Foundations of MIMO Communication

Understand the fundamentals of wireless and MIMO communication with this accessible and comprehensive text. Viewing the subject through an information theory lens, but also drawing on other perspectives, it provides a sound treatment of the key concepts underpinning contemporary wireless communication and MIMO, all the way to massive MIMO. Authoritative and insightful, it includes over 330 worked examples and 450 homework problems, with solutions and MATLAB® code and data available online. Altogether, this is an excellent resource for instructors and graduate students, and a great reference for researchers and practicing engineers.

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"*Foundations of MIMO Communication* by Heath and Lozano is a much needed tour-de-force on a critically important topic for wireless system design. The book provides an in-depth comprehensive treatment of the theoretical fundamentals in single-user, multiuser, and massive MIMO communications. It covers information-theoretic performance limits as well as estimation, channel modeling, matrix theory, and optimization techniques that are key to many aspects of MIMO system design. Hundreds of worked examples drive home the value of the theory in solving practical problems, and over one thousand important references are provided for further reading. This masterfully written book will serve as an invaluable textbook for students as well as a definitive reference for researchers and practicing engineers."

Andrea Goldsmith, *Stanford University*

"This masterpiece from two preeminent scholars and innovators was worth the wait: this book took over a decade to write. Heath and Lozano are renowned information and communication theorists who have made many pioneering contributions to the theory of MIMO. However, they are unusual in also having done much to bring MIMO into practice: Heath at a pioneering Silicon Valley startup and Lozano at Bell Labs. They hold dozens of fundamental patents on MIMO and their broad perspective gives this book a unique depth and relevance. The book is well-suited for teaching or self study, and stands apart from others in the beauty and completeness of the exposition."

Jeffrey Andrews, *The University of Texas at Austin*

"Remarkably, and almost three decades after its inception, MIMO technology is holding on to its status as one of the most impactful and versatile technologies, likely to underpin the design radio access networks for years to come still. With this highly complete textbook, Heath and Lozano are sharing their deep expertise and long-term experience in an area that has seen so many successes and revolutions. This material will be invaluable to teachers, students, and practitioners alike."

David Gesbert, *Eurecom*

"MIMO and Massive MIMO are key technologies for enhancing the capacity and coverage of 3G/4G/5G and Beyond 5G wireless systems across all frequency bands. Heath and Lozano provide a comprehensive treatment of this essential technology from both a theoretical and practical point of view. An excellent and must read for graduate students and researchers in the field of wireless communication."

Amitava Ghosh, *Nokia Bell Labs*

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To Pia and Rohan, for putting up with their busy Papa,
and to Nuria.

R. W. H.

For Ester, Carla and Bernat, beams of light in my life.
And for my parents and siblings.

A. L.

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Preface

The art of wireless communications is arguably one of the biggest technological revolutions in history, and a crowning achievement of modern engineering. Its impact on the functioning of contemporary society cannot be overstated. It seems utterly impossible to conceive today's world without the myriad devices that are wirelessly connected at all times.

In the decades since their inception, cellular systems have undergone five generational transitions. The first generation unfolded during the 1980s and offered only analog telephony. The second generation (2G), rolled out in the 1990s and whose dominant standard was the global system for mobile communications (GSM), saw the changeover to digital and the birth of text messaging. The third generation (3G), circa the 2000s, incorporated data and multimedia applications. Subsequently, the fourth generation (4G) brought about, besides faster bit rates and lower latencies, a complete adoption of packet switching as a platform for the mobile internet and the convergence of all standards worldwide into the long-term evolution (LTE) system. Honoring its name, LTE then evolved into the fifth generation (5G), augmented by another radio access interface termed new radio (NR) that allows operating on a much wider range of frequencies.

In the transitional period between 2G and 3G, as the soaring costs of spectrum collided with the pressures to increase bit rates so as to accommodate data and multimedia applications, the interest in radically improving the spectral and power efficiencies became acute. This propelled research initiatives that blossomed into major advances, chief among which stands multiple-input multiple-output (MIMO) communication, the subject of this book. In short, MIMO amounts to the transmission of concurrent signals from multiple antennas at one end of the link, with multiple antennas also at the receiving end.

Although MIMO is sometimes defined as the incorporation of the space domain to the communication process, this is not quite precise. The space domain has been at the crux of wireless systems since their onset, and in fact it is inherent to the concept of a cellular network: a region is tesselated into cells and the time/frequency signaling resources are reused repeatedly over such cells, i.e., they are spatially reused.

A more satisfying definition of MIMO might be the reusing of signaling resources, not over faraway cells, but rather across antennas—typically collocated—belonging to a common transmitter/receiver, such that new signaling dimensions are unlocked as antennas are added. The many advantages that this brings about are spelled out in detail throughout the text. By arranging these spatial dimensions, each seen as a scalar quantity, into a vector, it can be said that MIMO amounts to a vectorization of the transmission. This subsumes specific instances, such as *phased arrays* and antenna *diversity*, which predate the general formulation of MIMO.

Being in space rather than in time/frequency, the signaling dimensions harnessed with

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MIMO behave differently, and this book is about how to communicate over them. These new signaling dimensions multiply the signaling opportunities in time/frequency, hence it can be said that MIMO provides additional “bandwidth” without an increase in the actual electromagnetic bandwidth [1].

