesla who studied experimentally in 1898 the vibrations of an iron column and noticed that at certain frequencies specific pieces of equipment in the room would start to jiggle. Playing with the frequency, he was able to move the jiggle to another part of the room. Completely fascinated with these findings, he forgot that the column ran downward into the foundation of the building, and the vibrations were being transmitted all over Manhattan. The experiments started a sort of a small earthquake in his neighborhood with smashed windows, swaying buildings, and panicky people in the streets. For Tesla, the first hint of trouble came when the walls and floor began to heave [36]. He stopped the experiment when he saw police rushing through the door.

Generically, two types of resonance have to be distinguished: linear and nonlinear. From the physical point of view, they are defined by whether or not the external force coincides with the eigenfrequency of the system or not (linear and nonlinear resonance respectively). The condition of nonlinear resonance reads

$$\omega_n = \omega_1 + \omega_2 + \cdots + \omega_{n-1}, \tag{0.1}$$

with possibly different $\omega_i = \omega(\mathbf{k}_i)$ being the eigenfrequencies of the linear part of some nonlinear partial differential equation.

As will be explained in Chapter 1, the mathematical definition of resonance given above *does not coincide* with the physical one, which also includes resonance conditions on the wavevectors

$$\mathbf{k}_n = \mathbf{k}_1 + \mathbf{k}_2 + \cdots + \mathbf{k}_{n-1}. \tag{0.2}$$

Wavevectors have integer coordinates in resonators which are bounded or periodical domains, while unbounded domains lead to real-valued wavevectors. Notice that "bounded or periodical" domain does not mean "small," but rather indicates the importance of correlation between the domain size and the wavelengths. For instance, planetary waves in the Earth's atmosphere (wavelengths $\sim$1000 km, [122]), Stokes edge waves in the coastal zone (wavelengths $\sim$10 m, [63]), and capillary waves in a cylindrical container with radius 200 mm (wavelengths $\sim$1 mm, [205]) all have integer wavevectors. Conditions of nonlinear resonance (0.1),(0.2), being regarded in integers, usually yield to solving Diophantine equations with many variables with huge degrees. This is equivalent to Hilbert's 10th problem [89], which is proven to be algorithmically unsolvable [165]. Therefore, it is only in the last decade that nonlinear resonances have been studied independently in each application area. Analysis of nonlinear resonances presented in this book can be applied directly to the resonators of an *arbitrary* physical nature.

Nonlinear resonances are ubiquitous in physics. Euler equations, with various boundary conditions and specific values of some parameters, describe an enormous number of nonlinear dispersive wave systems (capillary waves, surface water waves, atmospheric planetary waves, drift waves in plasma, etc.), all possessing nonlinear resonances [260]. Nonlinear resonances appear in numerous typical mechanical systems, such as an infinite straight bar, a circular ring, and a flat plate [136]. The so-called "nonlinear resonance jump," important for the analysis of the turbine governor positioning system of hydroelectric power plants, can cause severe damage to mechanical, hydraulic, and electrical systems [91]. One tragic example is the collapse of the Sayano–Shushenskaya hydroelectric power station, Russia, on August 17, 2009, which cost not only enormous material losses but also 75 human lives. Nonlinear resonance is the dominant mechanism behind outer ionization and energy absorption in near-infrared laser-driven rare-gas or metal clusters [143]. Characteristic resonant frequencies observed in accretion disks allow astronomers to determine whether the object is a black hole, a neutron star, or a quark star [131]. Thermally induced variations of helium dielectric permittivity in superconductors are due to microwave nonlinear resonances [129]. Temporal processing in the central auditory nervous system analyzes sounds using networks of nonlinear neural resonators [5]. Nonlinear resonant response of biological tissue to the action of an electromagnetic field is used to investigate cases of suspected disease, e.g. cancer [234], etc.

While linear resonances in various physical systems are presently well studied [198, 199], it is quite a nontrivial problem to compute the characteristics of nonlinear resonances or just to predict their very appearance, even in the one-dimensional case. Thus, the notorious Fermi-Pasta-Ulam numerical experiments [246] with a nonlinear one-dimensional string (carried out more than 50 years ago) are still not fully understood [16]. In these experiments, Fermi, Pasta, and Ulam simulated the vibrating string with quadratic and cubic nonlinearity by solving the system of nearest-neighbor coupled oscillators (32 and 64 oscillators in different series of experiments). Fermi thought, after many iterations, that the system would exhibit thermalization, i.e. a state of equipartition of energy, and would "forget" about initially exited oscillators. Instead, the system exhibited a puzzling quasi-periodic behavior.

Keeping in mind the collapse of the Tahoma bridge, we can immediately see two main questions about nonlinear resonances we would like to have answers to: Where? and When? The answer to the first question is defined by the geometry of the physical system studied and is formulated mathematically in algebraic equations to be solved in integers. This part of the theory of nonlinear resonances is called *kinematics*. The answer to the second question is defined by the solutions to some systems of nonlinear ordinary differential equations; this part of the theory is called *dynamics*.

This book is the first attempt to present the theory of nonlinear resonances, both kinematic and dynamic, as a new scientific area, with its own computational methods, applications, and open questions. It is written for an interdisciplinary audience and is structured as follows. Each chapter begins as simply as possible with an elementary section presenting the general ideas of the complete chapter. Thus, to get a notion about the general ideas and results presented in this book, read only the first sections of each chapter. Basic knowledge of linear algebra, differential equations, and a bit of common sense would be enough for understanding. Deeper reading demands additionally some knowledge of Hamiltonian formalism, number theory, graph theory, and theory of integrable systems. In Chapter 5, we show how to plan simple laboratory experiments with pendulums for observing physical manifestations of the mathematical notions and constructions introduced to describe nonlinear resonances. In Chapter 6, nonlinear resonance analysis is used for identifying and describing novel regimes in wave turbulent systems; important open problems are formulated at the end.