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A brief historical account

> We usually imagine that invention occurs in a flash, with a eureka moment that leads a lone inventor towards a startling epiphany. In truth, large leaps forward in technology rarely have a precise point of origin. At the start, forces that precede an invention merely begin to align, often imperceptibly, as a group of people and ideas converge, until over the course of months or years they gain clarity and momentum and the help of additional ideas and actors. Luck seems to matter, and so does timing, for it tends to be the case that the right answers, the right people, the right place—perhaps all three—require a serendipitous encounter with the right problem. And then, sometimes, a leap. Only in retrospect do such leaps look obvious.
>
> Jon Gertner
> *The idea factory: Bell Labs and the great age of American innovation*
> Penguin Press, 2012.

MIMO builds on a long pedigree of the application of antenna arrays to communication, starting with Marconi's transatlantic experiment in 1901 [2]. While a comprehensive formulation and a broad understanding did arguably not materialize until the 1990s and 2000s, many of the constituent pieces were present in seemingly unconnected developments well before that. Let us briefly recount these various strands and how they came together.

A first thread relates to phased arrays, a subject of interest since the 1950s [3]. By adjusting the signal's phase at each antenna, either before transmission or upon reception, the overall radiation pattern of an array can be shaped into a beam and pointed in a desired direction, say toward a dominant propagation path. If, besides the phase, the amplitude can also be adjusted, then we have an *adaptive array*. In the 1980s, adaptive arrays evolved into smart antennas, whose array patterns can further null-out interference [4]. All of these are effectively spatial filters that rely on signal coherence across the arrays, an aspect discussed at length throughout the book and that has implications for the physical structure of those arrays and for the radio propagation. Precisely, this coherence is associated with specular propagation and with cleanly defined directions of arrival and departure.

In a second thread, arrays had long been a source of diversity. As detailed later, when multiple propagation paths exist, a signal conveyed over a wireless channel exhibits severe fluctuations, a deleterious phenomenon that can be mitigated by procuring independently fluctuating copies of that signal. An array can serve this purpose, as in such propagation conditions the antennas become uncorrelated—an opposite situation to that of adaptive arrays. Receive diversity was studied as early as 1954 [5] and implemented in early cellular generations already. Being more involved, transmit diversity is more recent [6–10].

Interestingly, diversity was shown to be compatible with the nulling-out of interference, provided the channel response was known [11].

A third thread is constituted by a variety of works that considered channels coupling multiple inputs into multiple outputs. These ranged from abstract formulations [12–14] to analyses motivated by the specific problem of communicating over coupled telephone lines [15–18]. These important precursors were the seed of MIMO, and also of multiuser detection for code-division multiple access (CDMA) [19, 20], two developments with substantial common ground. In drastic contrast with adaptive arrays and diversity, which in essence involve a single signal, these developments entail the concurrent transmission of multiple signals.

Particularly prescient was the contribution of Noach Amitay and Jack Salz, who, as early as 1984, considered a link with two orthogonally polarized antennas at each end [21]. This was extended by Jack Winters in a piece that featured multiantenna transmitters and receivers with many of the ingredients of contemporary MIMO communication: concurrent signals were transmitted from antennas collocated on a device—in time this would be known as *spatial multiplexing*—and a receiver equipped with multiple antennas recovered each of those signals by nulling-out the interference from the rest [22]. In hindsight, both of these milestone papers deserved more credit than they were given, but at the time there was no demand for enhanced wireless performance; in fact, funding for such research was being curtailed.

It was not until the 1990s that the atmosphere was primed for these ideas to coalesce. Arguably, the main catalyst was the work of Gerhard Foschini and Michael Gans, who set out to design the perfect antenna from an information-theoretic standpoint. Starting with an array and no preset conditions on how to use it, they found that, if the antennas were uncorrelated, the optimum strategy was to have each one radiate an independent signal, i.e., spatial multiplexing. This strategy was radically novel in that it sought to exploit, rather than counter or avoid, the fluctuations caused by multipath propagation. Foschini went further and proposed an architecture to effect spatial multiplexing, the so-called *layered architecture*, which was remarkable in that it could be built with off-the-shelf encoders and decoders and did not require the transmitter to know the channel's response [23]. Under the leadership of Reinaldo Valenzuela, a prototype with 12 transmit and 16 receive antennas confirmed the practicality of this proposition [24] and dispelled concerns about the feasibility of nulling-out interference from collocated antennas, i.e., concerns that in practice these signals would drown each other and be unrecoverable. Additional results by Emre Telatar consolidated the initial theoretical underpinnings of MIMO [25]. It behoves us to mention that Amitay, Salz, Winters, Foschini, Gans, Valenzuela, and Telatar were all associated with Bell Laboratories, placing this organization in a distinguished place in the history of MIMO. Angel Lozano's views on MIMO were strongly influenced by his interaction with these pioneers.

Another noteworthy advance in the 1990s was from Paulraj and Kailath, at Stanford University, who applied spatial multiplexing to TV distribution from multiple towers [26]. That idea became a core part of Iospan Wireless, an early developer of MIMO technology for fixed wireless access. Robert Heath's perspectives on MIMO were shaped by his role as an early employee of Iospan Wireless.

Also at Stanford, in yet another development in the 1990s, Raleigh and Cioffi provided a generalization to multiantenna channels of a technique (expounded in Section 4.4) that does require the transmitter to know the channel response [27, 28].

Altogether, after a slow ripening of ideas over an extensive period, a critical mass was reached. Perhaps because of the impetus provided by the soaring costs of spectrum or perhaps because, simply, the time was right, a chain reaction was sparked and spread rapidly through academia and industry. From an academic thought experiment, MIMO grew into a foundation of wireless communication standards. In the span of a few years, it was adopted by the IEEE 802.16 fixed wireless access system and by cellular standardization bodies. In particular, the 3G Partnership Project (3GPP) embraced it in a limited fashion for 3G and then as an integral part of the designs beginning with 4G LTE. In subsequent revisions of LTE, the numbers of supported antennas progressively increased and, for 5G, so-called *massive MIMO* deployments are intrinsic. However, the first commercial application of MIMO was not in the cellular arena.

In 1997, the IEEE 802.11 working group completed its original wireless local-area network (WLAN) standard. Successive revisions incorporated new features, with the earliest version to include MIMO being IEEE 802.11n, certified in 2007 and supporting four antennas (although two or three were more common at that time). IEEE 802.11n was succeeded by IEEE 802.11ac, supporting up to eight antennas, and further by IEEE 802.11ad, 802.11ax, and 802.11ay.

Cohorts of researchers have played a part in the advancement of MIMO, and it is nontrivial to apportion credit among individuals and institutions. A reasonable criterion to assign credit for any invention is the influence that each contribution ends up having [29], and this is the criterion applied in our exposition.

About this book

A wealth of texts on MIMO are available, including books with varying flavors and perspectives [30–40], edited books [41–43], monographs [44–46], and tutorial papers [47–49]. In addition, treatments of MIMO are included in general wireless communication books [50, 51]. The present volume, which builds on this diverse literature, is the result of 20 years of research and teaching. It is intended as a full-dress textbook for instructors and students at the graduate level, and a reference tool for researchers and practicing engineers. With this audience in mind, its aim is to be both accessible and comprehensive, and the conjunction of these objectives is the reason for the considerable length. We hope that readers will appreciate the organization of the contents, as well as the complementary features:

- 160 illustrations.
- 19 topical discussions.
- 339 examples. Some of these are titled examples, which encapsulate results of particular interest, making it easier for them to be identified and located. The rest of the examples are in the format of a problem accompanied by the corresponding solution.
- 463 homework problems proposed at the ends of the chapters.
- A companion website with the solutions to all the problems, and additional material.

There are various theoretical lenses under which MIMO, and in fact digital communication at large, can be seen. Here, the problem of MIMO communication is viewed through the lens of information theory, appropriately complemented with signal processing, estimation theory, channel modeling, optimization, linear algebra, and random matrix theory, and with a touch of stochastic geometry. The choice of information theory is not capricious, but rather the appreciation that, besides being digital, modern communications—certainly the forms that MIMO is relevant to—are built from the ground up with coding, operating very close to the fundamental limits that information theory delineates. Moreover, information theory yields surprisingly many design insights and the opportunity for extensive and informative analysis, greatly facilitating the exposition. As eloquently argued by James Massey [52], “information theory is the proper scientific basis for communication.”

In opening his treatise on quantum theory, the Nobel laureate Steven Weinberg declared that “there are parts of this book that will bring tears to the eyes of the mathematically inclined reader.” While not expecting to bring tears to the eyes of our readers, we also do not intend to sacrifice clarity or understanding at the altar of rigor. Rather, we abide by the principle that rigor should be at the service of the problem at hand, rather than the resolution of the problem being slaved to the formalisms of rigor.

Organization

The book is organized into three parts, plus a set of appendices.

- Part I, labeled *wireless communication theory*, exposes the pillars on which the edifice rests, with Chapters 1–3 successively introducing the perspectives of information theory and estimation, signal processing, and channel modeling. Along the way, the radio channel is in turn interpreted as a random transformation, as a linear impulse response, and as a stochastic process; three interpretations, each one fitting the perspective of its chapter. These perspectives converge in Chapter 4, which deals with non-MIMO communication, crisply presenting all the concepts that are then to be generalized.
- Part II is devoted to single-user MIMO communication, meaning a single transmitter and a single receiver. In this clean setting, we elaborate on how to transmit and receive with multiple antennas, first without receiver restrictions in Chapter 5, and then with a linearity restriction in Chapter 6.
- Although, with orthogonal multiplexing, single-user conditions can be created within a network, the signaling dimensions created by MIMO are best exploited when the multiuser aspects are brought into the picture. Part III deals with these aspects, first by introducing them broadly in Chapter 7, and then by delving into how to transmit to and receive from a plurality of multiantenna users. Again, this is first covered without restrictions, in Chapter 8, and then with a linearity restriction in Chapter 9. Finally, Chapter 10 broadens the scope in two respects: an entire cellular network is considered, and MIMO becomes massive MIMO.
- The appendices provide a compact tour of various mathematical results that are invoked throughout the book, conveniently couched in our notation, with the objective of rendering the text as self-contained as possible.

Based on the foregoing structure, a variety of itineraries can be defined in support of graduate courses. Some potential ones are as follows.

- A course on the information-theoretic principles of wireless communication, leading up to MIMO, can rest on Chapter 1, Chapter 2 (Sections 2.1, 2.2, and 2.6), and Chapter 4, with the necessary channel modeling taken from Sections 3.1–3.4.
- A basic course on MIMO communication can rely on Chapters 1–6 (with the possible exclusion of Sections 2.4, 2.5, and 2.7).
- An advanced course on multiuser MIMO, which could be concatenated with the one above, can rest on Chapters 7–9, with extension to massive MIMO via Chapter 10.
- A course specifically geared toward massive MIMO can be designed with Chapters 6, 9, and 10, plus whatever material is needed from Chapters 1–4 and 7.