The material presented in this book has been used since 2006 in a one-semester advanced course for undergraduates in pure and applied mathematics, and in computer science at J. Kepler University in Linz, Austria.

I am deeply grateful to Vladimir Zakharov who encouraged my work on the discrete effects in wave turbulent systems. This work gave me the inspiration necessary to realize that nonlinear resonance analysis is a useful mathematical tool, as basic as linear Fourier analysis, but having as the application area weakly nonlinear partial differential equations.

Very useful remarks, improvements and comments were received from Adrian Constantin, Jim Cooper, Vladimir Gerdjikov, Roger Grimshaw, Diane Henderson, Alexey Kartashov, German Kolmakov, Peter Lynch, Victor L'vov, Guenther Mayrhofer, Yuri Manin, Sergey Nazarenko, Mikhail Sokolovskiy, Efim Pelinovsky, Itamar Procaccia, Clemens Raab, Oleksii Rudenko, Veronika Retchitskaja, Jan Sanders, Michael Shats, Alexey Slunyaev, Lennart Stenflo, Vasilij Sotke, Igor Shugan, Rudolf Treumann, Mark Vilensky, and Erik Wahlén. I cordially thank them all.

I am particularly indebted to Wolfgang Schreiner for developing a Web service [216] for nonlinear resonance computations and writing the Appendix "Software" where corresponding computer programs are presented and also the web-based service interface is described providing on-line access to these programs.

The extracts included at the opening of Chapters 1–7 are reproduced, with permission, from M. Bulgakov, *The Master and Margarita*, translated by Richard Pevear and Larissa Volokhonsky (London: Penguin Classics, 2007), pp. 19, 28, 159, 179, 198, and 545.

I owe very much to my son Peter who suffered a lot from the lack of my attention during the accomplishing of this text – suffered but never complained.

Finally, it was a pleasure and a privilege to work in close collaboration with Simon Capelin, Laura Clark, Megan Waddington, and Sehar Tahir at Cambridge University Press.

All shortcomings of this book are my responsibility, of course.

CAMBRIDGE

Cambridge University Press
978-0-521-76360-8 - Nonlinear Resonance Analysis: Theory, Computation, Applications
Elena Kartashova
Frontmatter
More information

Glossary

wavevector	$\mathbf{k} = (m, n)$, with m, n being integers as indexes of Fourier harmonics
dispersion function	$\omega = \omega(\mathbf{k}),\ \omega_j = \omega(\mathbf{k}_j)$
three-wave resonance conditions	$\omega_1 + \omega_2 = \omega_3,\ \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3,\ (*)$
four-wave resonance conditions	$\omega_1 + \omega_2 = \omega_3 + \omega_4,\ \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4,\ (**)$
exact resonance	solution of (*) or (**)
resonance solution set	all solutions of (*) or (**)
a triad	an exact solution of (*)
a quartet	an exact solution of (**)
resonance cluster, primary	a triad in three-wave system, a quartet in four-wave system
resonance cluster, generic	a set of primary clusters connected *via* common wavevector(s)
size of a cluster	number of connected primary clusters within generic cluster
geometrical structure (**GS**)	each **k** is shown as a node of integer lattice (m, n); nodes corresponding to one solution are connected by lines
topological structure	all topologically equivalent elements of **GS** are shown as one subgraph (resonance cluster) with number of

		appearances for each subgraph shown on the side
NR-diagram		Graphical representation of a cluster in **k**-space; allows us to reconstruct uniquely its dynamical system
"slow" modes' amplitudes		
	physical variables	$A_j, \ j = 1, 2, 3,$
	canonical variables	$B_j, \ j = 1, 2, 3,$
	amplitude-phase	$C_j \exp(i\theta_j), \ j = 1, 2, 3,$
dynamical system, primary		
	three-wave case	$i\,\dot{B}_1 = ZB_2^*B_3, \ i\,\dot{B}_2 = ZB_1^*B_3,$ $i\,\dot{B}_3 = -ZB_1B_2.$
	four-wave case	$i\,\dot{B}_1 = ZB_2^*B_3B_4, \ i\,\dot{B}_2 = ZB_1^*B_3B_4,$ $i\,\dot{B}_3 = -ZB_4^*B_1B_2, \ i\,\dot{B}_4 = -ZB_3^*B_1B_2.$
coupling coefficient		Z
dynamical phase		
	of a triad	$\varphi_{12\|3} = \theta_1 + \theta_2 - \theta_3$
	of a quartet	$\varphi_{12\|34} = \theta_1 + \theta_2 - \theta_3 - \theta_4$
dynamical system, generic		few connected primary systems, corresponds to generic resonance cluster
resonance types		scale-resonances (three- and four-wave systems), angle-resonances (four-wave systems)
modes within a triad		**A**-mode, has maximal frequency ω_3, **P**-modes, have frequencies ω_1 and ω_2
mode pairs within a quartet		one-pairs: modes from one side of (**), (ω_1,ω_2) and (ω_3,ω_4) two-pairs: modes from different sides of (**); (ω_1,ω_3), (ω_2,ω_4), (ω_1,ω_4), (ω_2,ω_3),
connection types		
	cluster of triads	**AA**-, **AP**- and **PP**-connections
	cluster of quartets	**V**-connection (*via* one mode), **E**-connection (*via* one-pair), **D**-connection (*via* two-pair)
cluster's reduction		diminishing of the size of generic cluster due to the criterion of decay instability
	three-wave cluster	**PP**-reduction
	four-wave cluster	**V**-reduction