Requisites

The background assumed on the part of the readers corresponds to senior-level or first-year graduate-level courses on signals and systems, digital communication, linear algebra, and probability. A firm grasp of random variables is particularly desirable.

No expertise on cellular networks is presumed, beyond the basic notion that such networks are organized in cells, each featuring a base station that wirelessly communicates with the population of users within. The *forward link* or *downlink* embodies the communication from the base to its users, while the *reverse link* or *uplink* embodies the return communication from the users to the base. This naturally leads to the notion of *duplexing*, which refers to how these two directions of communication are arranged. If they are simultaneous over disjoint frequency bands, we speak of frequency-division duplexing (FDD), while, if they take place on alternating time intervals over a common frequency band, we speak of time-division duplexing (TDD). Full-duplexing, deemed unfeasible in the past, is now becoming possible thanks to advanced self-interference cancelation techniques [53–55], and thus it is also considered.

Among the numerous acronyms that—MIMO aside—sprinkle the text, two stand out and deserve introductory remarks.

- CSI, which stands for channel-state information, alludes to complete knowledge of the channel's response. Since a signature attribute of wireless channels is their variablity, acquiring and employing CSI is instrumental when communicating over such channels. In fact, the availability of CSI is one of the main axes of the exposition in the book.
- OFDM, which stands for orthogonal frequency-division multiplexing, is the signaling technique that has come to dominate communication, wireline and wireless [56, 57]. LTE, NR, and the mentioned WLAN standards all feature OFDM. Although the mainstream alternatives are entertained too, we acknowledge the dominance of OFDM and rely on it extensively for our formulations.

Notation

In a long text such as this one, notation can be a minefield. While striving for consistency and intuition, a modicum of flexibility becomes necessary at points. As part of the effort to convey meaning consistently, some points are worth noting.

- Whenever possible, variables are named to directly reminisce of the quantities they represent, e.g., SNR for the signal-to-noise ratio, MMSE for the minimum mean-square error, or N_{t} and N_{r} for the numbers of transmit and receive antennas.
- Bold symbols denote vectors and matrices, while nonbold symbols correspond to scalars.
- Capitalization distinguishes matrices from vectors, and large-scale from small-scale quantities. (The meaning of these scales is to become clear throughout the text.) Variables that are in general matrices retain their capitalization even in the special cases in which they may adopt a vector form.
- Frequency-domain quantities are represented with sans serif fonts, in contrast with time-domain quantities, which are denoted with serif fonts.
- Dummy variables and counters match, whenever possible, their respective quantities, e.g., we use $\mathbf{A}$ to denote a realization of the random matrix $\boldsymbol{A}$, and we use n to run a counter over N positions.

The common notational schemes and all the relevant symbols, excluding only variables appearing in intermediate derivations, are listed in the pages that follow. The notational

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schemes are, as much as possible, inspired by MATLAB®. For instance, $[\boldsymbol{A}]_{:,j}$ denotes the jth column of matrix $\boldsymbol{A}$ whereas $[\boldsymbol{A}]_{:,j:j'}$ denotes the submatrix containing columns j through j'.

A number of the variables are further indexed by a user indicator u whenever they are applied in multiuser contexts. For instance, in single-user settings, $\boldsymbol{H}$ is the time-domain normalized channel matrix; then, in multiuser settings, $\boldsymbol{H}_u$ indicates the normalized channel matrix for user u. If, besides multiple users, multiple cells are present, then the indexing is further augmented to identify the cells involved: $\boldsymbol{H}_{l;\ell,u}$ denotes the normalized channel matrix linking the base station at cell l with the uth user at cell ℓth.

Further scripting, not explicitly distinguished in the listings that follow, is applied to discriminate variations in the quantities. For instance, $\boldsymbol{W}$ is a generic linear receiver while $\boldsymbol{W}^{\text{MF}}$, $\boldsymbol{W}^{\text{ZF}}$, $\boldsymbol{W}^{\text{MMSE}}$ are specific types thereof, namely matched-filter, zero-forcing, and minimum mean-square error linear receivers.

Common notation

$\propto$	proportionality
$\approx$	approximation
$\simeq$	asymptotic equality
$\sim$	distribution
$\subseteq$	subset
$((\cdot))_K$	modulo-K
$\otimes$	Kronecker product
$\odot$	Hadamard (entry-wise) product
$*$	convolution

a, A	nonbold letters denote scalars
$\boldsymbol{a}$	bold lowercase letters denote column vectors
$\boldsymbol{A}$	bold uppercase letters denote matrices
$(\cdot)$	indexing for continuous signals
$a(t)$, $\boldsymbol{a}(t)$, $\boldsymbol{A}(t)$	time-domain continuous signals at time t
$\mathsf{a}(f)$, $\boldsymbol{\mathsf{a}}(f)$, $\boldsymbol{\mathsf{A}}(f)$	frequency-domain continuous signals at frequency f
$[\cdot]$	indexing for discrete signals
$a[n]$, $\boldsymbol{a}[n]$, $\boldsymbol{A}[n]$	time-domain discrete signals at time n
$\mathsf{a}[k]$, $\boldsymbol{\mathsf{a}}[k]$, $\boldsymbol{\mathsf{A}}[k]$	frequency-domain discrete signals at frequency k
$\{\cdot\}$	sequence

$[a]^+$	$= \max(0, a)$
$a\|_{\mathrm{dB}}$	$= 10 \log_{10} a$
$a\|_{3\,\mathrm{dB}}$	$= \log_2 a$
$\|a\|$	magnitude of a
$\\|\boldsymbol{a}\\|$	Euclidean norm of $\boldsymbol{a}$
$\\|\boldsymbol{A}\\|_{\mathrm{F}}$	Frobenius norm of $\boldsymbol{A}$
$\nabla_{\boldsymbol{x}}$	gradient with respect to $\boldsymbol{x}$

$\boldsymbol{A}^{\mathrm{T}}$	matrix transpose
$\boldsymbol{A}^*$	matrix conjugate transpose
$\boldsymbol{A}^{\mathrm{c}}$	matrix conjugate
$\boldsymbol{A}^{-1}$	matrix inverse
$\boldsymbol{A}^{\dagger}$	Moore–Penrose matrix pseudoinverse
$\boldsymbol{A}^{\star}$	value of $\boldsymbol{A}$ that solves an optimization problem
$\hat{\boldsymbol{A}}$	estimate of $\boldsymbol{A}$
$\tilde{\boldsymbol{A}}$	error in the estimation of $\boldsymbol{A}$

$[\boldsymbol{a}]_j$	jth entry of $\boldsymbol{a}$
$[\boldsymbol{A}]_{i,j}$	(i, j)th entry of $\boldsymbol{A}$
$[\boldsymbol{A}]_{:,j}$	jth column of $\boldsymbol{A}$
$[\boldsymbol{A}]_{:,j:j'}$	submatrix containing columns j through j' of $\boldsymbol{A}$

$[\boldsymbol{A}]_{:,-j}$	submatrix obtained by removing column j from $\boldsymbol{A}$
$\bar{\boldsymbol{A}}_{N,M}$	$N \times M$ matrix containing $\boldsymbol{A}$ at various times, frequencies or antennas
$\dot{f}(\cdot)$	first derivative of $f(\cdot)$
$\ddot{f}(\cdot)$	second derivative of $f(\cdot)$

Symbols

$\mathbf{0}_N$	all-zero matrix (the dimension N may be omitted)
$\mathbf{1}_N$	all-one matrix (the dimension N may be omitted)
$1\{\cdot\}$	indicator function
$A(\cdot)$	array factor
α	common (unprecoded) pilot symbol overhead
α_{d}	dedicated (precoded) pilot symbol overhead
α_{fb}	feedback overhead
$\boldsymbol{a}_{\mathrm{r}}(\theta)$	receive array steering vector for an angle θ
$\boldsymbol{a}_{\mathrm{t}}(\theta)$	transmit array steering vector for an angle θ
$\arg\max$	value that maximizes a function
$\arg\min$	value that minimizes a function
$\mathsf{b}[\mathsf{n}]$	nth bit of a message
$b_\ell[n]$	ℓth bit in the nth symbol of a codeword (n may be omitted)
b	excess bandwidth due to pulse shaping
B	bandwidth
B_{c}	channel coherence bandwidth
β	ratio of transmit-to-receive antenna numbers
$\upbeta$	fudge factor in the CESM, MIESM, and EESM methods
$\mathcal{B}_1^m$	subset of coded bits mapped to the mth constellation point that equal 1
c	speed of light
$c(\tau)$	continuous-time unnormalized pseudo-baseband impulse response
$c_{\mathrm{p}}(\tau)$	continuous-time unnormalized passband impulse response
$c_{\mathrm{b}}(\tau)$	continuous-time unnormalized baseband impulse response
$c_{\mathrm{b}}[\ell]$	discrete-time unnormalized baseband impulse response
$\mathsf{c}(f)$	frequency-domain unnormalized transfer function
$C(\mathsf{SNR}, \boldsymbol{H})$	capacity of a channel $\boldsymbol{H}$ as a function of SNR
$C(\mathsf{SNR})$	ergodic capacity or sum-capacity as a function of SNR
$C_\epsilon(\mathsf{SNR})$	outage capacity at outage level ϵ as a function of SNR
$\mathsf{C}(\frac{E_{\mathrm{b}}}{N_0})$	ergodic capacity as a function of $\frac{E_{\mathrm{b}}}{N_0}$
$\mathcal{C}$	set of cells reusing the same pilots as a cell of interest
$\boldsymbol{C}$	unnormalized channel matrix
χ	shadow fading
χ^2_{2M}	chi-square distribution with $2M$ degrees of freedom
Ξ	cross-polar discrimination
$\mathsf{CM}(\cdot)$	cubic metric

d	diversity order
d_{min}	minimum distance among constellation points
d_{r}	spacing between receive antennas in a ULA array
$d_{\mathrm{r},i,i'}$	spacing between receive antennas i and i'
d_{t}	spacing between transmit antennas in a ULA array
$d_{\mathrm{t},j,j'}$	spacing between transmit antennas j and j'
$\mathsf{d}(\boldsymbol{x},\boldsymbol{y})$	subspace distance between $\boldsymbol{x}$ and $\boldsymbol{y}$
$\mathsf{d}_{\mathsf{min}}(\cdot)$	minimum subspace distance of a codebook
D	distance between transmitter and receiver
D_{c}	coherence distance
D_{ref}	pathloss reference distance
D	distortion in the average SNR
$\mathcal{D}$	set of symbols bearing payload data
$\mathcal{D}(\boldsymbol{x}\|\boldsymbol{y})$	relative entropy (information divergence) between $\boldsymbol{x}$ and $\boldsymbol{y}$
$\delta(\cdot)$	delta function
Δ	equalizer delay
Δ_{D}	distance shift
Δ_{t}	time shift
$\det(\boldsymbol{A})$	determinant of $\boldsymbol{A}$
$\mathrm{diag}(\cdot)$	diagonal matrix
$\mathrm{DFT}_{N}\{\cdot\}$	N-point DFT

E	energy per symbol transmitted to a user
E_{s}	total energy per symbol
$E_{\mathrm{s}}^{\mathsf{r}}$	reverse-link total energy per symbol
E_{b}	energy per bit
$\mathbb{E}[\cdot]$	expectation
$\boldsymbol{E}$	MMSE matrix
$\mathcal{E}_{n}(\cdot)$	exponential integral of order n

F	frequency share in FDMA
$\mathcal{F}$	digital-feedback precoding codebook
$\boldsymbol{F}$	precoder
$f_{\boldsymbol{A}}(\cdot)$	PDF of $\boldsymbol{A}$
$F_{\boldsymbol{A}}(\cdot)$	CDF of $\boldsymbol{A}$
f_{c}	carrier frequency

$g(\tau)$	delay-domain pulse shape
$g_{\mathsf{rx}}(\tau)$	delay-domain receive pulse shaping filter
$g_{\mathsf{tx}}(\tau)$	delay-domain transmit pulse shaping filter
$\mathsf{g}(f)$	frequency-domain pulse shape
$\mathsf{g}_{\mathsf{rx}}(f)$	frequency-domain receive pulse shaping filter
$\mathsf{g}_{\mathsf{tx}}(f)$	frequency-domain transmit pulse shaping filter
G	large-scale channel gain

G_{r}	receive antenna gain
G_{t}	transmit antenna gain
$\mathrm{G}(N, M)$	Grassmannian manifold of M-dimensional subspaces on the Nth-dimensional space
$\Gamma(\cdot)$	gamma function
$\Gamma(\cdot, \cdot)$	upper incomplete gamma function
$\gamma(\cdot, \cdot)$	lower incomplete gamma function
γ_{EM}	Euler–Mascheroni constant
$h(t, \tau)$	continuous-time normalized impulse response
$\hbar(t, f)$	time-frequency normalized transfer function
$\hbar(\nu, \tau)$	Doppler-delay normalized spreading function
h_{b}	base station height
h_{m}	mobile user height
$\mathfrak{h}(\boldsymbol{x})$	differential entropy of $\boldsymbol{x}$
$\mathcal{H}(\boldsymbol{x})$	entropy of $\boldsymbol{x}$
$\boldsymbol{H}[n]$	discrete-time normalized channel (n may be dropped)
$\boldsymbol{H}_{\mathsf{ind}}$	normalized channel with IND entries
$\boldsymbol{H}_{\mathrm{w}}$	normalized channel with IID complex Gaussian entries
$\boldsymbol{H}_{\mathsf{LOS}}$	normalized LOS channel component
$\boldsymbol{H}_{\mathsf{vir}}$	virtual channel
$\boldsymbol{\mathsf{H}}[k]$	discrete-frequency normalized channel
$\upeta$	reciprocal of the water level in waterfilling
η	pathloss exponent
$i(\boldsymbol{x}; \boldsymbol{x})$	information density between $\boldsymbol{x}$ and $\boldsymbol{y}$
$I(\boldsymbol{x}; \boldsymbol{y})$	mutual information between $\boldsymbol{x}$ and $\boldsymbol{y}$
$I_n(\cdot)$	modified Bessel function of the first kind and order n
$\Im\{\cdot\}$	imaginary part
$\mathcal{I}(\mathsf{SNR})$	Gaussian-noise mutual information as a function of SNR
$\boldsymbol{I}_N$	identity matrix (the dimension N may be omitted)
$\mathrm{IDFT}_N\{\cdot\}$	N-point inverse DFT
j	imaginary unit
$J_n(\cdot)$	Bessel function of the first kind and order n
k	Boltzmann's constant
$\kappa(\cdot)$	kurtosis
K	number of OFDM subcarriers
$K_n(\cdot)$	modified Bessel function of the second kind
K_{ref}	pathloss intercept
K	Rice factor
L	channel order

L_{c}	length of the cyclic prefix
L_{cluster}	pilot reuse factor
L_{eq}	equalizer order
L_{netw}	number of cells in the network
L_{p}	pathloss
$\mathrm{L}_{\mathrm{A}}(b)$	a-priority L-value for bit b
$\mathrm{L}_{\mathrm{D}}(b)$	a-posteriority L-value for bit b
$\mathrm{L}_{\mathrm{E}}(b)$	log-likelihood ratio for bit b
$\mathcal{L}_{\infty}$	high-SNR power offset
$\mathsf{\lambda}$	Lagrange multiplier
λ_{c}	carrier wavelength
$\lambda_k(\boldsymbol{A})$	kth eigenvalue of $\boldsymbol{A}$ in decreasing order
$\lambda_{\mathsf{max}}(\boldsymbol{A})$	maximum eigenvalue of $\boldsymbol{A}$
$\lambda_{\mathsf{min}}(\boldsymbol{A})$	minimum eigenvalue of $\boldsymbol{A}$
$\boldsymbol{\Lambda}_{\boldsymbol{A}}$	square diagonal matrix containing the eigenvalues of $\boldsymbol{A}$
m	Nakagami fading parameter
M	constellation cardinality
$\mu_{\boldsymbol{A}}$	mean of $\boldsymbol{A}$
$\max(\cdot)$	maximum of various quantities
$\min(\cdot)$	minimum of various quantities
MMSE	MMSE
$\overline{\mathsf{MMSE}}$	local-average MMSE
N	number of symbols per codeword
N_0	noise spectral density
N_{a}	number of antennas
N_{bits}	number of bits per message
N_{c}	fading coherence
N_{f}	number of entries in a digital-feedback codebook
N_{max}	$= \max(N_{\mathrm{t}}, N_{\mathrm{r}})$
N_{min}	$= \min(N_{\mathrm{t}}, N_{\mathrm{r}})$
N_{p}	number of pilot symbols
N_{r}	number of receive antennas
N_{s}	number of signal streams
N_{t}	number of transmit antennas
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{R})$	real Gaussian with mean μ and covariance matrix $\boldsymbol{R}$
$\mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{R})$	complex Gaussian with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{R}$
$\boldsymbol{\Omega}$	matrix of variances
$\mathsf{P}(f)$	frequency-domain power allocation
$\mathbb{P}[\cdot]$	probability
$\boldsymbol{P}[n]$	time-domain power allocation (n may be omitted)

P_r	receive power
P_t	transmit power
$\mathcal{P}_\mathrm{r}(\theta)$	receive PAS as a function of θ
$\mathcal{P}_\mathrm{t}(\theta)$	transmit PAS as function of θ
$p_a(\cdot)$	PMF of a
p_e	error probability
p_out	outage probability
$\mathsf{PAPR}(\cdot)$	peak-to-average power ratio
ϕ	phase
$\psi(\cdot)$	digamma function

q	user weight
$Q(\cdot)$	Q-function

r	multiplexing gain
r	code rate
R	bit rate
$R_a(\cdot)$	autocorrelation of a
$\Re\{\cdot\}$	real part
$\mathcal{R}_m$	decision region for codeword m
$\boldsymbol{R}_\mathrm{r}$	receive correlation matrix
$\boldsymbol{R}_\mathrm{t}$	transmit correlation matrix
$\boldsymbol{R}_{\boldsymbol{x}}$	covariance/correlation matrix of $\boldsymbol{x}$
$\boldsymbol{R}_{\boldsymbol{x}\boldsymbol{y}}$	cross-covariance/cross-correlation matrix of $\boldsymbol{x}$ and $\boldsymbol{y}$
$\mathbf{R}_{\boldsymbol{A}}$	correlation tensor for $\boldsymbol{A}$
$\mathsf{\rho}$	forward–reverse power ratio
ρ_d	payload data power boosting coefficient
ρ_p	pilot power boosting coefficient

$\boldsymbol{s}[n]$	time-domain codeword symbol (n may be omitted)
$\mathbf{s}[k]$	frequency-domain codeword symbol
S_0	low-SNR slope
S_∞	number of spatial DOF
$S_a(\cdot)$	power spectrum of a
$S_h(\nu)$	Doppler spectrum
$S_\mathsf{h}(\tau)$	power delay profile
$S_\hbar(\nu, \tau)$	scattering function
$\mathcal{S}$	set of cells with pilots staggered relative to a cell of interest
$\mathcal{S}_0^\ell$	subset of constellation points whose ℓth bit is 0
$\mathcal{S}_1^\ell$	subset of constellation points whose ℓth bit is 1
SINR	local-average receive SINR
SIR	local-average receive SIR
SNR	local-average receive SNR
SNR^r	reverse-link local-average receive SNR

$\mathsf{SNR}_{\mathsf{eff}}$	effective SNR
$\mathsf{SNR}_{\mathsf{eq}}$	equivalent SNR
$\mathsf{SNR}_{\mathsf{xESM}}$	equivalent SNR in the xESM method
sir	output SIR of a signal stream
sinr	output SINR of a signal stream
snr	output SNR of a signal stream
$\overline{\mathsf{sir}}$	hardening-based output SIR of a signal stream
$\overline{\mathsf{sinr}}$	hardening-based output SINR of a signal stream
$\mathrm{sign}(\cdot)$	sign function
$\mathrm{sinc}(x)$	$= \frac{\sin(\pi x)}{\pi x}$
σ_a^2	variance of a
σ_{dB}	standard deviation of the shadow fading (in dB)
σ_θ	angle spread
$\sigma_k(\boldsymbol{A})$	kth singular value of $\boldsymbol{A}$ in decreasing order
$\boldsymbol{\Sigma}_{\boldsymbol{A}}$	rectangular diagonal matrix with the singular values of $\boldsymbol{A}$

t	time
τ	delay
T	symbol period in single-carrier transmission
T_{c}	coherence time
T_{d}	delay spread
T_{eff}	effective temperature
T_{OFDM}	OFDM symbol period
T	time share in TDMA
$\mathrm{tr}(\boldsymbol{A})$	trace of $\boldsymbol{A}$
θ	angle

U	number of active users
U_{tot}	total number of users
$\mathcal{U}$	subset of users
$\boldsymbol{U}$	unitary matrix
$\boldsymbol{U}_{\boldsymbol{F}}$	matrix containing the left singular vectors of $\boldsymbol{F}$

v	velocity
V	variance of the information density
$\boldsymbol{V}$	unitary matrix
$\boldsymbol{V}_{\boldsymbol{F}}$	matrix containing the right singular vectors of $\boldsymbol{F}$
$\boldsymbol{v}[n]$	discrete-time baseband noise vector (n may be omitted)
$\boldsymbol{v}_{\mathrm{p}}(t)$	continuous-time passband noise vector
$\boldsymbol{\mathsf{v}}[k]$	discrete-frequency baseband noise vector
ν_{M}	maximum Doppler frequency
ϑ	fractional power control exponent
$\mathrm{var}[\cdot]$	variance

$\mathrm{vec}(\boldsymbol{A})$	vector created by stacking the columns of $\boldsymbol{A}$
$\boldsymbol{W}$	linear receiver
$\mathcal{W}_N(M, \boldsymbol{R})$	N-dimensional central Wishart with M degrees of freedom and covariance $\boldsymbol{R}$
x_{i}	in-phase transmit signal component
x_{p}	passband transmit signal
x_{q}	quadrature transmit signal component
$\boldsymbol{x}[n]$	discrete-time transmit signal (n may be omitted)
$\boldsymbol{\mathsf{x}}[k]$	discrete-frequency transmit signal
y_{i}	in-phase received signal component
y_{p}	passband received signal
y_{q}	quadrature received signal component
$\boldsymbol{y}[n]$	discrete-time received signal (n may be omitted)
$\boldsymbol{\mathsf{y}}[k]$	discrete-frequency received signal (k may be omitted)

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More Information

Acronyms

2G	second generation
3G	third generation
3GPP	third-generation Partnership Project
4G	fourth generation
5G	fifth generation
APP	a-posteriori probability
ARQ	automatic repeat request
a.s.	almost surely
AWGN	additive white Gaussian noise
BC	broadcast channel
BCJR	Bahl–Cocke–Jelinek–Raviv
BICM	bit-interleaved coded modulation
BPSK	binary phase shift keying
CDF	cumulative distribution function
CDMA	code-division multiple access
CESM	capacity-effective SNR mapping
CM	cubic metric
COST	European Cooperation in Science and Technology
CSI	channel-state information
CSIR	channel-state information at the receiver
CSIT	channel-state information at the transmitter
DFT	discrete Fourier transform
DMT	diversity–multiplexing tradeoff
DOF	degrees of freedom
DPC	dirty-paper coding
DSL	digital subscriber line
EESM	exponential-effective SNR mapping
ESPAR	electronically steerable parasitic array radiators
FDD	frequency-division duplexing
FDMA	frequency-division multiple access
FFT	fast Fourier transform
FIR	finite impulse response
GSM	global system for mobile communications
H-ARQ	hybrid-automatic repeat request

IA	interference alignment
IEEE	Institute of Electrical and Electronics Engineers
IID	independent identically distributed
IIR	infinite impulse response
IND	independent nonidentically distributed
INR	interference-to-noise ratio
ISI	intersymbol interference
ITU	International Telecommunications Union
JSDM	joint spatial division and multiplexing
LDPC	low-density parity check
LGB	Linde–Buzo–Gray
LMMSE	linear minimum mean-square error
LOS	line-of-sight
LTE	long-term evolution
MAC	multiple-access channel
MAP	maximum a-posteriori
MCS	modulation and coding scheme
MIESM	mutual-information-effective SNR mapping
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
ML	maximum likelihood
MMSE	minimum mean-square error
MU-MIMO	multiuser MIMO
MU-MISO	multiuser MISO
MU-SIMO	multiuser SIMO
MU-SISO	multiuser SISO
NLOS	non-line-of-sight
NOMA	non-orthogonal multiple access
NR	new radio
OFDM	orthogonal frequency-division multiplexing
OFDMA	orthogonal frequency-division multiple access
PAM	pulse-amplitude modulation
PAPR	peak-to-average power ratio
PARC	per-antenna rate control
PAS	power angle spectrum
PDF	probability density function
PDP	power delay profile

PHY	PHYsical layer
PMF	probability mass function
PPP	Poisson point process
PSK	phase shift keying
QAM	quadrature amplitude modulation
QPSK	quadrature phase shift keying
RMS	root mean-square
SC-FDE	single-carrier frequency-domain equalization
SCM	spatial channel model
SDMA	space-division multiple access
SIC	successive interference cancelation
SIMO	single-input multiple-output
SINR	signal-to-interference-plus-noise ratio
SIR	signal-to-interference ratio
SISO	single-input single-output
SLNR	signal-to-leakage-plus-noise ratio
SNR	signal-to-noise ratio
SUI	Stanford University Interim
SU-MIMO	single-user MIMO
SU-SISO	single-user SISO
SVD	singular value decomposition
TCM	trellis-coded modulation
TDD	time-division duplexing
TDMA	time-division multiple access
UCA	uniform circular array
ULA	uniform linear array
WLAN	wireless local area network
WSSUS	wide-sense stationary uncorrelated scattering
ZF	zero-forcing